PRICING OF INDEX INSURANCE USING BLACK-SCHOLES FRAMEWORK: A CASE STUDY OF GHANA

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1. Abstract

This paper focuses on pricing index-based insurance using the Black-Scholes framework. Pricing of index-based insurance is unique because it responds to independent and objective parameters. These parameters are expected to correlate, as accurately as possible, with the loss of a specific crop suffered by the policy holder. The index parameter used in this paper is rainfall. The rainfall data is proven to have a high correlation with the yield of maize. The yield and rainfall data used in this paper was collected from the Tamale district in Ghana. The rainfall data was obtained from the Ghana Meteorological Agency and the crop yield from the Statistics, Research and Information Directorate (SRID).

The Black-Scholes framework assumes a lognormal distribution; therefore to price the index-based insurance, the rainfall data is proven to follow a lognormal distribution. This study also considers possible index triggers and their effect of the prices on an index-based insurance contact.

2. Introduction

Sustained growth and significant poverty reduction over the recent two decades have made Ghana an African success story. GDP has grown between four to eight percent annually over the past decade and is expected to continue in the coming years. This puts Ghana on track to reach the first Millennium Development Goal to cut poverty in half by 2015. In fact, Ghana has achieved an overall reduction in the poverty rate from 52 percent to 28 percent over the past ten years. Many factors have contributed to this impressive performance including improvement in policies and the investment climate.

Agriculture has been the backbone of Ghana’s economy in the entire post-independence history (McKay and Aryeetey, 2004). Agriculture dominates the economy contributing close to 30% of total GDP, and provides employment on a formal and informal basis. This sector is the largest source of employment for Ghanaians and is dominated by smallholder farmers producing food and cash crops. While policy and political failure caused decline in per capita GDP growth until the 1980s, the agricultural sector had been affected far less due to reduced government intervention. Analysis based on the last three runs of National Representative Household surveys shows that, agricultural crop production is the most important activity for a majority of rural households, as it serves as the major income generating activity.

While agricultural productivity growth has started to pick up in the recent two decades, the main driving factor behind the rapid agricultural growth is the crop subsector (excluding cocoa),accounting for more than two-thirds of the agricultural economy. Staple crops such as maize, sorghum, rice, cassava, yam, plantain, pulses and oilseeds dominate this subsector. Cocoa is Ghana's most important traditional export crop and has received special attention from the government in terms of financial support.

Adverse weather events such as drought, excessive rains, storms and hurricanes cause heavy losses to farmers. While these disasters can often not be prevented from happening, they can, to some extent, be predicted and arrangements can be made to reduce their impact. Unfortunately, in cases where disasters cannot be predicted, farmers will have to cope with major losses after the event occurs.

Agricultural insurance, including livestock, fisheries and forestry, is especially geared to covering losses from adverse weather and similar events beyond the control of farmers. It is one of the most quoted tools for managing risks associated with farming. While many pilot programs have been developed over the years, targeting especially small-scale farmers in developing countries, agricultural insurance remains primarily a business for farmers in developed countries. Only a small percentage of global premiums are paid in the developing world, where insurance is primarily available to larger and wealthier farmers.

Insurance spreads risk across the farming industry and the economy or, in the case of international reinsurance, to the international sphere. It is not the universal solution to the risk and uncertainties that farmers face. Furthermore, it can only address part of the losses resulting from some perils and is not a substitute for good on-farm risk- management techniques, sound production and farm management practices and investments in technology.

This paper focuses on pricing index-based insurance using the Black-Scholes framework. The initial focus is pricing an index insurance for managing risk of agricultural loss to maize for northern Ghana using this framework. Although the initial focus is on maize from northern Ghana, the implications of this paper are more far-reaching.

3. Review of existing index insurance used in agriculture.

Crop Insurance is common in most developed countries and major efforts are being made to extend it to other parts of the world especially developing countries like Ghana. Traditional agricultural insurance like crop insurance is not readily available in developing countries like Ghana because the cost of insurance can be economically unfeasible for insurers as a result of the smaller farm lots, lower limits of liability and subsequent lower premiums (Katie School of Insurance, 2011).

Other reasons that traditional crop or livestock insurance would be challenging in developing countries like Ghana include:

- Traditional crop or livestock insurance rely on direct measurement of the loss or damage suffered by the farmer. However, field loss assessment is normally costly or not feasible, particularly where there are a large number of small-scale farmers or where insurance markets are undeveloped.
- Traditional crop or livestock insurance does not address correlated risks nor works best where there are correlated risks. With traditional products, perils such as drought are challenging to insure.
- Traditional crop or livestock insurance has high operational and transaction costs. Both require massive individual underwriting (client assessment) and claims can be settled at a higher cost.

The main difference between index-based insurance and traditional agricultural insurance is that loss estimates for the farmer is based on an index or a parametric trigger for the loss rather than the individual loss of each policyholder as is the case with the latter (Skees, Hartell, & Murphy, 2007). Examples of indexed insurance used in agriculture include:

- A Malawi index-based crop insurance which measures the amount of rain recorded at local meteorological stations. The insurance pays off farmers loans in whole or in part in case of severe drought if the index hits the specified contract threshold at the end of the contract.
- A Normalized Difference Vegetation Index (NDVI) constructed from data from satellite images which indicate the level of vegetation available for livestock to consume in northern Kenya. When values (which typically range from 0.1 to 0.7) fall below a certain threshold, the insurance is triggered.

