ALM – Managing Interest Rate Risk

Wei Hao FSA, MAAA

September 2010

Outline

Background

- Duration and convexity
- Duration management

Liability Duration

- Assumptions
- Summary of results

Asset Duration Target

- Effect of interest rate changes
- Surplus protection

Background

Variations of Duration

- Macaulay Duration
- Modified Duration
- Effective Duration (option-adjusted)
- Key Rate Duration

Interpretations of Duration

- Weighted average time where CFs from a FIS are received (Macaulay D)
- First derivative of P-Y relationship of FIS (Modified D)
- Measure of sensitivity of bond price to small changes in parallel yield curve shift (Effective D)

Variations on Duration (I)

- Macaulay Duration: weighted average time-to-maturity of the cash flows of a bond. (the weight of each cash flow is based on its discounted present value)

$$Mac\ D = \sum_{t=1}^{n} w_t * t = \sum_{t=1}^{n} \frac{PV_t}{\Pr ice} * t$$

$$\% \Delta P = \frac{\Delta P}{P} \approx -Mac\ D * \frac{\Delta r}{1+r}$$

Variations on Duration (II)

- Modified Duration: an adjusted measure of Macaulay duration that produces a more accurate estimate of bond price sensitivity

Modified Duration (MD)= $\dfrac{Mac\ D}{\left(1+\dfrac{r}{m}\right)}$

m is the # of compounding period per year

$$\frac{\Delta P}{P} = \Delta\% P \approx -MD * \Delta r$$

Variations of Duration (III)

- A duration/convexity measure that includes the effect of embedded options on a bond's price behavior
  - Effective Duration $D_E = \dfrac{P_- - P_+}{2P_0 \Delta r}$
  - Effective Convexity $C_E = \dfrac{P_- + P_+ - 2P_0}{2P_0 \Delta r^2}$
  - Bond price to interest rate change:

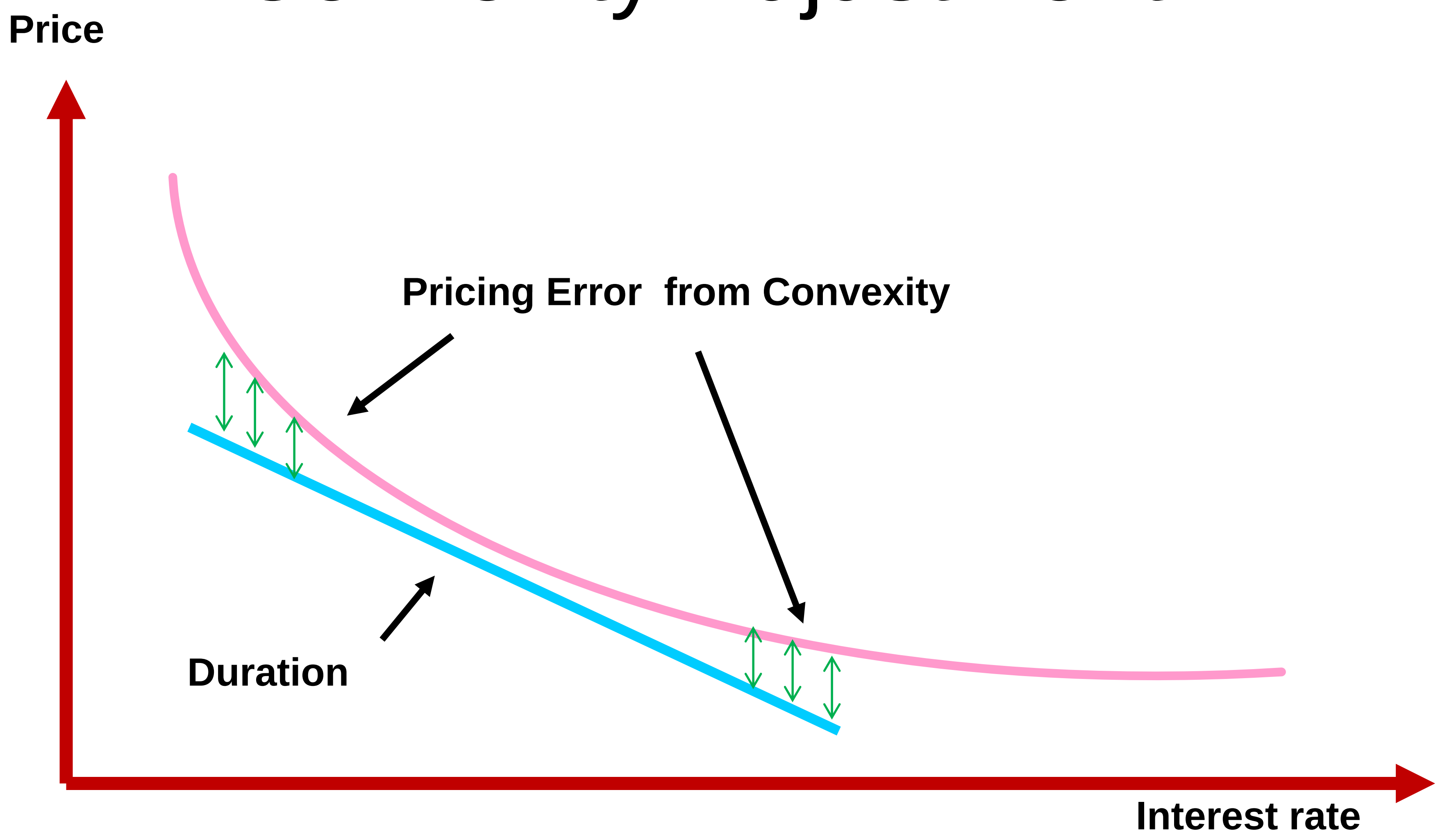
$$\%\Delta P = \frac{\Delta P}{P} \approx -D_E \Delta r + \frac{1}{2} C_E \Delta r^2$$

