



ILLINOIS STATE UNIVERSITY  
Department of Mathematics

**THE IMPACT OF INCREASE IN RETIREMENT AGE  
ON THE SOCIAL SECURITY TRUST FUND IN THE  
MIDST OF DECLINING BIRTH RATE**

Author: Akwasi Sarpong

Supervised by: Dr. Krzysztof Ostaszewski, FSA, CERA, FSAS, CFA, MAAA  
(Director, Actuarial program)

RESEARCH PAPER SUBMITTED TO THE DEPARTMENT OF MATHEMATICS,  
ILLINOIS STATE UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENT  
FOR THE AWARD OF A MASTER OF SCIENCE IN MATHEMATICS

May 8, 2014

## **Abstract**

This paper seeks to estimate how the social security trust fund will be affected by an increase in retirement age in the midst of declining birth rate. The model used is based on the Gompertz-Makeham law of mortality and assumptions of fixed OASDI (Old Age, Survivors, and Disability Insurance) tax rates and fixed replacement ratio at retirement. Four scenarios of retirement ages are considered (ages 65, 66, 67 and 68) and the trend of the trust fund given each of these retirement ages is estimated. Results obtained from the analysis indicate that:

1. As life expectancy continues to rise, the rate of increment of total amount of benefits in a year will always be higher than the rate of premium increase.
2. An increase in retirement age can only delay the decline of the social security trust fund, since in the long run, total amount paid as benefits will outrun the total premiums paid into the social security account.

## **Introduction**

The Social Security is a retirement system which was established in 1935 to provide retirement benefits to retirees and disabled workers. It is primarily a pay-as-you-go system; that is, benefit payments to each generation of retirees are funded by contributions from contemporaneous workers<sup>1</sup>. The social security account is specifically referred to as the social security trust fund<sup>2</sup>. Over the years, changes in some macroeconomic indicators have posed threats to the sustainability of the social security program. Among these indicators is the decline in birth rate which began in the 1960s. During the mid-to-late-1960s, fertility began to decline dramatically,

---

<sup>1</sup> American Academy of Actuaries

<sup>2</sup> Social Security Administration

shrinking from 3.0 children per woman in 1947 to a low of just 1.74 per woman by 1976<sup>3</sup>. The rate has, however, been stable around 2 children per woman since then<sup>4</sup>. This situation has led to fewer workers paying into the system as compared to the number of retirees. In fact, when the Social Security began in 1935, the ratio of workers to retirees was 45 to 1. However, as at 2005, this ratio had dropped to approximately 3 to 1<sup>5</sup>. The reduction in the ratio of workers to retirees has caused a considerable decline in the trust fund. Research has shown that if no action is taken, the social security trust fund will run out by 2037<sup>6</sup>. Major studies have been going on over the past years to find ways of improving the financial position and long term sustainability of the social security. It is in this regard that this model is developed, to estimate the pattern of the social security trust fund, given a fixed premium rate and replacement ratio.

## **Model building**

This analysis was done by using Microsoft Excel to estimate the balance in the social security trust fund. The aim is to determine the pattern of the trust fund when different scenarios of retirement ages (ages 65, 66, 67 and 68) are considered. In order to estimate the balance in the trust fund, the total premiums paid by workers and total benefits received by retirees will be determined for each year. The total premiums and benefits can be evaluated from the population and workers' salaries, at a given OASDI (Old Age, Survivors, and Disability Insurance) tax rate and replacement ratio. Also, the population can be derived from the survival and birth rates. The survival rate used in this analysis assumes the Gompertz-Makeham law of mortality. Figures and rates are arbitrarily determined and are based on a hypothetical economy. For the purpose of our analysis, the following assumptions have been made.

