Abstracts for Invited Speakers

What follows is a list of abstracts for the invited speakers in the order that they will be presented. These can also be found individually via the conference website's participant list.

Extending Fisher's inequality to coverings

Daniel Horsley

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A (v, k, λ)-*design* is a collection of k-element subsets, called *blocks*, of a v-set of *points* such that each pair of points occurs in exactly λ blocks. *Fisher's inequality* is a classical result that states that every nontrivial (v, k, λ)-design has at least v blocks (equivalently, has $v \geq \frac{k(k-1)}{\lambda}+1$). An elegant proof of Fisher's inequality, due to Bose, centres on the observation that if X is the incidence matrix of a nontrivial design, then XX^T is nonsingular. This talk is about extending this proof method to obtain results on coverings.

A (v, k, λ)-*covering* is a collection of k-element blocks of a v-set of points such that each pair of points occurs in at least λ blocks. Bose's proof method can be extended to improve the classical bounds on the number of blocks in a (v, k, λ)-covering when $v < \frac{k(k-1)}{\lambda} + 1$. Specifically, this is accomplished via bounding the rank of XX^T, where X is the incidence matrix of a (v, k, λ)-covering, using arguments involving diagonally dominant matrices and m-independent sets in multigraphs.

A Tale of Universal Cycles in Two REU-Environments

Anant Godbole

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I regularly supervise undergraduate research projects as part of my tenured position at ETSU, where we *require* undergraduate research of all our majors via the course MATH 4010. At the same time I have run an NSF-sponsored REU site during most years since 1991. This talk will outline the symbiotic nature of these undertakings. I will provide concrete examples of how great NSF-REU work led to important work by an ETSU undergraduate and, conversely, how a project started in MATH 4010 led to important REU work in the summer. In both cases, the students in question were working on *Universal Cycles*, introduced into the literature by Chung, Diaconis, and Graham.

Edge colouring multigraphs

Penny Haxell

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While edge colouring in graphs is well understood (by Vizing's classical theorem), the chromatic index $\chi'(G)$ of a multigraph G can fall anywhere in the range $[\Delta(G), 3\Delta(G)/2]$ (where $\Delta(G)$ denotes the maximum degree of G), or in terms of the maximum edge multiplicity $\mu(G)$ the range $[\Delta(G), \Delta(G)+\mu(G)]$. We discuss results showing that if $\chi'(G)$ is significantly larger than $\Delta(G)$, then G contains a small subgraph that is very dense. This idea is at the core of the famous open problem of Goldberg and (independently) Seymour, which seeks to identify the properties of G that influence $\chi'(G)$.

3-Flows with Large Support

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Tutte's 3-Flow Conjecture says that every 4-edge-connected graph should have a nowhere-zero 3-flow. The 4-edge-connectivity assumption cannot be weakened—K_4 is an example of a 3-edge-connected graph that does not have a nowhere-zero 3-flow. However, K_4 is minimal in the sense that K_4^- has a nowhere-zero 3-flow. Since K_4 has 6 edges in total, this means that we are able to give K_4 a 3-flow in which 5/6 of the edges are nonzero. With DeVos, Pivotto, Rollova, and Samal, we can show that this is the worst case in general—that is, if G is any 3-edge-connected graph, then G has a 3-flow with support size at least $\frac{5}{6}|E(G)|$. As a corollary, this implies that every planar graph has an assignment of three colours to its vertices so that at most a sixth of its edges join vertices of the same colour.

Hamilton decompositions

Barbara Maenhaut

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The problem of partitioning the edge set of a graph into Hamilton cycles is a classical problem, with results dating back to the work of Walecki in the 1890s. Over the last 50 years, much work has been done on several open conjectures and problems that deal with Hamilton decompositions of certain families of graphs. I will survey some of this work and discuss a few recent developments, especially on Hamilton decompositions of vertex-transitive graphs, line graphs, and infinite graphs.

Abstracts for Contributed Talks

What follows is a list of abstracts for the contributed talks organized by lead presenter's last name (as listed in the schedule). These can also be found individually via the conference website's participant list.

Combinatorial-based Pairwise Event Sequence Generation for Automated GUI Testing of Android Apps

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Mobile apps are Event-Driven Systems (EDS) that often rely on Graphical User Interfaces (GUI) as the primary means of interaction with end-users. These systems are tested using event sequences that exercise the system's functionality while covering as much of its source code as possible. In this work, we develop a greedy pairwise approach to automate event sequence generation for Android app testing. The proposed approach dynamically traverses an app by identifying and executing GUI events that maximize the coverage of pairwise event combinations. We implement two variants of a greedy pairwise algorithm. The first considers the order of the event pairs while the second one does not. We conduct experiments on five Android apps by comparing the proposed approach to random event sequence generation. The results show that given the same parameters, test suites generated using a greedy pairwise approach tend to achieve higher block coverage than test suites with randomly generated event sequences. The results also show that pairwise test suites tend to find a higher number of faults than randomly generated test suites. However, the results show that there was no significant difference between pairwise test suites generated using ordered pairs and pairwise test suites generated using unordered pairs in terms of block coverage and number of faults found.

On Enumeration of Paths in Catalan–Schröder Lattices*

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We address the problem of enumerating paths in square lattices, where allowed steps include $(1,0)$ and $(0,1)$ everywhere, and $(1,1)$ above the diagonal $y = x$. We consider two such lattices differing in whether the $(1,1)$ steps are allowed along the diagonal itself. Our analysis leads to explicit generating functions and an efficient way to compute terms of many sequences in the Online Encyclopedia of Integer Sequences, proposed by Clark Kimberling over a decade and a half ago.

*The work is supported by the National Science Foundation under grant No. IIS-1462107.

Avoiding subsystems in cycle systems

John Asplund[1,*], Michael Schroeder[2], Venkata Dinavahi[3]

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A k-cycle decomposition of G is a partition of the edge set of G such that each element of the partition induces a k-cycle. If $G = K_n$ then it is called a k-cycle system of order n. The necessary and sufficient conditions for the existence of a k-cycle system of order n have already been determined. We aim to take this one step further. We will show how minute changes to systems can impact the structure of the k-cycle system in interesting ways. In particular, this talk will show there exists a k-cycle system $\mathcal{P}$ of order n such that *no* subset of $\mathcal{P}$ forms a k-cycle system of order t where $2 < t < n$ and both n and k are odd; if we can show this, we say that $\mathcal{P}$ contains no subsystems.