Effective Duration and Convexity

- Duration calculation is valid for small changes in interest rate. It is less accurate for large changes.

- The duration approximation – a straight line relating change in bond price to change in interest rate – always understates the price of the bond; it underestimates the increase in bond price when rates fall, and overestimates the fall in price when rates rise.

Convexity Adjustment



Duration Management

introduction

- An *immunized* portfolio is largely protected from fluctuations in market interest rates.
    - seldom possible to eliminate interest rate risk completely
    - a portfolio's immunization can wear out, requiring managerial action to reinstate
    - continually immunizing a portfolio can be time-consuming and costly

Duration Management (cont'd)

duration matching

- *Duration matching* selects a level of duration that minimizes the combined effects of reinvestment rate and interest rate risk

- Two versions of duration matching
  - bullet immunization (target date immunization)
  - bank immunization (surplus immunization)

Duration Management (cont'd)

bullet immunization

- Seeks to ensure that a predetermined sum of money is available at a specific time in the future regardless of interest rate movement
- Objective is to get the effects of interest rate and reinvestment rate risk to offset
  - If interest rates rise, coupon proceeds can be reinvested at a higher rate
  - If interest rates fall, coupon proceeds can be reinvested at a lower rate

Duration Management (cont'd)

surplus immunization

- Addresses the problem that occurs if interest-sensitive liabilities are included in the portfolio
- Interest rate changes cause changes in both assets and liabilities and hence in surplus. Can we eliminate this effect?
- Yes: match dollar duration of assets and liabilities!

Duration Management (cont'd)

surplus immunization

- To immunize surplus, must reorganize balance sheet such that:

$$\$_A \times D_A = \$_L \times D_L$$

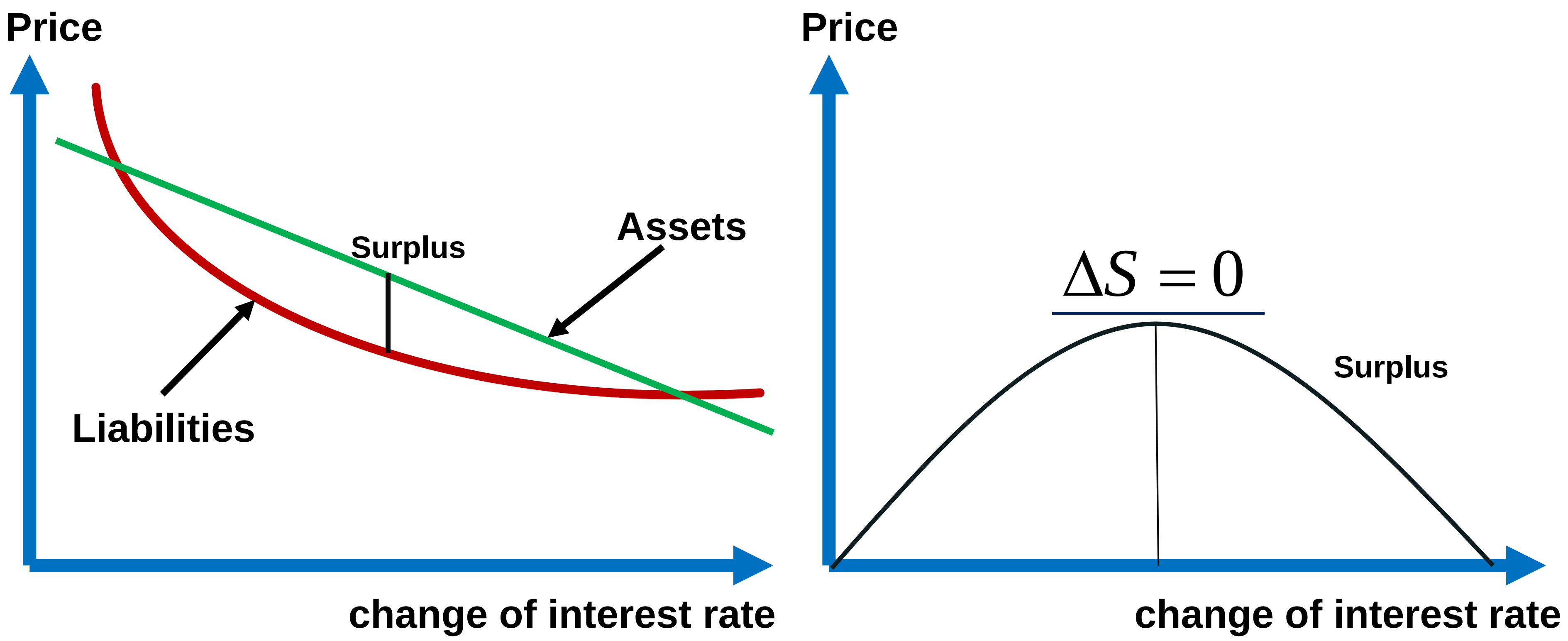
where

$\$_{A,L}$ = dollar value of assets or liabilities

$D_{A,L}$ = dollar-weighted duration of assets or liabilities

Duration Management (cont'd)

surplus immunization



Duration Management (cont'd)

surplus immunization

Assets $$\Delta A = -D_A \cdot A \cdot \Delta y + \frac{1}{2} C_A \cdot A \cdot (\Delta y)^2$$

Liabilities $$\Delta L = -D_L \cdot L \cdot \Delta y + \frac{1}{2} C_L \cdot L \cdot (\Delta y)^2$$

Surplus change
(assuming asset convexity $C_A \approx 0$)

$$\Delta S = \Delta A - \Delta L = -D_A \cdot A \cdot \Delta y + D_L \cdot L \cdot \Delta y - \frac{1}{2} C_L \cdot L \cdot (\Delta y)^2$$

Duration Management (cont'd)

surplus immunization

Surplus percentage change
recognizing $A - S = L$

$$\frac{\Delta S}{S} = (D_L - D_A) \cdot \frac{A}{S} \cdot \Delta y - D_L \cdot \Delta y - \frac{1}{2} C_L \cdot \frac{L}{S} \cdot (\Delta y)^2$$

For a given surplus percentage change $\frac{\Delta S}{S}$

$$D_A - D_L = \frac{-\frac{\Delta S}{S} - D_L \cdot \Delta y - \frac{1}{2} C_L \cdot \frac{L}{S} \cdot (\Delta y)^2}{\frac{A}{S} \cdot \Delta y}$$

Liability Duration Calculation

assumptions

- Corporate model used as the baseline.
- Due to low interest rate environment, a shift of 10bp is used.
- Durations are calculated for each line of business and they are dollar-weighted.

Liability Duration Calculation

summary of results

Left Blank Intentionally

Interest Rate Risk on Surplus

$$\frac{\Delta S}{S} = (D_L - D_A) \cdot \frac{A}{S} \cdot \Delta y - D_L \cdot \Delta y - \frac{1}{2} C_L \cdot \frac{L}{S} \cdot (\Delta y)^2$$

Given currently $D_A = 5$, $D_L = 5.5$

rate change	Surplus % change
0.25%	-1.7%
0.50%	-4.1%
0.75%	-7.3%
1.00%	-11.2%
1.25%	-15.9%
1.50%	-21.4%
1.75%	-27.6%
2.00%	-34.5%

Asset Duration Target

For a given surplus percentage change $\frac{\Delta S}{S}$, and interest rate change Δy

$$D_A - D_L = \frac{-\frac{\Delta S}{S} - D_L \cdot \Delta y - \frac{1}{2} C_L \cdot \frac{L}{S} \cdot (\Delta y)^2}{\frac{A}{S} \cdot \Delta y}$$