---

<sup>3</sup> Social Security Advisory Board (December 2010)

<sup>4</sup> The World Bank

<sup>5</sup> Pethokoukis (2005)

<sup>6</sup> Social Security Advisory Board (December 2010)

## Assumptions

1. Birth rate is assumed to be 30 per 1000 of the population from the period 1800 to 1969 and declines to 20 per 1000 from 1970 to 2100
2. Mortality decreases steadily over the years.
3. The only increment in population is birth and the only decrement is death.
4. Everyone starts work at age 18 and retires at the retirement age.
5. Starting salary is fixed for all people who start work in a particular year and increases by 0.5% every year. For instance, if starting salary for people who turn 18 in year 2000 is \$50,000, then the starting salary for people who turn 18 in 2001 will be  $\$50,000 * 1.005 = \$50,250$
6. Once starting salary is determined, salary increases by 2% annually.
7. The only decrements for number of workers are death and retirement.
8. OASDI tax rates are fixed at 6.5%.
9. Pension benefit at year of retirement is 50% of last salary and increases annually by 2%.
10. Every worker contributes to the social security program and every retiree receives retirement benefits.
11. The only deduction from the fund is benefits paid to retirees and the only additions are premiums paid by workers.
12. The social security trust fund does not accrue any interest.

## Analysis

This analysis assumes a retirement age of 65. In a similar way, the same procedure can be used for the other scenarios of retirement ages (i.e. ages 66, 67 and 68).

## Survival rates

Survival rate is the probability that a person born in a particular year would survive to age  $t$  (where  $t$  is from 0 to 130). It is represented by  ${}_tP_x$  and is derived using the Makeham<sup>7</sup>

distribution ( $\mu_x = A + BC^x$ ), where  ${}_tP_x = \exp\left(-\int_x^{x+t} \mu_s\right)$

$$A = 0.0001$$

$$B = 0.00035$$

$$C = 1.075 + 0.0001 * (2010 - \text{year}), \text{ "year" represents the year of birth}$$

$$x = 0$$

The derivation results in the equation:

$${}_tP_0 = \exp\left(-\left(0.0001t + \frac{0.00035 * ((1.075 + 0.0001 * (2010 - \text{year}))^t - 1)}{\ln(1.075 + 0.0001 * (2010 - \text{year}))}\right)\right)$$

“ $t$ ” varies from 0 to 130

## Population

The population for each year is determined using the estimated survival rates and assumed birth rates. It is evaluated as the sum of the number of survivors to a particular age for people born in a particular year.

Given:

$b_x$  as birth rate per 1000 population in year  $x$

$P_{x-1}$  as Population in year  $x-1$

$B_x = P_{x-1} * b_x / 1000$  as number of babies born in year  $x$

${}_tP_{0(x)}$  as probability that a baby born in year  $x$  will survive to age  $t$ .

${}_tL_x = B_x * {}_tP_{0(x)}$  as babies born in year  $x$  that survived after  $t$  years,

$$\text{The Population in year } y = \sum_{t=0}^{130} {}_tL_{y-t}$$

---

<sup>7</sup> The constants A and B from the Makeham distribution are obtained from the Society of Actuaries, and C accounts for the steady decline in mortality

For instance, the population in 2100 is the sum of babies born in 2100, babies born in 2099 that survived after one year, babies born in 2098 that survived after two years, babies born in 2097 that survived after three years... up to babies born in 1970 that survived after 130 years (at which point the survival rate is 0). Table 2 in the appendix is a snapshot depicting the number of people born in a particular year and have survived to age  $t$ . The highlighted values are the summands needed for calculating the population in year 2100.

### **Salaries**

The salaries for people born in a particular year and are  $t$  years of age can be determined, since it is assumed that the starting salaries for people who begin work in the same year is fixed, and salaries increase by 2% per annum.  ${}_tS_x$  will be used to represent salaries for people born in year  $x$  and are  $t$  years of age. For instance, assuming a person born in 2010 receives a starting salary of \$46,534.03 at the age of 18; then his salary at age 60 will be:

$${}_{60}S_{2010} = \$46,534.03 * 1.02^{60-18} = \$106,900.04$$

### **Total premiums in a year**

Total premium in a year refers to the total amount of contributions made by workers in a particular year. Per the estimated yearly population and salaries, the total premiums can be calculated, at a given OASDI tax rate. For the purpose of this analysis, OASDI tax rates are assumed to be 6.5% of annual salary. Premium paid by an individual in a year is the product of the individual's annual salary and the OASDI tax rate of 6.5%. Since salaries for people born in a particular year are assumed to be the same, the premium paid by each person born in year  $x$  and is  $t$  years of age =  ${}_tS_x * 0.065$  (where  ${}_tS_x$  represents salaries for people born in year  $x$  and are  $t$  years of age). For example, if a person born in 1970 receives a salary of \$58,929.39 at age 40 (in 2010), then the premium paid in year 2010 by this person =  $\$58,929.39 * 0.065 = \$3830.41$

The total premium for a year is calculated as the sum of premiums paid by all workers in that year.