4. Why indexed insurance might work

Index-based insurance is an agricultural insurance scheme that pays for losses based on an index, an independent and objective measure that is highly correlated with losses such as extreme weather. Index-based insurance contracts, such as rainfall insurance, circumvent the moral hazard and adverse selection problems that plague traditional insurance (Skees, 2008).Poor rural people in developing countries are vulnerable to a range of risks and constraints that impede their socio-economic development. Weather and rainfall risk are pervasive in agriculture. Weather shocks can trap farmers and households in poverty, but the risk of shocks also limits the willingness of farmers to invest in measures that might increase their productivity and improve their economic situation. Nonetheless, formal and informal coping measures have been developed at farmer, community, market and government levels.

Careful studies into crop insurance has shown that the popularity of Index-based insurance can be attributed to the failures of the traditional insurance. In Joseph W. Glauber's article (August 2004) on crop insurance, he espoused the history of crop insurance in a developed country like U.S.A and emphasized particularly on its failures and measures that were adopted to salvage the crop insurance program. Jerry R. Skees (1994) agreed with Halcrow (1949) and Miranda (1991) that much of the interest in area yield insurance has been motivated by concerns with problems with the traditional farm-loss crop insurance offered through the Federal Crop Insurance Corporation (FCIC). These problems which include high cost of premiums and low participation rate are also well documented by Goodwin and Smith; Just and Calvin; Congressional Commission for the improvement of the Federal Crop insurance program; U.S General Accounting Office (1991, 1992).

The International Fund for Agricultural Development (IFAD) has been working around the clock on index insurance as part of its commitment to reduce vulnerabilities of poor rural smallholders and open their access to a range of financial services with the sole aim of improving their livelihoods. In 2008, IFAD joined forces with the World Food Program (WFP) to launch the Weather Risk Management Facility (WRMF). The WRMF has conducted global research in government and donor best practice in weather index-based insurance, while supporting Weather Index Insurance in China and Ethiopia (IFAD and WFP 2010).

In comparison with traditional agricultural insurance, Index-based insurance lowers the threshold of insurability i.e. the economic size of an insurance transaction that can be reasonably serviced by an insurer. The simplified nature of the product offers additional opportunities to reach a wider range of households and for innovative design to target the poor. However, the most likely target group will be emergent and commercial farmers, as it is unlikely that the majority of poor smallholders would directly purchase insurance on a sustainable basis.

Index-based Insurance works best where it forms part of an integrated approach to risk management, where constraints such as lack of access to finance, improved seed, inputs and markets can be addressed. This type of insurance is well suited for covariate risks such as drought or flood and thus more difficult to address with household-level coping strategies or traditional market-based risk transfer than are localized risks such as hail. The strongest relationships typically involve a single crop, a marked rainy season and no irrigation.

Index-based Insurance relies on historical and current weather data. Historical data are used as the basis for data analysis in product design and pricing. Current data – as measured by local weather stations – provide the information needed in the operational phase. Unfortunately, both historical and current data are not always plentiful. The completeness of the historical dataset is highly variable for different areas, particularly for daily data, which are needed for index design. Similarly, the density of weather stations forming the national network varies considerably from country to country. In order to meet requirements for a commercial Index-based Insurance and reinsurance transaction, it is recommended, as a guideline, that there be at least 20 years of historical daily data and that missing data should not exceed 3 per cent of the total daily dataset (IFAD 2011)

In developing countries, Index-base insurance can be considered for two broad purposes (IFAD and WFP 2010).

- Index-based insurance can be used as a tool to promote agricultural and rural development. It can help households, financial service providers and input suppliers manage low-to-medium-frequency covariate risks such as drought or excess rainfall.
- Index -based insurance can provide an alternative method of funding disaster recovery assistance or relief programs.

5. Black-Scholes framework.

In 1973, the Chicago Board of Options Exchange began trading options in exchanges, although previously options had been regularly traded by financial institutions in over the counter markets. In the same year, Black and Scholes (1973), and Merton (1973), published their seminal papers on the theory of option pricing. Since then the growth of the field of derivative securities has been phenomenal. In recognition of their pioneering and fundamental contribution to option valuation, Scholes and Merton received the Award of the Nobel Prize in Economics in 1997. Unfortunately, Black was unable to receive the award since he had already passed away.

In essence, the Black-Scholes model states that by continuously adjusting the proportions of stocks and options in a portfolio, the investor can create a riskless hedge portfolio, where all market risks are eliminated. The ability to construct such a portfolio relies on the assumptions of continuous trading and continuous sample paths of the asset price. In an efficient market with no riskless arbitrage opportunities, any portfolio with a zero market risk must have an expected rate of return equal to the risk-free interest rate. This approach led to the differential equation, known in physics as the "heat equation". Its solution is the Black-Scholes formula for pricing European put and call options on non-dividend dividend paying stocks.