The Symmetric Arctic Rank of Boolean Matrices and the Clique Covering Content of Graphs

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Let G be a simple undirected graph on n vertices and let $A = A(G)$ be the (Boolean) $(0,1)$ adjacency matrix of G. Let $\rho_{sa}(A)$ be the minimum $|B|$ over all possible factorizations of A of the form BB^t for some (Boolean) $n \times k$ $(0,1)$ matrix B and some k where $|B|$ denotes the number of nonzero entries in B. This function, $\rho_{sa}(A)$, is called the *symmetric arctic rank of* A. Let $\mathcal{C}(G)$ denote the set of all clique covers of G, and let $\mathbb{C}(G)$ be the minimum over $\mathcal{C}(G)$ of sum of the orders of the cliques composing the clique cover. This function, $\mathbb{C}(G)$, is called the *clique cover content of* G. It is shown that $\mathbb{C}(G) = \rho_{sa}(A(G))$ and some properties and problems are presented.

On Path-Hamiltonian Tournaments

Zhenming Bi

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A Hamiltonian graph G is *j-path Hamiltonian* if every path of order j can be extended to a Hamiltonian cycle of G. The *Hamiltonian extension number* $he(G)$ of G is the largest integer k such that G is j-path Hamiltonian for every integer j with $1 \leq j \leq k$. Hamiltonian extension in graphs has been studied. Here, we study corresponding concepts for Hamiltonian tournaments. Results and open questions are presented in this area of research. This is joint work with F. Fujie and P. Zhang.

Re-Visiting Rainbow Connection in Graphs

Alexis Byers

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A rainbow coloring of a connected graph G is an edge coloring of G, where adjacent edges may be colored the same, with the property that for every two vertices u and v of G, there exists a $u-v$ rainbow path (no two edges of the path are colored the same). The minimum number of colors in a rainbow coloring of G is the rainbow connection number of G. This topic has been studied by many. We revisit this concept from a different point of view. Recent results and problems in this area of research are presented. This is joint work with Z. Bi, G. Chartrand, and P. Zhang.

Perfect matchings in bipartite graphs, Mersenne primes and 2-rooted primes

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We give graph theoretic characterizations of two important classes of prime numbers: Mersenne primes and primes p for which 2 generates the multiplicative group of the field of p elements. Our characterizations of these prime numbers are based on perfect matchings in circulant bipartite graphs on $2p$ vertices. [arXiv:1404.4096]

On Balanced Factorial Designs of Resolution Ten and Balanced Arrays of Strength Nine

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Balanced arrays (B-arrays) are generalizations of orthogonal arrays (O-arrays) and have been extensively used in the construction of balanced factorial designs. An array T with two levels (say, 0 and 1), m constraints (factors), and N treatment-combinations (runs) is merely a matrix T of size $(m \times N)$ with two elements 0 and 1. T is said to be of strength t $(\leq m)$ if, in each t-rowed submatrix T^* of T, the following condition is satisfied: every vector $\underline{\alpha}$ of T^* of weight i $(0 \leq i \leq t)$ (the weight of a vector $\underline{\alpha}$ is defined to be the number of 1's in it) appears with the same frequency μ_i (say, $0 \leq i \leq t$). The set $\{m; \mu_0, \mu_1, \ldots, \mu_t\}$ is called the set of parameters for the array T. If $\mu_i = \mu$, then T is called an O-array with parameter μ. It is quite obvious that N is known if the μ_i's are given. It is well known that B-arrays of strength t, under certain conditions, give rise to balanced factorial designs of resolution $(t+1)$ in which each of the m factors is at two levels, denoted in design theory by 2^m designs. In this paper, we derive some existence conditions for B-arrays with $t = 9$. We also use these conditions for obtaining $\max(m)$ for a given set $(\mu_0, \mu_1, \ldots, \mu_9)$.

Dominating Parameters of Certain Graphs

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In this talk we will discuss about various number of dominating parameters including the dominating number, independent dominating number, clique dominating number, connected dominating number, strong dominating number and weak dominating number of total graph of $\mathbb{Z}_n \times \mathbb{Z}_m$.

C_z frames of $M(b, n)$, where z is even

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Let $M(b, n)$ be the complete multipartite graph with b parts $B_0, \ldots, B_{b-1}$ of size n. A z-cycle system of $M(b, n)$ is said to be a C_z-frame if the z-cycles can be partitioned into sets $S_1, \ldots, S_k$ such that for $1 \leq j \leq k$, where S_j induces a 2-factor of $M(b, n) - B_i$ for some $i \in Z_b$. The existence of a C_z-frame of $M(b, n)$ has been settled when $z \in \{3, 4, 5, 6\}$. Here, we consider C_z-frames when $z \geq 8$ is even.

Tridiagonal and Hessenberg Matrices Representing Recursive Number Sequences

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We present a general method for constructing sequences of certain tridiagonal matrices from a second-order recursive number sequence with initial condition $a_0 = 0$ and $a_1 = 1$, so that the permanents of the matrices give values of the sequence elements. Then the result is extended to the case of the second-order recursive number sequences with arbitrary initials by using a type of tridiagonal matrices. Finally, the permanents of certain upper-Hessenberg matrices are used to represent some n^{th}-order recurrence number sequences.

Square-free Rank of Integers

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For any positive integer n, there are positive integers a, b such that a^2 is the largest square divisor of n, and $n = a^2b$: the *square-free rank* of n is the number of prime divisors of b. For instance, $242 = 11^2 \cdot 2$ and $243 = 9^2 \cdot 3$ lie in a run of 5 consecutive integers all of square-free rank 1. Below 10^{10}, the longest run of integers with constant square-free rank has 20 members, all of square-free rank 3.

For each $k \geq 1$, we give an explicit upper bound on the size of any run of integers of square-free rank k, and show (at least for $k \leq 7$) there are infinitely many pairs of consecutive integers of square-free rank k. For distinct primes p, q there is a smallest positive integer $d(p,q)$ such that $0 < a^2p - b^2q = d(p,q) \leq p$ holds for infinitely many integers a, b.