Given:

${}_t l_x$  as people born in year x that survived after t years

${}_t S_x$  as salaries for people born in year x and are t years of age.

The total premiums paid in year y =  $0.065 * \sum_{t=18}^{64} {}_t l_{y-t} * {}_t S_{y-t}$

For instance, the total premium paid in 2100 is the sum of premiums paid by people born in 2082 and are now 18 years of age, people born in 2081 and are now 19 years of age, people born in 2080 and are now 20 years of age... up to people born in 2036 and are now 64 years of age. Table 3 in the appendix is a snapshot depicting the total premiums collected from people born in a particular year and have survived to age t. The highlighted values are the summands needed for calculating the total premiums paid by workers in year 2100.

### **Total benefits in a year**

The total benefit in a year refers to the total amount of benefits received by retirees in a particular year. Retirement benefit at age 65 is assumed to be 50% of the final year's salary which also increases every year by 2%. i.e. Benefits received at age t =  $0.5 * \text{Salary at age 64} * 1.02^{t-64}$ .

Total benefit received in a year can now be calculated as the sum of benefits received by all living retirees in that year.

Given:

${}_{64} S_x$  as salary at age 64 for people born in year x

${}_t l_x$  as people born in year x that survived after t years

${}_t RB_x = 0.5 * {}_{64} S_x * 1.02^{t-64}$  represents retirement benefits received by retirees born in year x and are t years old.

The total benefits received in year  $y = \sum_{t=65}^{130} l_{y-t} * RB_{y-t}$

For instance, the total benefits received in 2100 is the sum of benefits received by people born in 2035 and are now 65 years of age, people born in 2034 and are now 66 years of age, people born in 2033 and are now 67 years of age... up to people born in 1970 and are now 130 years of age (at which point the survival rate is 0). Table 4 in the appendix is a snapshot depicting the total benefits paid to people born in a particular year and have survived to age  $t$ . The highlighted values are the summands needed for calculating the total benefits paid in year 2100.

### **Trust fund balance**

The trust fund balance is the amount of money in the social security account. It is estimated as the total amount of premiums paid by policy holders in that year minus total amount of benefits paid to retirees in that year plus the previous year's balance in the trust fund. This means:

$$\text{Trust fund balance for year } y = P_y - B_y + O_{y-1}$$

$P_y$  = Total premiums for year  $y$

$B_y$  = Total benefits for year  $y$

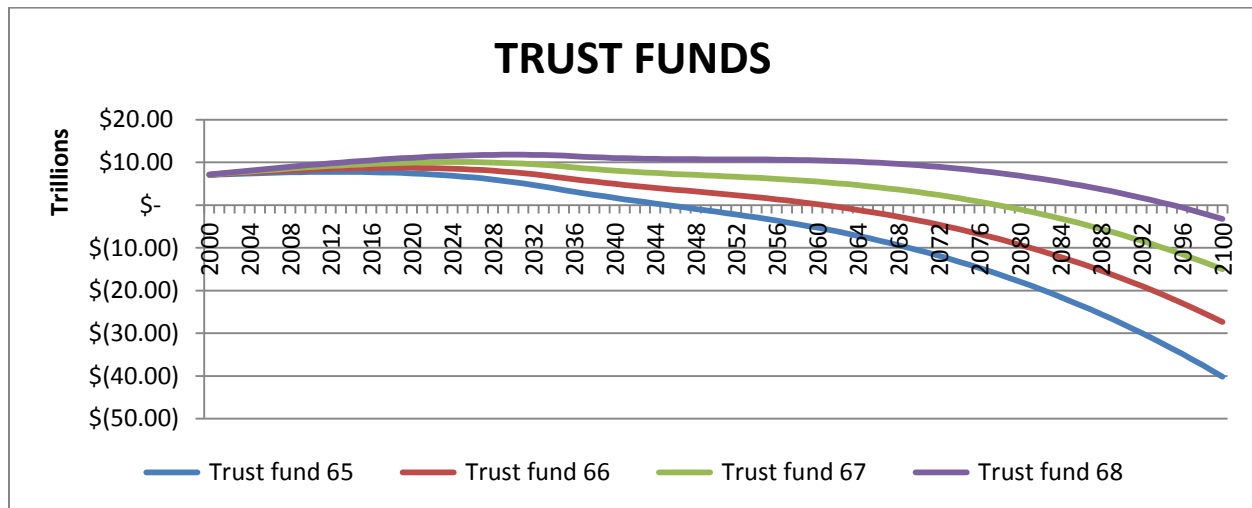
$O_{y-1}$  = Outstanding Balance of year  $y-1$