Let:

- $C(S,t)$= the price of a European call option
- $P(S,t)$= the price of a European put option.
- S= the price of the stock, which will sometimes be a random variable and other times a constant.
- K = strike price of the option.
- r =the annualized risk-free interest rate, continuously compounded (the force of interest).
- $\mu =$ The drift rate of S, annualized.
- σ =the standard deviation of the stock's returns; this is the square root of the quadratic variation of the stock's log price process.

- t= a time in years; generally: now=0, expiry=T.

- δ= dividend rate, continuously compounded

- $N(x)$= standard normal cumulative distribution function,

$$C(S,t) = Se^{-\delta t}N(d_1) - Ke^{-rt}N(d_2)$$

$$P(S,t) = Ke^{-rt}N(-d_2) - Se^{-\delta t}N(-d_1)$$

Where;

$$d_1 = \frac{ln\left(\frac{Se^{-\delta t}}{Ke^{-rt}}\right) + (0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln\left(\frac{S}{K}\right) + (r - \delta + 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{ln\left(\frac{Se^{-\delta t}}{Ke^{-rt}}\right) - (0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln\left(\frac{S}{K}\right) + (r - \delta - 0.5\sigma^2)t}{\sigma\sqrt{t}}$$

6. Conceptual Framework

Cash-or-nothing options

Let $S(t)$ be the price of a stock at time t.Then a cash-or-nothing put option is one which the purchaser receives c at expiry time T if $S(t) < K$, 0 otherwise. The cash-or-nothing call option is one which the purchaser receives c at expiry time T if $S(t) > K$. Where c denote the payoff.

Black-Scholes proved that we can use the risk-neutral probability rather than the true probability to evaluate the price of an option, as long as we then discount at the risk-free rate instead of the true rate. The Black-Scholes also assume S_T/S_0 is lognormally distributed (Weishaus, 2009).

- $N(d_2)$ is the risk-neutral probability that $S(t)$ will exceed K. Consequently, $N(-d_2)$is the risk-neutral probability that the stock price will be less than K at time T.
- The price of a cash-or-nothing put option is $ce^{-rt}\,N(-d_2)$, where c denote the payoff when stock price is less than K at time T.
- The price of a cash-or-nothing call option is $ce^{-rt}\,N(d_2)$, where c denote the payoff when stock price is less than K at time T.

There are a number of similarities between index insurance and cash-or-nothing options. Therefore, index-based insurance can be priced just like a cash-or-nothing option.
To price an index insurance using Black-Scholes, we consider the following;

- The trigger measurement in the index insurance is R_T
- The payout structure for an index insurance is a lump sum.
- The index follow lognormal distribution.

The essential feature of index-based insurance is that the insurance contract responds to an objective parameter (e.g. measurement of rainfall or temperature) at a defined weather station during an agreed time period. The purchaser of and index insurance policy receive a payoff if the rainfall measurements fall below the trigger measurement which is calculated based on historical rainfall data. Let P denote the lump sum payment, then the premium of the index insurance is;

$$Premium = Pe^{-rt}\,N(-d_2)$$

Index-based insurance can also be used to insure farmers against excessive rainfall. In this case, the premium of the insurance contract can be calculated as

$$Premium = Pe^{-rt}\, N(d_2)$$

Where;

- $N(-d_2)$ = the true probability that Rainfall is less than the trigger rainfall measurement
- $N(d_2)$ = the true probability that Rainfall is greater than the trigger rainfall measurement
- r=continuously compounded interest rate over duration t.

7. Application

a. Case study site

Ghana produces a variety of crops in various climatic zones which range from dry savanna to wet forest. This research is primarily focused on the Tamale district of the northern region of Ghana where there is substantial farming activity. The northern region of Ghana is considered the major bread basket of the country, and is also the most susceptible to the vagaries of weather, especially the lack of rainfall. Unfortunately past agricultural growth and development has been accompanied by increased income inequality, and poverty abatement is lagging in Northern Ghana (Al Hassan and Diao, 2007).

This northern part of Ghana is made up of three main regions: Upper West Region, the Upper East Region and the Northern Region (Al-Hassan & Diao, 2007). The largest of these is the Northern region which incidentally is the largest region in Ghana covering a land area of about 70,383 square kilometers. However, it is has the lowest population density of all ten regions in the country (PPMED Ghana, 1991) with 80% of its people dependent on farming. The major crops grown here are yam, millet, rice, maize, sorghum, soybeans, groundnut and cassava. This paper focuses on maize production in Tamale which is the administrative capital of the Northern region.

b. Data Sources and Data limitations.