We show the sequence $(3n^2 \mid n \in \mathbb{N})$ has no member in the interval $[2a^2, 2(a+1)^2]$ with $a \in \mathbb{N}$ if and only if a is in the Beatty sequence $\left(\lfloor (3+\sqrt{6})n \rfloor \mid n \in \mathbb{N}\right)$. Finally, we study sequences $(a_1, a_2, \ldots, a_k)$ for square-free rank 1 integers $2a_1^2 > 3a_2^2 > \cdots > p_k a_k^2$ with $p_i(a_i+1)^2 > 2a_1^2$ for $1 < i \leq k$, and the corresponding sequences in which all the inequalities are reversed and $p_i(a_i-1)^2 < 2a_1^2$ for $1 < i \leq k$.

Decompositions of Complete Digraphs into Small Tripartite Digraphs

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The paw graph consists of a triangle with a pendant edge attached to one of the three vertices. We obtain a multigraph by adding exactly one repeated edge to the paw. Now, let D be a directed graph obtained by orientating the edges of that multigraph. For 12 of the 18 possibilities for D, we establish necessary and sufficient conditions on n for the existence of a (K_n^*, D)-design. Partial results are given for the remaining 6 possibilities for D. This research is supported by grant number A1359300 from the Division of Mathematical Sciences at the National Science Foundation.

Incomplete tournaments with handicap two

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A *d-handicap distance antimagic labeling* of a graph G with vertex set $V = \{x_1, x_2, \dots, x_n\}$ and edge set E is a bijection $f\colon V \to \{1, 2, \dots, n\}$ with induced function $w\colon V \to \mathbb{N}$ defined as

$$w(x_i) = \sum_{x_i x_j \in E} f(x_j),$$

having the property that $f(x_i) = i$ and the weight sequence $w(x_1), w(x_2), \dots, w(x_n)$ forms an increasing arithmetic progression with difference d. A graph G is a *d-handicap distance antimagic graph* or simply *d-handicap graph* if it allows a d-handicap distance antimagic labeling.

The spectrum of all pairs (n, r) for which there exists an r-regular 1-handicap distance antimagic graph with n vertices has been completely determined for even n by Froncek, Kovar, Kovarova, Krajc, Kravcenko, Shepanik, and Silber. For odd n, some sporadic infinite classes are known due to the first author, who also showed at MCCCC 29 that for every feasible n odd there exists at least one r-regular 1-handicap graph.

I will present a complete spectrum of regularities for which r-regular 2-handicap graphs of order $n \equiv 0 \pmod{16}$ exist, and show a similar construction for $n \equiv 8 \pmod{16}$ which covers the spectrum except for extremely small and large values of r.

On Hamada's conjecture in affine spaces

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In 1973, Hamada made the following conjecture: Let D be a geometric design, and let r be the p-rank of D. If D' is a design with the same parameters as D, then the p-rank of D' is greater than or equal to r. The equality holds if and only if D' is isomorphic to D.

In 1986, Tonchev and more recently Harada, Lam and Tonchev (2005); Jungnickel and Tonchev (2009); and Clark, Jungnickel and Tonchev (2011) found designs having the same parameters and p-rank as certain geometric designs, hence provide counter-examples to the "only if" part of Hamada's conjecture. In this talk, we discuss some properties of the three known nonisomorphic 2-$(64, 16, 5)$ designs of 2-rank 16, one being the design of the planes in the 3-dimensional affine geometry over the field of order 4, and try to find an algebraic way to use the similarities between these designs in a search for counter-examples to Hamada's conjecture in affine spaces of higher dimension.

Cyclic Analogues to Generating Functions for Various Integer Compositions

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Integer compositions and related enumerative questions have been extensively studied. The cyclic analogues of these questions, however, have been surprisingly insufficiently considered. Using the cyclic construction of Flajolet and Soria, we obtain the generating function for cyclic compositions with parts under modulo conditions. With this generating function we present some statistics of the parts in cyclic compositions, and we will give other generating functions for various types of cyclic integer compositions.

Maximal packings of the complete graph with stars

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For graphs G and H, a maximal G-packing of H is a set S of edge-disjoint subgraphs of H all isomorphic to G, such that S does not contain any subgraph G. In this talk, we discuss maximal S_k-packings of K_n and see some results on possible leave graphs.

Some Colorings That Are Sum Colorings

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For a positive integer-valued vertex coloring c of a nontrivial connected graph G, where adjacent vertices may be colored the same, the color sum $s(v)$ of a vertex v is the sum of the colors $c(u)$ of all vertices u of G belonging to the closed neighborhood $N[v]$ of v. If $s(x) \neq s(y)$ for every two adjacent vertices x and y for which $N[x] \neq N[y]$, then c is a closed sigma coloring of G. The minimum number of colors required of a closed sigma coloring of G is the closed sigma chromatic number of G. Results, conjectures and open questions are presented in this area of research.

Set-Defined Colorings in Graphs

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For a nontrivial connected graph G, Frank Harary and Mike Plantholt introduced an edge coloring $c\colon E(G) \to [k] = \{1, 2, \ldots, k\}$, where adjacent edges may be colored the same, that gives rise to a vertex coloring $c'\colon V(G) \to \mathcal{P}([k])$, where $c'(v)$ is the set of colors of the edges of G incident with v and where $u \neq v$ implies that $c'(u) \neq c'(v)$. We take another look at this concept and present recent results in this area of research. This is joint work with Z. Bi, G. Chartrand, S. English and P. Zhang.

New $\mathrm{LS}[3][2,3,2^8]$ Geometric Large Sets

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Let V be an n-dimensional vector space over the field of q elements. By a *geometric* $t-[q^n,k,\lambda]$ design we mean a collection $\mathcal{D}$ of k-dimensional subspaces of V, called blocks, such that every t-dimensional subspace T of V appears in exactly λ blocks in $\mathcal{D}$. A large set $\mathrm{LS}[N][t,k,q^n]$ of geometric $t-[q^n,k,\lambda]$ designs is a decomposition of the collection of all k-dimensional subspaces (k-spaces) of V into N mutually-disjoint $t-[q^n,k,\lambda]$ geometric designs. In this work we compute the Kramer-Mesner incidence matrices between the orbits of 2-spaces and 3-spaces under G, construct geometric large sets of parameters $\mathrm{LS}[3][2,3,2^8]$, using the L^3 algorithm for lattice basis-reduction as was used by Braun, Kohnert, Östergard, and Wasserman in 2013. We also construct geometric large sets of the same parameters using linear programming, prove that these large sets are all non-isomorphic to each other, and prove the automorphism groups of all these geometric large sets is G alone.