As aforementioned, this analysis assumed retirement age of 65. In the same vein, this approach can be used for all other scenarios of retirement ages (i.e. ages 66, 67 and 68).

## **Results**

Fig. 1 (below) shows the pattern of the social security trust fund from year 2000 to 2100 when the Gompertz-Makeham law of mortality is assumed, OASDI tax rates fixed at 6.5%, replacement ratio fixed at 0.50 and birth rate declines from 30 to 20 per 1000 population.



**Fig 1**

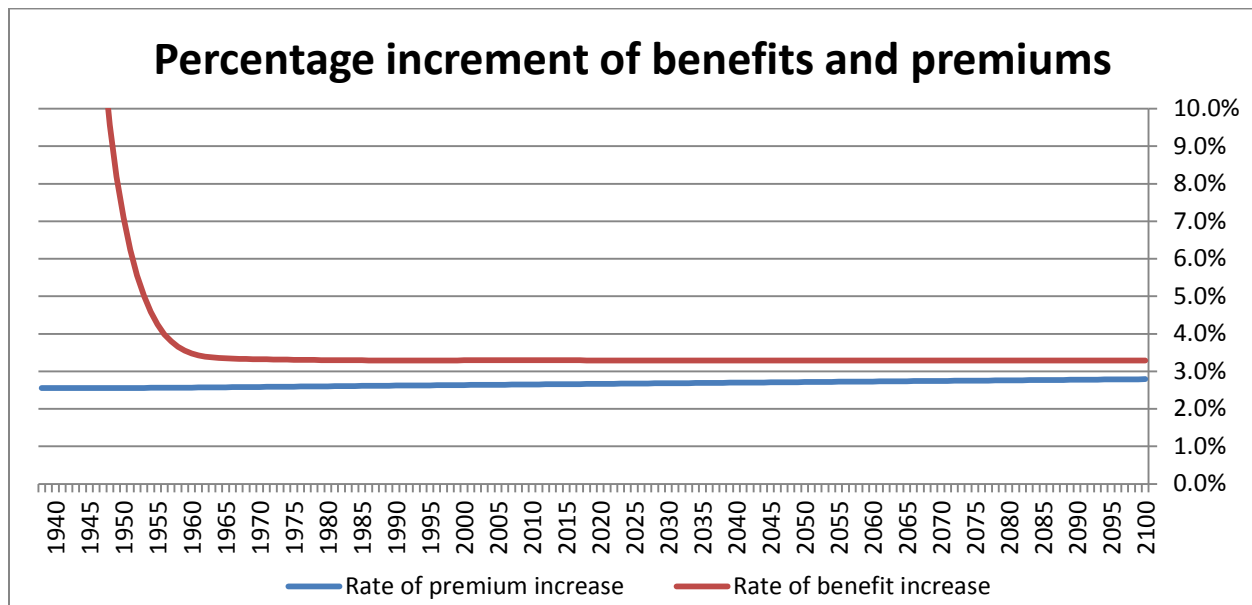
It can be seen from fig 1 (above) that the balance in the trust fund increases up to a particular point and then begins to fall. This situation happens in all four cases of retirement ages. This means an increase in retirement age does not sustain the social security forever. It only delays the decline in the trust fund balance up to a particular point. At retirement age 65, the trust fund begins to fall in the year 2014 and ultimately runs out in 2046. Similarly at retirement age 68, the trust fund starts declining in the year 2031 and ultimately runs out in 2095. Table 1 (below) shows the years in which the trust fund starts to decline and runs out in all four cases of retirement ages:

**Table 1**

Retirement age	Trust fund starts to decline	Trust fund runs out
Age 65	2014	2045
Age 66	2020	2061
Age 67	2025	2078
Age 68	2031	2095

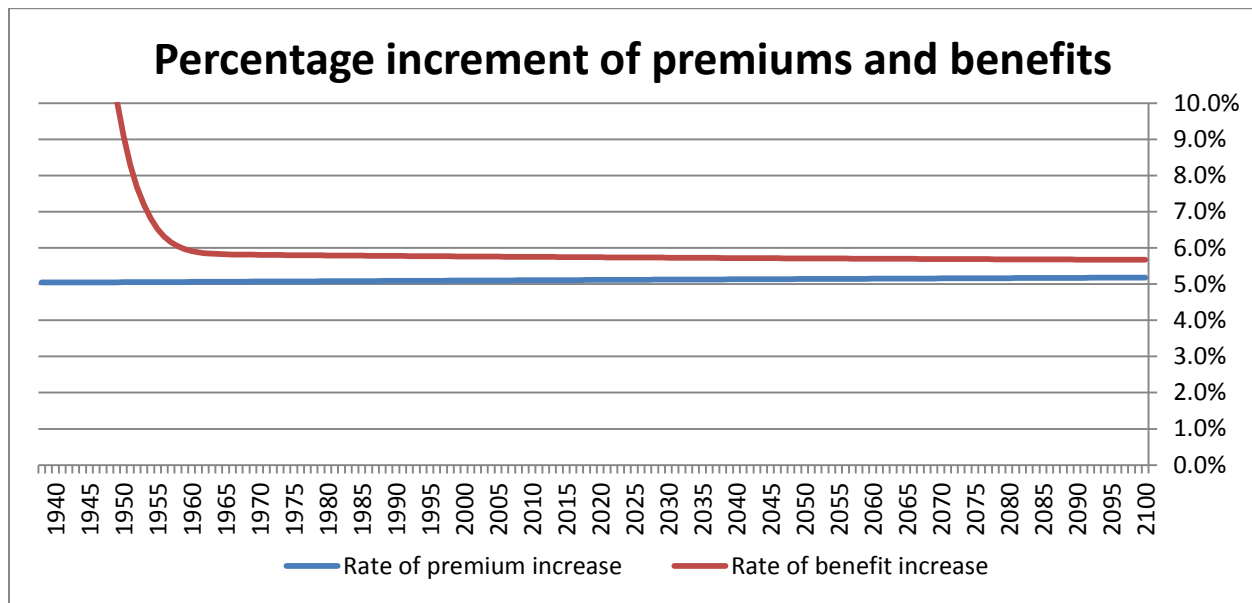
The inevitability of the decline of the trust fund is attributed to the fact that total benefits paid to retirees rises at a faster rate than premiums. Given that all assumptions hold and birth rate fixed at 30 per 1000 of the population, the rate of increase of yearly benefits is stable around 3.3% while the rate of increase in premiums is stable around 2.7%. This is displayed in the fig 2 (below).

**Fig 2**



It is worth noting that even when these fixed assumptions of rates are changed, benefits always rise faster than premiums. For instance, when birth rate is increased to 50 per 1000, OASDI tax rate increased to 10% of salary and replacement ratio reduced to 0.30, the rate of benefit increase becomes stabilized around 5.7% and rate of premium increase, stabilized around 5.1%. This is depicted in the fig 3 (below).