The rainfall data for Tamale was obtained from the Ghana meteorological Agency and the crop data was obtained from the Statistics, Research and Information Directorate (SRID).To design and price an insurance contract based on a rainfall index, the data for the index must be sufficient. The data used in this research spanned a period of about fifteen years, from 1992 to 2007; this would have been sufficient for accessing correlations if rainfall patterns were consistent. Due to the variability encountered, a longer time period, of about forty years, is preferred.

c. Analysis of Crop yield and rainfall

In this study, maize yield is chosen because it is one of the important cash crops in Ghana and is highly correlated with the food security in Ghana. In general rainfall is expected to correlate with production of crops and therefore the yield especially in areas like northern Ghana do not rely on irrigation. Monthly rainfall data from the Tamale district from 1992 to 2007 was collected for the analysis. Due to the uneven distribution of daily rainfall, monthly rainfall data was used to identify the correlation. Maize is a major crop and most of it is grown during the rainy season, which runs from April through September.

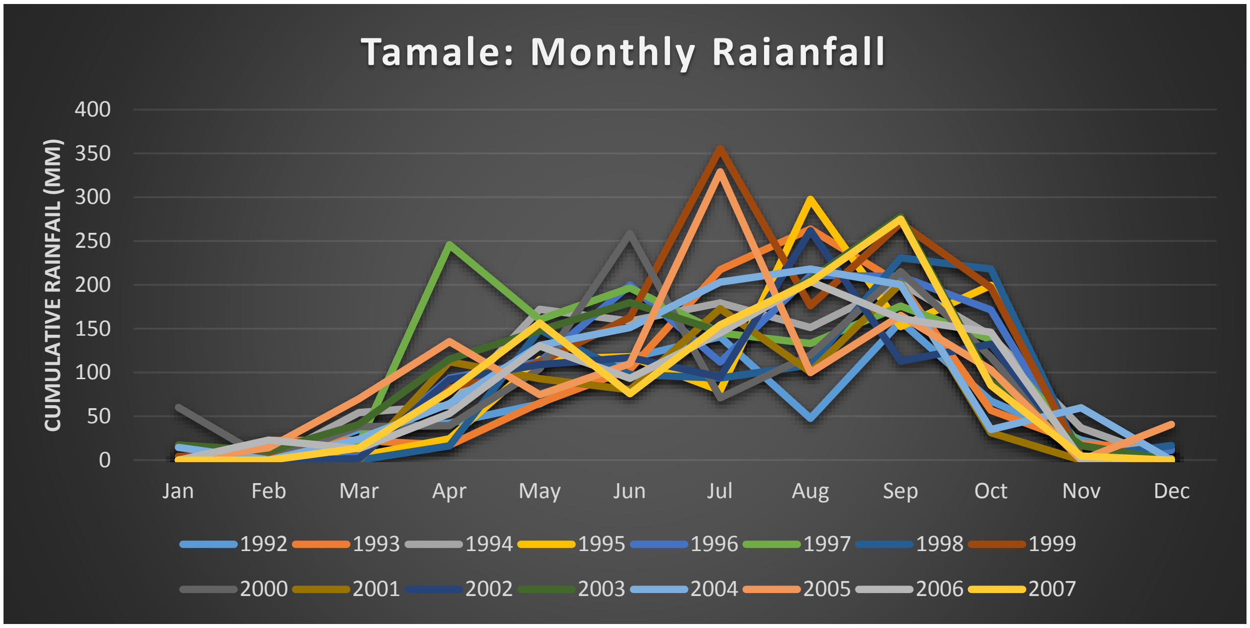
Descriptive statistics of maize and cumulative monthly rainfall data are presented in Table 1 below. Average maize yield for the Tamale district is 1.09 mt/ha. Average monthly rainfall is highest in September (200.68 mm).

Table 1: Descriptive statistics of maize and cumulative monthly rainfall data.