On Subgraphs With Proper Connection Number 2

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An edge coloring of a connected graph G, where adjacent edges can be colored the same, is a proper-path coloring if every two vertices of G are connected by a properly colored path. The minimum number of colors required of a proper-path coloring of G is its proper connection number $pc(G)$. For many graphs, this number is 2. If H is a connected spanning subgraph of G, then $pc(H) \geq pc(G)$. For several classes of graphs G for which $pc(G) = 2$, the minimum size of a connected spanning subgraph H of G with $pc(H) = 2$ is determined. This is a joint work with Z. Bi, G. Chartrand, and P. Zhang.

Twin Edge Colorings of Trees

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For a connected graph G of order at least 3 and an integer $k \geq 2$, a twin edge k-coloring of G is a proper edge coloring of G using elements of $\mathbf{Z}_k$ so that the induced vertex coloring in which the color of a vertex v in G is the sum (in $\mathbf{Z}_k$) of the colors of the edges incident with v is a proper vertex coloring. The minimum k for which G has a twin edge k-coloring is called the twin chromatic index of G. It has been conjectured that the twin chromatic index of every connected graph G of order at least 3 (except C_5) lies between the maximum degree of G and 2 plus the maximum degree of G. In this talk, we present recent progress on this conjecture for trees as well as other new results in this area of research.

Signed edge domination numbers of complete tripartite graphs: Part One

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The closed neighborhood $N_G[e]$ of an edge e in a graph G is the set consisting of e and of all edges having an end-vertex in common with e. Let f be a function on $E(G)$, the edge set of G, into the set $\{-1, 1\}$. If $\sum_{x\in N[e]} f(x) \geq 1$ for each edge $e \in E(G)$, then f is called a signed edge dominating function of G. The signed edge domination number of G is the minimum weight of a signed edge dominating function of G. In this talk, we present the signed edge domination number of the complete tripartite graph $K_{m,n,p}$, where $1 \leq m \leq n \leq p \leq m + n$.

On Square Graceful Graphs

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A (p, q)-graph $G(V, E)$ is said to be a square graceful graph if there exists an injection $f\colon V(G) \to \{0, 1, \ldots, q^2\}$ such that the induced mapping $f^*\colon E(G) \to \{1, 4, 9, \ldots\}$ defined by $f^*(uv) = |f(u)f(v)|$ is a bijection of $\{1^2, 2^2, 3^2, \ldots, q^2\}$. The function f is called a square labeling of G. Some square graceful graphs, forbidden subgraphs and the star square graceful deficiency number s of some graphs are determined.

Hamiltonian Cycles in k-Partite Graphs

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Chen, Faudree, Gould, Jacobson, and Lesniak determined the minimum degree threshold for which a balanced k-partite graph has a Hamiltonian cycle, extending a result of Moon and Moser about Hamiltonian cycles in balanced bipartite graphs. However, when $k \geq 3$ a Hamiltonian k-partite graph is not necessarily balanced. We determine some minimum degree thresholds for Hamiltonian cycles in (not-necessarily-balanced) k-partite graphs.

A characterization of trees with equal 2-domination and 2-independence numbers

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A set S of vertices in a graph G is a 2-dominating set if every vertex of G not in S is adjacent to at least two vertices in S, and S is a 2-independent set if every vertex in S is adjacent to at most one vertex of S. The 2-domination number $\gamma_2(G)$ is the minimum cardinality of a 2-dominating set in G, and the 2-independence number $\alpha_2(G)$ is the maximum cardinality of a 2-independent set in G. Chellali and Meddah [*Trees with equal 2-domination and 2-independence numbers,* Discussiones Mathematicae Graph Theory **32** (2012), 263–270] provided a constructive characterization of trees with equal 2-domination and 2-independence numbers. Their characterization is in terms of global properties of a tree, and involves properties of minimum 2-dominating and maximum 2-independent sets in the tree at each stage of the construction. We provide a constructive characterization that relies only on local properties of the tree at each stage of the construction.

Pattern Packing and Superpatterns in Circular Permutations

Charles Lanning

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Pattern avoidance in permutations is a classic enumerative combinatorial problem that is recently considered in circular permutations. The opposite of pattern avoidance can be considered in two different ways. The first is pattern packing. That is to contain as many copies of a given pattern as possible in a single permutation of given length. The second is finding superpatterns. That is to find a permutation (or pattern), as short as possible, that contains all possible patterns from a given set. We introduce these questions for circular permutations in this talk. Some interesting observations and questions will be presented.

On Square Graceful Deficiencies of Cycles

C. C. Chen, Z. B. Gao, Sin-Min Lee*

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A (p, q)-graph $G(V, E)$ is said to be a square graceful graph if there exists an injection $f : V(G) \to \{0, 1, \ldots, q^2\}$ such that the induced mapping f^*: $E(G) \to \{1, 4, 9, \ldots\}$ defined by $f^*(uv) = |f(u)f(v)|$ is a bijection of $\{1^2, 2^2, 3^2, \ldots, q^2\}$. The function f is called a square labeling of G. A new parameter called star square graceful deficiency number of a graph is defined by Murugam recently, and the star square graceful deficiency number s of some cycles is determined.

On minimum balanced bipartitions of triangle-free graphs

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A *balanced bipartition* of a graph G is a partition of $V(G)$ into two subsets V_1 and V_2 that differ in cardinality by at most 1. A *minimum balanced bipartition* of G is a balanced bipartition V_1, V_2 of G minimizing $e(V_1, V_2)$, where $e(V_1, V_2)$ is the number of edges joining V_1 and V_2 and is usually referred to as the *size* of the bipartition. In this paper, we show that every 2-connected graph G admits a balanced bipartition V_1, V_2 such that the subgraphs of G induced by V_1 and by V_2 are both connected. This yields a good upper bound to the size of minimum balanced bipartition of sparse graphs. We also present two upper bounds to the size of minimum balanced bipartitions of triangle-free graphs, which sharpen the corresponding bounds of [G. Fan, B. Xu, X. Yu and C. Zhou, Upper bounds on minimum balanced bipartitions, *Discrete Math.* **312** (2012) 1077–1083].