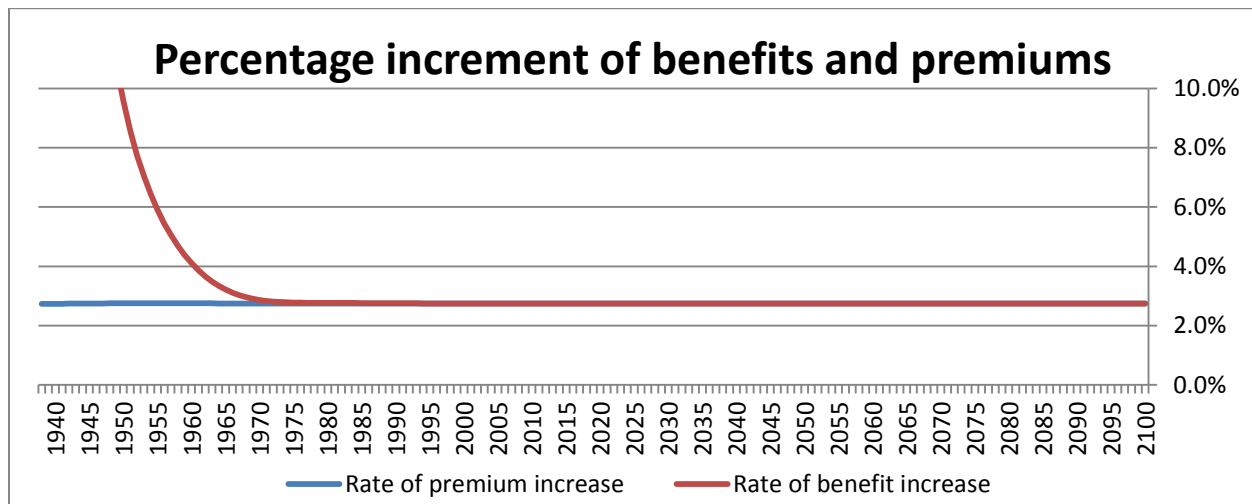
**Fig 3**



This result is justified by the fact that mortality is assumed to be declining over the years. This means a person born in the year 1900 is more likely to die before a particular age  $t$  than a person born in the year 2000. A decline in mortality means an increase in life expectancy, which results in retirees living longer, hence an increase in total benefits paid to retirees.

To validate the argument that increased life expectancy will lead to total benefits rising faster than total premiums, the same mortality rates were assumed for people born in all years (i.e. from 1800 to 2100). By changing  $C$  in the mortality assumptions from  $C = 1.075 + 0.0001 \cdot (2010 - \text{year})$  to  $C = 1.075$ , the rate of change stayed the same for both total premiums and benefits. This is shown in Fig 4 (below).

**Fig 4**



## Conclusion

As seen from the results, an increase in the retirement age can improve the financial position and long term sustainability of social security program. However, because benefits always increase faster than premiums, the social security trust fund will run out eventually, if the necessary measures are not taken. It is also important to note the negative ramifications of increasing retirement age. An increase in retirement age is essentially a benefit cut<sup>8</sup>. Thus, it creates financial difficulties for people who cannot continue to work, but do not qualify for any form of insurance. It also hurts workers in physically demanding jobs, since they would have to work for longer years<sup>9</sup>. It is therefore necessary to consider other factors that can also improve the financial position of the social security. Reducing benefits, increasing the OASDI tax rate and measures to correct the declining birth rate are some alternatives worth considering.

<sup>8</sup> National senior citizens law Centre.

<sup>9</sup> United States Government Accountability Office

## References

1. American Academy of Actuaries. 2010. 'Raising the retirement age for social security'. Issue brief. Washington, DC: American Academy of Actuaries
2. National senior citizens law center (2011) Increasing the social security retirement age cuts benefits and drives up administrative cost, Protecting the rights of low-income older adults, Policy issue brief
3. Official Social Security Website, "What are the trust funds?"
4. Pethokoukis, J. M. (2005) 'Boomer Burden', U.S. News & World Report 138(2): p46
5. Social security advisory board, Social security: Why action should be taken soon, December 2010
6. Social Security Reform: Raising retirement ages would have implications for older workers and SSA disability rolls. GAO-11-125. Washington, D.C.: November 18, 2010
7. Society of Actuaries Fundamentals of Actuarial Practice Online Study Modules, Mortality Table from exercise on social insurance, 2006