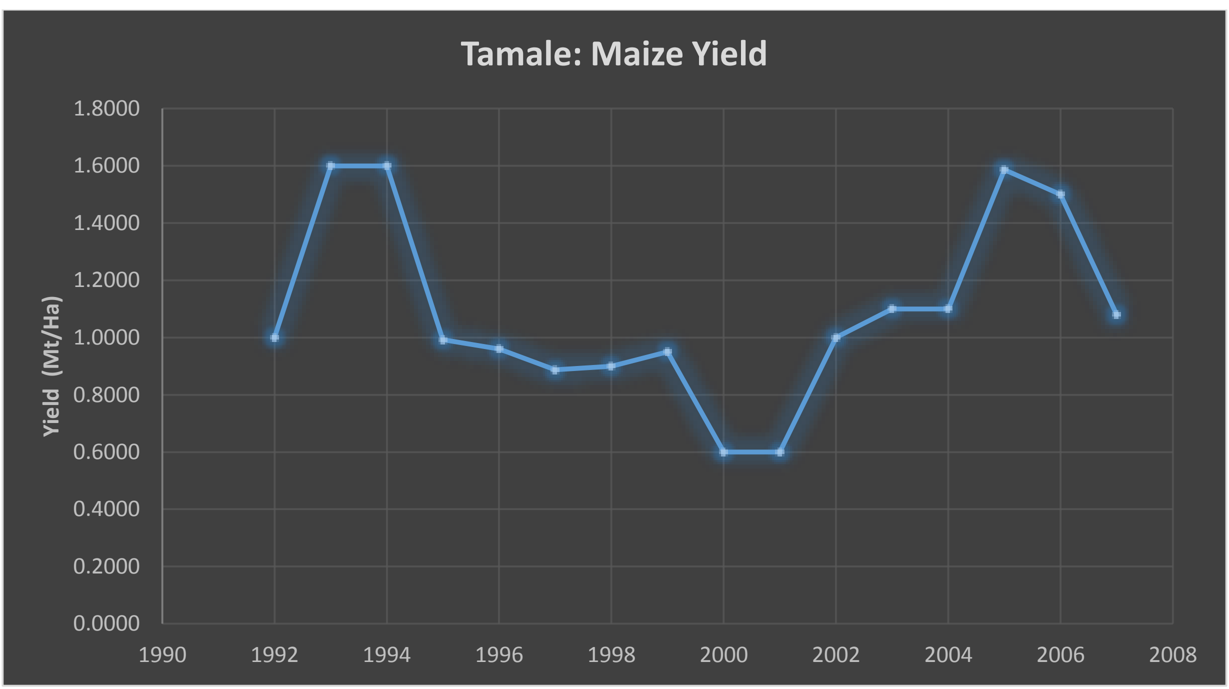
Variable	N	Mean	Std Dev	Minimum	Maximum
Maize_Y	16	1.09	0.32	0.60	1.60
Jan_R	16	6.02	15.39	0.00	60.00
Feb_R	16	6.43	9.09	0.00	22.90
Mar_R	16	22.22	20.45	0.00	69.90
Apr_R	16	79.53	56.93	16.32	245.79
May_R	16	118.46	33.58	64.15	171.98
Jun_R	16	138.76	50.96	75.64	258.75
Jul_R	16	165.03	81.08	70.96	355.84
Aug_R	16	175.22	69.94	47.69	297.54
Sep_R	16	200.68	46.76	113.18	278.13
Oct_R	16	120.67	58.02	31.11	218.17
Nov_R	16	11.70	16.73	0.00	59.78
Dec_R	16	5.15	10.97	0.00	40.90

Note: Month_R is the monthly cumulative rainfall in mm for that month

Graph 1: Monthly Rainfall for Tamale



Graph 2: Historical Maize yield for Tamale



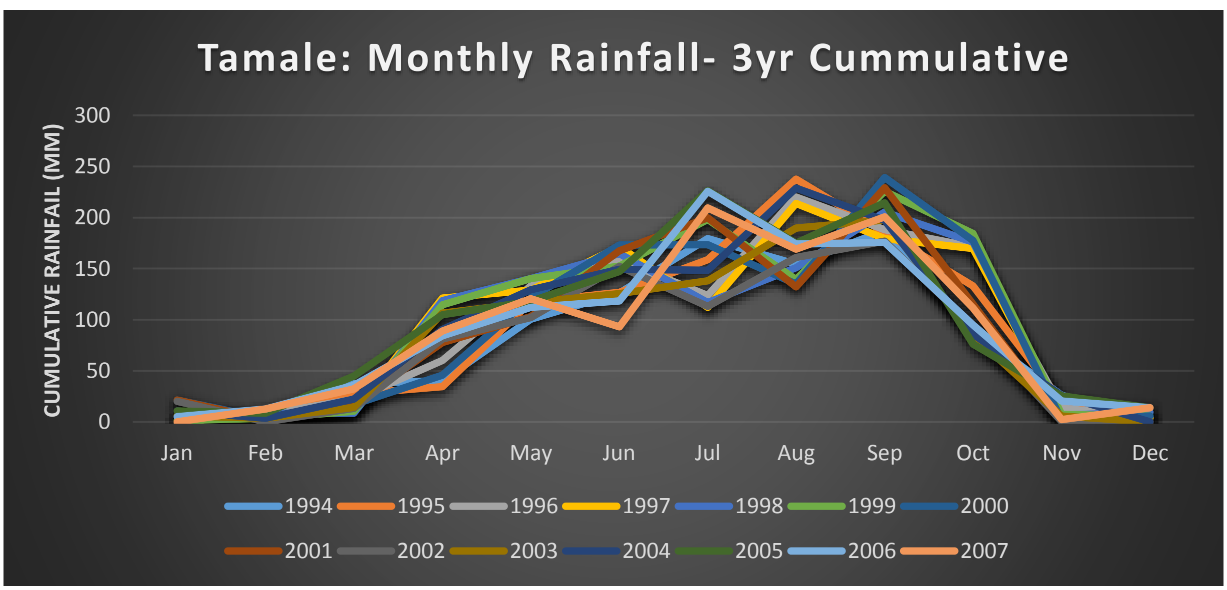
It is observed that there is an upward trend in the monthly rainfall data until October. To find out whether the monthly rainfall data correlates with the maize yield data, the correlations between monthly rainfall data and yield data is calculated in table 2. It is found that there are both positive and negative correlations with the cumulative monthly rainfall data and maize yield. The two highest correlation values are 0.53 and 0.51 for February and March, respectively.