Fractional Separation Dimension

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Given a linear ordering σ of $V(G)$, say that a pair of nonincident edges is *separated* by σ if both vertices of one edge precede both vertices of the other. The *separation dimension* is the minimum size of a set of vertex orders needed to separated every pair of non-incident edges. The *t-separation dimension* $\pi_t(G)$ of a graph G is the minimum size of a multiset of vertex orders needed to separate every pair of non-incident edges of G t times. The *fractional separation dimension* $\pi_f(G)$ of a graph G is $\liminf_t \pi_t(G)/t$.

We show that $\pi_f(G) \leq 3$ for every graph G, with equality if and only if $K_4 \subseteq G$. On the other hand, there is no sharper upper bound; we show $\pi_f(K_{m,m}) = \frac{3m}{m+1}$.

A stability version for a theorem of Erdős on nonhamiltonian graphs

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In 1962, Erdős proved an upper bound for the number of edges in a nonhamiltonian graph with n vertices and minimum degree d. For each n and d, he also provided an extremal example that meets the bound. In particular, for $d < n/6$, the extremal example is a graph $H_{n,d}$. We show that for $d < n/6$, every nonhamiltonian graph on n vertices with minimum degree at least d with "close" to the maximum number of edges is either a subgraph of $H_{n,d}$ or a subgraph of $K'_{n,d}$ where $K'_{n,d}$ is composed of edge-disjoint copies of K_{n-d} and K_{d+1} sharing one vertex. Thus we give a classification of all nonhamiltonian graphs with minimum degree d and "a lot" of edges. Furthermore, this yields a polynomial-time algorithm that determines if a graph with minimum degree d and sufficiently many edges is hamiltonian.

On disjoint cycles in a graph and a theorem of Dirac and Erdős

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For an integer $k \geq 2$ and a graph G, let $H_k(G)$ be the set of vertices with degree at least $2k$ and $L_k(G)$ be the set of vertices of degree at most $2k-2$. A seminal result of Corrádi and Hajnal from 1963 proves that if $|V(G)| \geq 3k$ and $H_k(G) = V(G)$ (i.e. $\delta(G) \geq 2k$), then G contains k disjoint cycles. In the same year, Dirac and Erdős proved that G contains k disjoint cycles whenever $|H_k(G)| - |L_k(G)| \geq k^2 + 2k - 4$. We prove a stronger version of this theorem, showing that a graph has k disjoint cycles whenever the difference is at least $3k$ and that this bound is sharp. We also show that a difference of $2k$ is sufficient when G is large, G is planar, or G contains at most two disjoint triangles.

Completing Partial Latin Squares with Two Filled Rows and b Filled Columns

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In this paper, we give an alternate proof for completing partial Latin squares with two filled rows and two filled columns, which simplifies the current proof by Adams, Bryant, and Buchanan significantly. We accomplish this by developing the concept of replacing a symbol by the content of a column and row, and then using a well known completion result by Smetaniuk. We also show when we can guarantee that a partial Latin square with two filled rows and b filled columns can be completed.

A High Dimensional Version of Frobenius Numbers

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A numerical semigroup G is a subset of $\mathbb{N}$ that is closed under addition. If the generators of G are coprime, then there exists a number $\mathrm{Fr}(G)$ such that for all $n > \mathrm{Fr}(G)$, $n \in G$. That is, $\mathrm{Fr}(G)$ is the largest integer not in G. This number is widely studied, and many generalizations have been considered. In this talk, we discuss the analog of the Frobenius number to semigroups of $\mathbb{N}^n$. If $G \subseteq \mathbb{N}^n$ is a semigroup, is there is an element $v \in \mathbb{N}^n$ such that for all $n > v$, $n \in G$? This version of the problem seems to be missing from the literature. We will give the $\mathbb{N}^n$-analogues of the problem under which the answer is affirmative.

Graph Fall-colouring: Operators and Heredity

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A fall-colouring is a partition of the vertices of a graph into independent dominating sets. In this talk, we will briefly look at the effect of various operators (both binary and unary) on the fall-colourability of graphs. We will also present a new NP-complete problem in fall-colouring.

Triangle-tilings in graphs without large independent sets

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We study the minimum degree necessary to guarantee the existence of perfect and almost-perfect triangle-tilings in an n-vertex graph G with sublinear independence number. In this setting, we show that if $\delta(G) \geq n/3 + o(n)$ then G has a triangle-tiling covering all but at most four vertices. Also, for every $r \geq 5$, we asymptotically determine the minimum degree threshold for a perfect triangle-tiling under the additional assumptions that G is K_r-free and n is divisible by 3.

On Strongly Chromatic Choosable Graphs with an Application to List Coloring the Cartesian Product of Graphs

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The list chromatic number of the Cartesian product of graphs is not well understood. The best result is by Borowiecki, Jendrol, Kral, and Miskuf (2006) who proved that the list chromatic number of the Cartesian product of two graphs can be bounded in terms of the list chromatic number and the coloring number of the factors, implying a bound exponential in the list chromatic number of the factors. We show how to improve this bound for certain large classes of graphs.

We generalize the notion of strong critical graphs, introduced by Stiebitz, Tuza, and Voigt in 2008, to strong k-chromatic choosable graphs, and we show that it gives a strictly larger family of graphs that includes odd cycles, cliques, join of a clique with any other such graph, and many more families of graphs. Our main result gives a sharp bound on choosability of the Cartesian product of a strong k-chromatic choosable graph satisfying an edge bound and a traceable graph. This result can be applied to find chromatic choosable families of graphs improving the existing bounds on their choosability. The proof uses the notion of unique-choosability as a sufficient condition for list colorability, discovered by Akbari, Mirrokni, and Sadjad in 2006, to set up a loaded inductive statement that guarantees non-unique list colorings.

Decompositions of K_n^* into orientations of $K_4 - e$

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Let K_n^* denote the complete digraph on n vertices, where each pair of distinct vertices has an arc in each direction. Let D be a digraph formed by orienting the edges of $K_4 - e$, the complete graph on 4 vertices with an edge deleted. Necessary and sufficient conditions are presented for the existence of a D-decomposition of K_n^* for eight such digraphs, and partial negative results are given for the remaining two. This research is supported by grant number A1359300 from the Division of Mathematical Sciences at the National Science Foundation.