## Appendix

**Table 2:** Estimating population for year 2100

<b>t</b>	<b><math>l_{2100}</math></b>	<b><math>l_{2099}</math></b>	<b><math>l_{2098}</math></b>	<b><math>l_{2097}</math></b>	<b><math>l_{2096}</math></b>	<b><math>l_{2095}</math></b>
<b>0</b>	10,839,320.87	10,742,908.61	10,647,608.63	10,553,404.55	10,460,279.98	10,368,218.56
<b>1</b>	10,834,320.47	10,737,952.50	10,642,696.31	10,548,535.50	10,455,453.72	10,363,434.60
<b>2</b>	10,829,064.06	10,732,742.26	10,637,531.70	10,543,416.02	10,450,378.84	10,358,403.82
<b>3</b>	10,823,534.96	10,727,261.31	10,632,098.35	10,538,029.72	10,445,039.06	10,353,110.04
<b>4</b>	10,817,715.45	10,721,492.01	10,626,378.70	10,532,359.16	10,439,417.04	10,347,536.01
<b>5</b>	10,811,586.65	10,715,415.62	10,620,354.12	10,526,385.80	10,433,494.33	10,341,663.38
<b>6</b>	10,805,128.49	10,709,012.15	10,614,004.73	10,520,089.89	10,427,251.30	10,335,472.63
<b>7</b>	10,798,319.61	10,702,260.37	10,607,309.42	10,513,450.41	10,420,667.03	10,328,942.97
<b>8</b>	10,791,137.28	10,695,137.68	10,600,245.70	10,506,445.00	10,413,719.30	10,322,052.27
<b>9</b>	10,783,557.36	10,687,620.05	10,592,789.66	10,499,049.89	10,406,384.43	10,314,777.00

**Table 3:** Estimating total premiums for year 2100

<b>t</b>	<b><math>l_{2082}^* \cdot S_{2082}</math></b>	<b><math>l_{2081}^* \cdot S_{2081}</math></b>	<b><math>l_{2080}^* \cdot S_{2080}</math></b>	<b><math>l_{2079}^* \cdot S_{2079}</math></b>
<b>18</b>	608,644,026,004.56	600,450,771,717.92	592,377,728,336.73	584,422,658,427.40
<b>19</b>	620,023,901,511.32	611,676,207,202.08	603,451,002,983.44	595,346,008,929.91
<b>20</b>	631,566,096,005.87	623,061,570,223.77	614,681,850,866.22	606,424,614,815.07
<b>21</b>	643,268,286,228.29	634,604,542,820.37	626,067,959,479.64	617,656,169,190.85
<b>22</b>	655,127,633,342.58	646,302,295,524.02	637,606,508,847.85	629,037,861,694.07
<b>23</b>	667,140,734,896.67	658,151,439,684.38	649,294,124,206.78	640,566,331,529.01
<b>24</b>	679,303,573,096.41	670,147,976,130.92	661,126,825,051.28	652,237,616,896.01
<b>25</b>	691,611,459,175.76	682,287,239,958.41	673,099,970,332.31	664,047,100,596.72
<b>26</b>	704,058,973,643.12	694,563,841,217.26	685,208,199,587.41	675,989,451,600.87
<b>27</b>	716,639,902,183.96	706,971,601,290.52	697,445,369,787.83	688,058,562,359.53

**Table 4:** Estimating total benefits for year 2100

<b>t</b>	<b><math>t_{2035}^* \text{ } t\text{RB}_{2035}</math></b>	<b><math>t_{2034}^* \text{ } t\text{RB}_{2034}</math></b>	<b><math>t_{2033}^* \text{ } t\text{RB}_{2033}</math></b>	<b><math>t_{2032}^* \text{ } t\text{RB}_{2032}</math></b>
<b>65</b>	264,211,654,498.56	260,516,266,334.47	256,878,904,500.37	253,296,848,370.70
<b>66</b>	260,385,220,262.43	256,689,421,345.37	253,052,004,824.98	249,470,266,399.13
<b>67</b>	255,977,027,014.62	252,286,078,380.27	248,653,824,101.68	245,077,580,506.81
<b>68</b>	250,973,389,491.92	247,292,995,583.35	243,671,559,950.20	240,106,423,986.32
<b>69</b>	245,364,897,815.82	241,701,224,482.66	238,096,721,047.64	234,548,758,507.77
<b>70</b>	239,147,155,348.05	235,506,844,251.89	231,925,857,745.99	228,401,601,036.19
<b>71</b>	232,321,535,640.67	228,711,713,651.45	225,161,308,945.44	221,667,765,573.28
<b>72</b>	224,895,939,565.31	221,324,221,471.30	217,811,945,784.15	214,356,600,022.37
<b>73</b>	216,885,528,893.91	213,360,011,480.48	209,893,887,883.74	206,484,693,652.24
<b>74</b>	208,313,407,576.11	204,842,652,873.51	201,431,162,888.26	198,076,525,648.01