Table 2: Correlation of monthly cumulative rainfall (mm) with Yield

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maize_Y(year)	-0.40	0.53	0.51	-0.13	-0.02	-0.32	0.42	0.24	-0.16	-0.07	0.28	0.41

These results provide some insights into the correlations between crop yield and cumulative rainfall. Since rainfall is the only index used in this paper, the correlations figures above are not significant enough. In attempt to have a higher correlation, the 3-year moving average is calculated for both the monthly cumulative rainfall data and maize yield. The graph below shows the rainfall pattern of the 3-year moving average rainfall data.

Graph 2: 3-yr Moving Average for rainfall



From the graph above, after the 3-year moving average the trend in the rainfall data becomes smoothed out and clearer. From table 3 below, the correlations between the 3-year moving average values of rainfall and yield has improved.

Table 3: Correlation between 3-year moving average values for monthly rainfall and maize yield.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Maize_Y(year)	-0.62	0.80	0.82	-0.34	-0.09	-0.73	0.43	0.40	-0.43	-0.31	0.70	0.59

The highest positive correlation is 0.82 in March. This implies yield is higher when rainfall values in March are higher. The rainfall trigger for an insurance contract that protect farmers against low rainfall can be calculated from the March rainfall data since it has the highest positive correlation to maize yield. The highest negative correlation is -0.73 in June. This means yield is lower when rainfall values are higher. The rainfall trigger for the insurance contact that insures farmers against excessive rainfall can be calculated from the June rainfall data.

d. March and June Rainfall data follows lognormal distribution

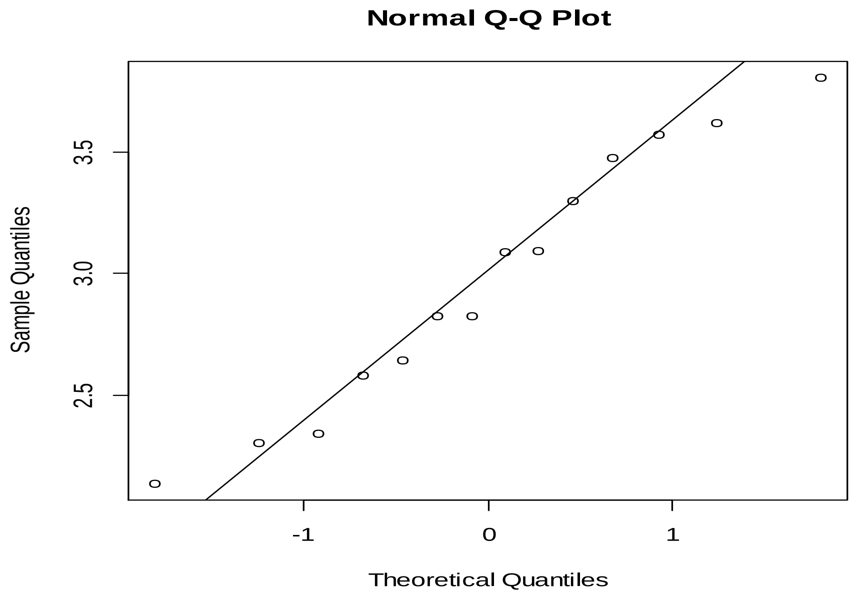
To use the Black-Scholes framework in pricing, the index should follow a lognormal distribution. The Q-Q plot and the Shapiro-Wilk test will be employed to prove that the rainfall data from March and June follows a lognormal distribution.

The Q-Q Plot

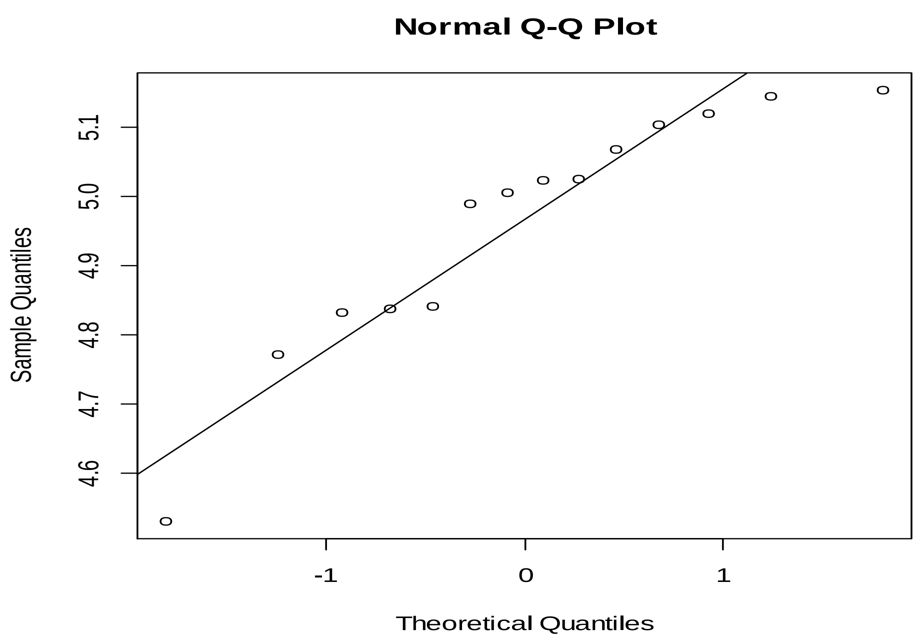
The quantile-quantile plot, or Q-Q plot, is a simple graphical method for comparing two sets of sample quantities. Plot the pairs of order statistics$(X_{(k)}, Y_{(k)})$. If the points are from the same distribution, the points should lie roughly on a line through the origin with slope 1. If the distributions are linearly related, the points in the Q–Q plot will approximately lie on a line, but not necessarily on the line *y* = *x.* Very often the dataset is compared to the Normal distribution, a theoretical population. The R programming language has a function that plots the order statistics of a sample against the corresponding quantiles of the standard normal distribution. If the plot is roughly linear, then the data are approximately normally distributed.