Embedding factorizations in complete uniform hypergraphs

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We look at when it is possible to embed an r-factorization of a complete h-uniform hypergraph on m vertices into an s-factorization of a complete h-uniform hypergraph on n vertices. There are some "obvious" necessary conditions. Häggkvist and Hellgren showed that for $r = s = 1$ these are sufficient. We prove that when $r = s$ the obvious necessary conditions together with $\gcd(n, h) \mid m$ are sufficient (which includes the result for $r = s = 1$). Furthermore when $r < s$ the obvious necessary conditions together with $\gcd(n, h) \mid m$ and $n \geq 2m$ and $\frac{s}{r} \leq \frac{m}{k}\left[1 - \binom{m-k}{h}/\binom{m}{h}\right]$ are sufficient. Neither of these is an exact characterization, but there is scope for improvement. From another point of view, one can attempt to find an exact characterization for small values of h. We do this for $h = 3$ and already discover that the exact characterization has one "less-obvious" necessary condition, namely that if $s\binom{m-1}{2} = r\binom{n-1}{2}$ and $m(s-r)$ is odd then $rm\binom{n-m}{2} \geq \binom{m-1}{2}$.

Counting multigraph pairs of orders 2 and 3

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Let λK_n be the complete λ-fold multigraph of order n. Let G and H be multigraphs on n vertices. We call (G, H) a *λ-fold multigraph pair of order n* if all of the following hold:

1. G and H have no isolated vertices,
2. $G \ncong H$, and
3. $E(G) \cup E(H) = E(\lambda K_n)$.

Shown are the general results for finding how many λ-fold multigraph pairs there are on both 2 and 3 vertices, by finding generating functions with respect to λ.

Party Problems & Ramsey Numbers

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In a red-blue coloring of a graph G, every edge of G is colored red or blue. For two graphs F and H, the Ramsey number $R(F, H)$ of F and H is the smallest positive integer n such that every red-blue coloring of the complete graph K_n of order n results in either a subgraph isomorphic to F all of whose edges are red or a subgraph isomorphic to H all of whose edges are blue. It is well known that Ramsey numbers have a connection with certain party problems. While the study of Ramsey numbers has been a popular area of research in graph theory, over the years a number of variations of Ramsey numbers have been introduced. We look at some of those introduced more recently and consider questions dealing with related party problems. This is joint work with Z. Bi, G. Chartrand and P. Zhang.

Optimizing Designs for the Arbitrary Factorial Term Model Space

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Consider experiments involving k factors. While a fractional factorial design may be used, such a design may be too large and not properly account for simultaneous interactions between more than two factors. To each model, we associate a hypergraph on k vertices, each hyperedge of size s corresponding to the s-way interaction between a specified s factors and the response variable. We study alternative designs when the experimenter has reasons for providing upper bounds on how many significant s-way interactions exist for each of $s = 1, 2, 3, \ldots$. The Kruskal-Katona theorem sometimes implies improvements for the experimenter's upper bounds. Beyond that, we strategize to find designs that reasonably maximize estimation capacity while limiting the variance of the parameters.

Hamiltonicity of the Preferential Attachment model

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The preferential attachment model is perhaps the best-known model of complex real-world networks. In this model, vertices are added to the graph one by one, and each time a new vertex is created it establishes a connection with m random vertices selected with probabilities proportional to their current degrees. We prove that if $m \geq 1{,}260$, then asymptotically almost surely there exists a perfect matching; and if $m \geq 29{,}500$, then asymptotically almost surely there exists a Hamiltonian cycle. One difficulty in the analysis comes from the fact that vertices establish connections only with vertices that are "older" (i.e., are created earlier in the process). In view of that, we consider a simpler setting—sometimes called the uniform attachment model—in which vertices are added one by one and each vertex connects to m older vertices selected uniformly at random. In this talk, we will outline the proof of the statements for the uniform attachment model; to get the result for the preferential attachment model involves quite technical work.

On Friendly Index Set of Hypercube Q_n

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For a graph $G = (V, E)$, a binary vertex labeling (coloring) $f\colon V(G) \to \mathbb{Z}_2$, is said to be friendly if the number of vertices labeled 0 is almost the same as the number of vertices labeled 1. The friendly labeling $f\colon V(G) \to \mathbb{Z}_2$ induces an edge labeling $f_*\colon E(G) \to \mathbb{Z}_2$ defined by $f_*(xy) = |f(x) - f(y)|\ \forall xy \in E(G)$. Let $e_f(i) = |{f_*}^{-1}(i)|$ be the number of edges labeled i. The friendly index set (or cordial set) of the graph G, denoted by $C(G)$, is defined by

$$C(G) = \{\, |e_f(1) - e_f(0)| : f \text{ is a friendly vertex labeling of } G \,\}.$$

In this talk, among other facts, we investigate the friendly index set of n-cubes Q_n and improve the conjecture that is already stated in this regard. In addition, without computer aid, we completely determine the friendly index sets of Q_n for $n \leq 6$.

Graphs without independent cycles and chorded cycles

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In 1963, Corrádi and Hajnal proved a conjecture of Erdős showing that every graph G on at least $3k$ vertices with $\delta(G) \geq 2k$ contains k disjoint cycles. A chorded cycle analogue was proven by Finkel in 2008, who showed that every graph G on at least $4k$ vertices with $\delta(G) \geq 3k$ contains k disjoint chorded cycles. Both results are best possible, leading Kierstead, Kostochka, and Yeager to characterize the sharpness examples to Corrádi-Hajnal, and Molla, Santana, and Yeager to characterize the sharpness examples to Finkel's result.

In 2010, Chiba, Fujita, Gao, and Li proved a mixed version of the aforementioned results. In particular, they show that for integers r and s with $r + s \geq 1$, every graph G on at least $3r + 4s$ vertices with $\delta(G) \geq 2r + 3s$ contains r disjoint cycles and s disjoint chorded cycles. In this talk we present a characterization of the sharpness examples to this statement, which provides a transition between the results of Kierstead et al. and Molla et al.

On Edge Decompositions of a Complete Graph into Smaller Complete Graphs

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The problem of decomposing the edges of a complete graph K_n to form a given number of K_3's and K_4's has been studied extensively. We further investigate the possibility of decomposing the edges of a complete graph K_n into a given number of smaller graphs for all possible values of i and j through elementary combinatorial techniques.

A Bijection between Schröder Paths and Permutations

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A permutation (as a word) is *stack sortable* if the entries can be rearranged in increasing order through the use of pop and push operations with a specified number of stacks. Knuth showed that permutations that avoid the pattern 231 are those that can be sorted with a single stack, which are in correspondence to Dyck paths. We consider the following variation: let $\mathcal{P}_n$ be the set of permutations that avoid the patterns 3142 and 3241 and $\mathcal{S}_n$ be the set of Schröder paths of order n, which are the lattice paths from $(0,0)$ to (n,n) composed of East steps $(1,0)$, North steps $(0,1)$, and Diagonal steps $(1,1)$ that travel weakly below the line $y = x$. The permutations of $\mathcal{P}_n$ are precisely those that are sortable by a DI-sorting machine, that is, two stacks in series where at all times entries in the first stack must be in decreasing order from top to bottom. In this talk, we present a bijection between $\mathcal{P}_n$ and $\mathcal{S}_{n-1}$, which arises "naturally" from the process by which a permutation in $\mathcal{P}_n$ is sorted by the DI-machine, where symbols are moved through the two stacks according to a specified algorithm.

Existence of Some Signed Magic Arrays

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We consider the notion of a *signed magic array*, which is an $m \times n$ rectangular grid with the same number of filled cells s in each row and the same number of filled cells t in each column, filled with a certain set of numbers that is symmetric about the number zero, such that every row and column has a zero sum. We attempt to make progress toward a characterization of for which (m, n, s, t) there exists such an array. This characterization is complete in the case where $n = t$ and in the case where $n = m$; we also characterize three-fourths of the cases where $n = 2m$.

A New Approach to the Total Chromatic Number Conjecture

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Total Chromatic Number Conjecture (TCNC) is an open problem in graph theory, which after more than 50 years still challenges many researchers. This classic conjecture provides an upper bound $\Delta(G) + 2$ for the total chromatic number of an arbitrary simple graph G with maximum degree $\Delta(G)$, where total chromatic number $\chi^{''}(G)$ is defined as the minimum number of colors required to color both vertices and edges of G with the three natural coloring conditions. Thus far the best upper bound provided in the literature for $\chi^{''}(G)$, under a preassigned condition, is $\Delta(G) + 10^{26}$. In this paper, we introduce an infinite class of graphs for each of which 10^{26} is reduced to 5. The technique presented here is also used to prove that for an arbitrary graph G, $\chi^{''}(G) \leq 2\Delta(G) + 5$. Further investigation regarding adaptation of this new approach to other classes of graphs is highly advisable for more research.

New ideas for tabulating Baille-PSW pseudoprimes

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A Baille-PSW pseudoprime is a composite number n that is simultaneously a base-2 Fermat pseudoprime and a Lucas pseudoprime whose discriminant Δ satisfies $(\Delta \mid n) = -1$. An open question is whether any such number exists. I will discuss a new algorithm for tabulating such pseudoprimes that exploits their divisibility structure.

A graph descriptor encoded by the Ihara zeta function

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The Ihara zeta function of a graph is a function of complex argument defined as

$$Z(u) = \prod_{[C]} \left(1 - u^{|C|}\right)^{-1}$$

where $[C]$ runs over all the prime cycles of the graph and $|C|$ denotes the length of $[C]$. The Ihara zeta function of a connected graph that has no pendant vertices encodes several graph invariants, including its order, size, number of loops, girth, and number of spanning trees. In this talk we introduce a new graph descriptor that is encoded by the Ihara zeta function.

The spectra of several multigraphs with 4 vertices, 6 edges, and edge multiplicity at most 2

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Let G be a multigraph on 4 vertices with 6 edges and edge multiplicity at most 2. Let ${}^{\lambda}K_n$ denote the complete multigraph on n vertices where each edge is repeated λ times. Necessary and sufficient conditions on n and λ for the existence of a G-decomposition of ${}^{\lambda}K_n$ are presented for several such multigraphs. This research is supported by grant number A1359300 from the Division of Mathematical Sciences at the National Science Foundation.

On a Family of the Super Edge-Graceful Trees

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A (p,q)-graph G is said to be edge graceful if the edges can be labeled by $1,2,\ldots,q$ so that the vertex sums are distinct, modulo p. It is shown that if a tree T is edge-graceful then its order must be odd. Lee conjectured that all trees of odd orders are edge-graceful. J. Mitchem and A. Simoson introduced the concept of super edge-graceful graphs that is a stronger concept than edge-graceful for some classes of graphs. A graph $G=(V,E)$ of order p and size q is said to be super edge-graceful if there exists a bijection

$$f\colon E \to \begin{cases} \left\{0,+1,-1,+2,-2,\ldots,\frac{q-1}{2},-\frac{q-1}{2}\right\} \text{ if } q \text{ is odd;} \\ \left\{+1,-1,+2,-2,\ldots,\frac{q}{2},-\frac{q}{2}\right\} \text{ if } q \text{ is even.} \end{cases}$$

such that the induced vertex labeling f^* defined by $f^*(u)=\sum f(u,v) \mid (u,v)\in E$ has the property:

$$f^*\colon V \to \begin{cases} \left\{0,+1,-1,+2,-2,\ldots,\frac{p-1}{2},-\frac{p-1}{2}\right\} \text{ if } p \text{ is odd;} \\ \left\{+1,-1,+2,-2,\ldots,\frac{p}{2},-\frac{p}{2}\right\} \text{ if } p \text{ is even.} \end{cases}$$

is a bijection. Lee conjectured that all odd trees are super edge-graceful. The conjecture is still unsettled. In this paper we exhibit a family of trees of odd orders which are super edge-graceful.

On Gracefulness of Union of Mongolian Tents with Some Graphs

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The disjoint union of two graceful graphs does not need to be graceful. We construct infinitely many new graceful graphs by Mongolian tents through union with graphs C_3, C_4, star $\mathrm{St}(t)$ and lotus graph $L(t)$.

"Sparse" decompositions of even-regular graphs

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Let G be a $4k$-regular graph with $k\geq 2$. We show that G can be decomposed into k 4-regular spanning subgraphs $G_1,G_2,\ldots,G_k$, each of which does not contain a subgraph that is isomorphic to K_5-e. In the case when m is even, our results give a more transparent proof of a result by Oksimets that states that every connected $2m$-regular graph G with $m\geq 2$ and $|E(G)|$ divisible by 3 can be decomposed into paths of length 3.