The graph below (Graph 3), is the Q-Q Plot for Log (Mar) data using the R programming language. Log (Mar) is the natural logarithm of the rainfall data for March. The points approximately lie on the line. This shows that Log (Mar) follows a normal distribution hence the rainfall data for March follows a lognormal distribution. Also, graph 4 is the Q-Q plot for Log (Jun) data. Log (Jun) is the natural logarithm of the rainfall data for June. The points approximately lie on the line. This shows that Log (Jun) follows a normal distribution hence the rainfall data for June follows a lognormal distribution.

Graph 3. Q-Q Plot of March Rainfall Data



Graph 4. Q-Q Plot of June Rainfall Data



The Shapiro-Wilk Test

The Shapiro-Wilk test calculates a *W* statistic that tests whether a random sample, $x_1, x_2 \ldots x_n$comes from a normal distribution. Small values of *W* are evidence of departure from normality.

The *W* statistic is calculated as follows:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}\left(x_{(i)} - \bar{x}\right)^2}$$

Where the $x_{(i)}$ are the ordered sample values ($x_{(i)}$ is the smallest) and the a_i are constants generated from the means, variances and covariances of the order statistics of a sample of size *n* from a normal distribution. To use the Shapiro-Wilk test, the null and alternative hypothesis are defined below:

$$H_0 = The\ data\ follows\ Normal\ Distribution$$
$$H_1 = The\ data\ do\ not\ follow\ Normal\ Distribution$$

For the log (Mar) data the W statistic=0.9568 and its associated P-values is 0.67. Assuming an alpha level of 0.05, the null hypothesis cannot be rejected which concludes that the Log (Mar) follows a Normal distribution. This implies the March rainfall data follows a lognormal distribution.

Shapiro-Wilk Test:

```
        Shapiro-Wilk normality test

data:  log(Mar)
W = 0.9568, p-value = 0.67
```

For the log (Jun) data the W statistic=0.8887 and its associated P-values is 0.08. Assuming an alpha level of 0.05, the null hypothesis cannot be rejected which concludes that the Log (Jun) follows a Normal distribution. This implies the June rainfall data follows a lognormal distribution.

Shapiro-Wilk Test:

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        Shapiro-Wilk normality test

data:  log(Jun)
W = 0.8887, p-value = 0.07736
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e. Pricing using Black-Scholes Framework: Insurance for Drought

Consider a rainfall index insurance product which pays out 100 currency units when the rainfall R for the Tamale district in March falls below the trigger rainfall measurement R_T.

$$Payout = \begin{cases} 100 \ \ if \ R < R_T \\ 0 \quad otherwise \end{cases}$$

Table 5: Maize yield data and March rainfall data.

Year	Yield	Mar_R
1994	1.40	37.28
1995	1.40	27.12
1996	1.18	21.92
1997	0.95	10.01
1998	0.92	8.50
1999	0.91	10.40
2000	0.82	16.91
2001	0.72	16.91
2002	0.73	13.23
2003	0.90	14.10
2004	1.07	22.08
2005	1.26	44.84
2006	1.40	35.57
2007	1.39	32.21

The premium will be calculated for R_T= 10th Percentile, R_T=25th Percentile and R_T=50th Percentile. From the March rainfall data: 10th Percentile= 10.13, 25th Percentile=13.45 and the 50th Percentile=19.42. The Rainfall data highlighted in yellow is R_0 because it is the most recent rainfall data.

$$Premium = 100 * e^{-rt} N(-d_2)$$

$$d_2 = \frac{ln\left(\frac{R_0}{R_T}\right) + (\mu)t}{\sigma\sqrt{t}}$$

Where;

$$\hat{\mu}(t) = \left(\alpha - \delta - 0.5\sigma^2\right)(t)$$

$$\hat{\mu} = \frac{1}{n}\ln\frac{R_t}{R_0} \quad \hat{\sigma} = \sqrt{p}\sqrt{\frac{n}{n-1}\left(\frac{\sum_{i=1}^{n} x_i^2}{n} - \overline{x}^2\right)}$$