Some Remarks about Semigroups of Contraction Mappings of a Finite Chain

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The study of various (sub)-semigroups of transformations/mappings has made a significant contribution to semigroup theory. The most notable classes are the three fundamental semigroups of transformations: the *full symmetric semigroup*, the *partial symmetric semigroup* and the *symmetric inverse semigroup*. The subsemigroups of *order-preserving* transformations, *order-decreasing (extensive)* transformations and their intersections were arguably the most studied. Others are the *Baer-Levi* and *Croissot-Teissier* semigroup.

It is now established how counting certain natural equivalence classes in various semigroups of partial transformations of an n-set leads to very interesting enumeration problems. Many numbers and triangle of numbers regarded as combinatorial gems like the Fibonacci number, Catalan number, Schröder number, Stirling numbers, Eulerian numbers, Narayana numbers, Lah numbers, etc. have all featured in these enumeration problems. These enumeration problems lead to many numbers and triangle of numbers in the Online encyclopaedia of integer sequences (OEIS), but there are also others that are not yet or have just been recently recorded in OEIS.

In this talk, we are going to focus on the combinatorial (enumerative) aspects of the classes of *contraction* transformation semigroups, which for some curious reason(s) until very recently little is known about.

On Gracefulness of Disjoint Union of Complete Bipartite Graphs and Generalized Lotus Graphs

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The disjoint union of two graceful graphs does not need to be graceful. Rosa proved that all complete bipartite graphs are graceful. Lee showed that Lotus graph $L(n)$ is k-graceful. We investigate under what condition the union of complete bipartite graph $K(a, b)$ and generalized lotus graph $L(n, k)$ is graceful.

ℓ-Connectivity and ℓ-edge-connectivity of random graphs

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For an integer $\ell \geq 2$, the ℓ-connectivity $\kappa_\ell(G)$ of a graph G is defined to be the minimum number of vertices of G whose removal produces a disconnected graph with at least ℓ components or a graph with fewer than ℓ vertices. The ℓ-edge-connectivity $\lambda_\ell(G)$ of a graph G is the minimum number of edges whose removal leaves a graph with at least ℓ components if $|V(G)| \geq \ell$, and $\lambda_\ell(G) = |E(G)|$ if $|V(G)| < \ell$. In this paper, we establish sharp threshold functions for the ℓ-connectivity and ℓ-edge-connectivity of random graphs, which generalize the result of Erdős and Rényi, and Stepanov. In fact, further strengthening our results, we show that in the random graph process, with high probability the hitting times of minimum degree at least k and of ℓ-connectivity (or ℓ-edge-connectivity) at least $k(\ell - 1)$ coincide. This can be seen as a generalization of the results of Bollobás and Thomassen.

On a Packing Problem of Alon and Yuster

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Two graphs G_1 and G_2, each on n vertices, *pack* if there exists a bijection f from $V(G_1)$ onto $V(G_2)$ such that $uv \in E(G_1)$ only if $f(u)f(v) \notin E(G_2)$. In 2014, Alon and Yuster proved that, for sufficiently large n, if $|E(G_1)| < n - \delta(G_2)$ and $\Delta(G_2) \leq \sqrt{n}/200$, then G_1 and G_2 pack. Recently, we characterized the pairs of graphs for which the theorem of Alon and Yuster is sharp. We also prove the stronger result that for sufficiently large n, if $|E(G_1)| \leq n$, and $\Delta(G_2) \leq \sqrt{n}/60$, and $\Delta(G_1) + \delta(G_2) \leq n - 1$, then G_1 and G_2 pack whenever there is a vertex $v_1 \in V(G_1)$ such that $d(v_1) = \Delta(G_1)$ and $\alpha(G_1 - N[v_1]) \geq \delta(G_2)$.

On Automorphisms of Order Three of a Putative $[72, 36, 16]$ Binary Self-Dual Code

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The existence of a $[72, 36, 16]$ binary self-dual code, C, is a long standing open problem. It is known that if C exists, then the order of its automorphism group is at most five. We assume that C has automorphism σ of order three. Let F be the subspace of vectors in C fixed by σ and let E be the subspace of vectors in C having even weight on each cycle of σ. Then $C = F \oplus E$, the natural projection of F is the $[24, 12, 8]$ Golay code, and E is an Hermitian self-dual code of length 24 over the field with four elements $\{0, e, \omega, \omega^2\}$ where $\omega + \omega^2 = e$, $e^2 = e$. We show that E does not have a generator matrix with entries from $\{0, e\}$.

Group Divisible Designs of Three Groups and Block Size Five with Configuration $(1, 2, 2)$

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We present the group divisible designs with three groups and block size five in which each block has Configuration $(1, 2, 2)$, that is, each block has exactly one point from one of the three groups and two points from each of the other two groups. We provide necessary and sufficient conditions of the existence of a GDD $(n, 3, 5; \lambda_1, \lambda_2)$ with Configuration $(1, 2, 2)$. A highlight of this study is a technique which uses two and then three idempotent MOLS consecutively to construct a required family of GDDs.

On clustering detection based on a quadratic programming in hypergraphs

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A proper cluster is usually defined as maximally cohenrent groups from a set of objects using pairwise or more complicated similarities. In general hypergraphs, clustering problem refers to extraction of sub-hypergraphs with a higher internal density, for instance, maximal cliques in hypergraphs. The determination of clustering structure within hypergraphs is a significant problem in the area of data mining. There have been various works of detecting clusters on graphs and uniform hypergraphs that were published in the past decades. Recently, it has been shown that the maximum $\{1,2\}$-clique size in $\{1,2\}$-hypergraphs is related to the global maxima of a certain quadratic program based on the structure of the given non-uniform hypergraphs. In this talk, we give a vertex-weighted extension and also extend this result to relate strict local maxima of this program to certain maximimal cliques including 2-cliques or $\{1,2\}$-cliques. This is joint work with Drs. Tang and Zhang.

Duals of Bernoulli Numbers and Polynomials and Euler Numbers and Polynomials

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A sequence inverse relationship can be defined by a pair of infinite inverse matrices. If those two matrices are the same, they define a dual relationship. We generate a unified approach to construct dual relationships using pseudo-Riordan involutions. Then we give four dual relationships for Bernoulli numbers and Euler numbers, from which the corresponding dual sequences of Bernoulli polynomials and Euler polynomials are constructed. Some applications in the construction of identities of Bernoulli numbers and polynomials and Euler numbers and polynomials are discussed based on the dual relationships.