μ *and* σ are the parameters of the lognormal distribution. n is the number of ratios, $x_i = \ln(R_i / R_{i-1})$ and p is number of periods per year. From the rainfall data:

$$t = 1\ year$$
$$\mu = \frac{1}{13}\ln\left(\frac{32.21}{37.28}\right) = -0.01125$$
$$\sigma = 0.3972$$
$$R_0 = 32.21$$
$$R_T = 10th\ Percentile = 10.13$$
$$r = 5\%\ \ (assumed)$$

$$d_2 = \frac{ln\left(\frac{32.21}{10.13}\right) + (-0.01125)}{0.3972} = 2.884$$
$$N(-2.884) = 0.00196$$

$$Premium = 100 * e^{-0.05}\ (0.00196) = 0.186\ currency\ units.$$

Table 5 below summarizes the premium for different trigger rainfall measurements. From the table, it is clear that the trigger is very crucial in pricing. There was a 667.2% increase in premium when the Trigger rainfall measurement increased from 10.13 mm to 13.45mm and there was a 609.7% increase in premium when the trigger changed from 13.45 mm to 19.42 mm.

Table 5: Premium for different Trigger rainfall measurements.

Trigger		Payout	Premium
10th Percentile	10.13	100	0.186
25th Percentile	13.45	100	1.427
50th Percentile	19.42	100	10.128

f. Pricing using Black-Scholes Framework: Insurance for Excess Rainfall

Consider a rainfall index insurance product which pays out 100 currency units when the rainfall R for the Tamale district for June falls above the trigger rainfall measurement R_T.

$$Payout = \begin{cases} 100 & if\ R > R_T \\ 0 & otherwise \end{cases}$$

Table 6: Maize yield data and June rainfall data.

Year	Yield	Jun
1994	1.40	126.29
1995	1.40	126.79
1996	1.18	158.98
1997	0.95	171.58
1998	0.92	164.69
1999	0.91	152.37
2000	0.82	173.13
2001	0.72	167.31
2002	0.73	151.98
2003	0.90	125.66
2004	1.07	149.41
2005	1.26	147.05
2006	1.40	118.18
2007	1.39	115.15

The premium will be calculated for R_T= 75th Percentile, R_T=85th Percentile and R_T=95th Percentile. From the June rainfall data: 75th Percentile= 163.27, 85th Percentile=167.52 and the 95th Percentile=172.12. The Rainfall data highlighted in yellow is R_0 because it is the most recent rainfall data.

$$Premium = 100 * e^{-rt}\, N(d_2)$$

$$d_2 = \frac{ln\left(\frac{R_0}{R_T}\right) + (\mu)t}{\sigma\sqrt{t}}$$

Where;

$$\hat{\mu}(t) = \left(\alpha - \delta - 0.5\sigma^2\right)(t)$$

$$\hat{\mu} = \frac{1}{n}\ln\frac{R_t}{R_0} \text{ and } \quad \hat{\sigma} = \sqrt{p}\sqrt{\frac{n}{n-1}\left(\frac{\sum_{i=1}^{n} x_i^2}{n} - \overline{x}^2\right)}$$

Where μ *and* σ are the parameters of the lognormal distribution. n is the number of ratios, $x_i = \ln\left(R_i / R_{i-1}\right)$ and p is number of periods per year. From the rainfall data:

$$t = 1\ year$$
$$\mu = \frac{1}{13}\ln\left(\frac{115.15}{126.29}\right) = -0.00711$$
$$\sigma = 0.1309$$
$$R_0 = 115.15$$
$$R_T = 75th\ Percentile = 163.27$$
$$r = 5\%\ \ (assumed)$$

$$= \frac{ln\left(\frac{115.15}{163.27}\right) + (-0.00711)}{0.1309} = -2.7218$$
$$N(-2.7218) = 0.003247$$

$$Premium = 100 * e^{-0.05}\ (0.003247) = 0.309\ currency\ units$$

Table 6 below summarizes the premium for different Trigger rainfall measurements.

Trigger		Payout	Premium
75th Percentile	163.27	100	0.309
85th Percentile	167.52	100	0.168
90th Percentile	172.12	100	0.085

Table 6: Premium for different Trigger rainfall measurements.

8. Conclusion

This study found that there is a strong correlation between maize yields and rainfall measurements in the Tamale district and index insurance can be introduced in Tamale district. The highest positive correlation (0.82) was in the month of March and the highest negative correlation (-0.73) was in June. This is an important finding because the planting date for major season of maize in Ghana is the end of March. This finding reinforce the knowledge that good amount of rainfall is needed in the flowering stages of maize production.

The study also found that over the past two decades there exist an upward trend in rainfall data from January to October and a downward trend from October to December in Tamale. This suggest the reason behind the major planting season being at the end of March.

As a way of pricing index-based insurance a case was made for the Black-Scholes framework. This framework eliminates the disadvantage of high premiums associated with traditional insurance considering a farmer has to pay 10.128 currency units in order to receive 100 currency units when rainfall in March falls below the 50^{th} percentile of Rainfall data.

The study also found that trigger measurement and payout structure have a huge effect on the premium of the contract. Designing index insurance products will be technically challenging hence insurance companies need to invest in a number of research activities in order to effectively capture the relationship between the index variable and crop loss. It must be noted that introducing index insurance requires the support of stakeholders: insurers, Ghana Meteorological Agency and government which provide the regulatory environment.

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