ENTERPRISE RISK MANAGEMENT: EVOLUTION OF A MODERN THEORY

Christian Wirtz

96 Pages August 2006

This thesis provides an overview of the evolution of enterprise risk management and its importance for any company. The phases of the modern risk management process are discussed separately with a focus on different techniques of risk measurement. The discussion goes along with several illustrations and sample calculations.

APPROVED:

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Recent historical and economical events have been perpetually proving that earth can be dangerous place. Negative occurrences seriously hit those who are unaware, while they are less critical to those who are prepared.

COSO (2004), Standards Australia and Standards New Zealand (2004), and Basel Commitee on Banking Supervision (2005) have offered frameworks for companies to help them identify and manage their risks. The goal of this risk management is to improve the likelihood of meeting a company's objectives.

This thesis gives information on the evolution of early approaches of a company's risk management to a sophisticated modern theory. It describes the details of the management process and provides further insights into the measurement of risk. The properties of several risk measures are discussed and proven. Numeric and graphical examples illustrate the results.

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C. W.

CONTENTS

TABLES

FIGURES

Figure | Page

CHAPTER I

INTRODUCTION

Risk is the exposure to a possible loss in the negative case or a possible benefit in the positive case, depending on a present process or a future event with an uncertain outcome (Holton , 2004). In everyday language, "risk" often refers just to negative events and is used synonymously with "danger" or "threat".

In history people have been willingly accepting high risks. Illustrative examples for this might be rulers waging a war and risking the lives of their soldiers to conquer new territories, as well as traveling merchants importing valuable goods such as spices or silk from Asia to Europe in the Middle Ages. The risk of losing the goods or even their lives on the journey was always present, but since most people long for security and stability in their lives, they often tried to avoid or eliminate risks, especially those that were threatening the means of their existence. As we can see there were always incentives to handle risk for every entity, including individuals as well as companies, or countries.

In the modern world simply avoiding risk is not viewed as an efficient solution. Risk and return are both taken into account. Investors, banks, and especially insurers are interested in assuming risks as long as they are able to diversify these risks and get higher returns as trade-off. In order to perform this trade-off correctly, the entity must have a complete understanding of its own exposure to risk. This thesis tries to provide a complete overview of all elements of this process of understanding and managing of risk - from a company's perspective.

Chapter II defines "risk management" and shows the historical expansion of the concept that is leading to modern risk management implemented by companies. Feldblum (2006) gives an idea of how important it is to recognize the incentives prevalent in management. Understanding incentives is closely related to controlling risk. In order to control the risks of a firm, several individual effects have to be related to each other. Risks are often dependent on other events and cannot be viewed separately.

Chapter III breaks down the risk management process of a firm into stages and addresses each of them separately. The author's intention is to provide an overview of the actions and structural decisions that need to be made to establish a risk management process in a company. The focus of the analysis lies on the measurement of risks. Section "Measurement of Risk" describes and analyzes properties of the most common measures used in the industry and gives a perspective on more recent and less popular approaches. "Risk Allocation" extends the idea of measuring the company's complete exposure to risk. Every line of business of the company can be made liable for a specific part of the exposure. The techniques presented to perform a fair allocation are derived from the results of the game theory (Osborne and Rubinstein , 1994).

Chapter IV concludes the thesis and provides a critical review of the potential that risk management can provide for a company.

CHAPTER II

CONCEPTS OF ENTERPRISE RISK MANAGEMENT

Risk Management

Risk management is "the process concerned with the identification, measurement, control, and minimization of [...] risks in information systems to a level commensurate with the value [...] protected" (ATIS , 2001). Although its foundations reach back to the early years of the last century when Keynes (1921) and Knight (1921) published their writings on risk, uncertainty, and probability, risk management as practiced today developed as a tool in the 1960s. A survey conducted among 221 firms, published in 1961, and later presented by Greene (1968), revealed that 25% of the companies had full-time and 10% part-time managers concerned with risk management. 96% of the companies with less than 10,000 employees had not recognized the function of a risk manager yet. These managers focused their attempt on the handling of "pure risks" only; risks that involve the possibility of loss with no chance for a gain such as fire hazard or accidents (Greene , 1968). Especially insurers - willing to assume pure risks - were interested in protecting themselves against catastrophes or unaffordable potential losses from their portfolio of policies.

These early efforts have been inspired by the ideas of Gallagher (1956) and Barlow, who developed the idea of "cost-of-risk" in the 1960's (Oshins , 1990). Cost of risk is the sum of self-funded losses, insurance premiums, risk control costs, and other administrative costs. One idea is that the value of a company is given by the difference of the hypothetical value without any risk less the cost of risk.

$$\text{value(with risks)} = \text{value(without risks)} - \text{cost of risk}$$

Therefore, the approach to maximize the true value is equivalent to minimizing cost of risk.

Example 1

The hypothetical value of a firm based on revenue, assets, and equities, without any risk, is assumed to be $1 MM. The only source of risk is machine breakdown leading to $200,000 losses due to stoppage of production and repair expenses of $50,000. The probability of a breakdown is 10%. Now the company has 4 options.

The first option is to do nothing assuming an expected loss of $25,000 and an additional cost of uncertainty estimated as $15,000. This results in $40,000 total cost of risk and $960,000 firm value.

Alternatively the company can choose to control the risk, spending an amount of $9,000 to decrease the frequency of breakdown to 5%. New expected losses and estimated cost of uncertainty are $12,500 and $10,000, respectively. The $9,000 are called loss control costs. The total cost of risk therefore is $31,500.

Spending another $9,000 for loss control reduces the probability of breakdown to 3% and expected losses to $7,500. This is not favorable in a scenario where the cost of uncertainty remains high at $7,000. The total cost of risk accumulates to $32,500.

The last option is to buy insurance. An insurer offers full insurance for a premium of $32,000. The firm realizes the expected loss of $25,000 and has to finance the loading of $7,000.

In this example, we can see that not the minimization of risk, but the second option leads to the lowest cost or risk, and therefore the highest true firm value.

This idea paved the way for a broader concept of risk management where larger companies can reduce their reliance on insurance by internal activities that control the impact of risk and uncertainty on the organization. Strazewski (1996) describes the results of a cost of risk survey that was conducted by Tillinghast-Towers Perrin and the Risk & Insurance Management Society every year. To allow for comparison between companies, the cost of risk is related to the revenue of the company and stated in $1 per $1,000 revenue.

Financial Risk Analysis

The last decades showed that risk management focusing on pure risks is much too limited.

1. In July 1944, delegates from 45 nations gathered at the United Nations Monetary and Financial Conference in Bretton Woods, New Hampshire. Putman and Wilford (1986) describe the Bretton Woods system that was decided to be

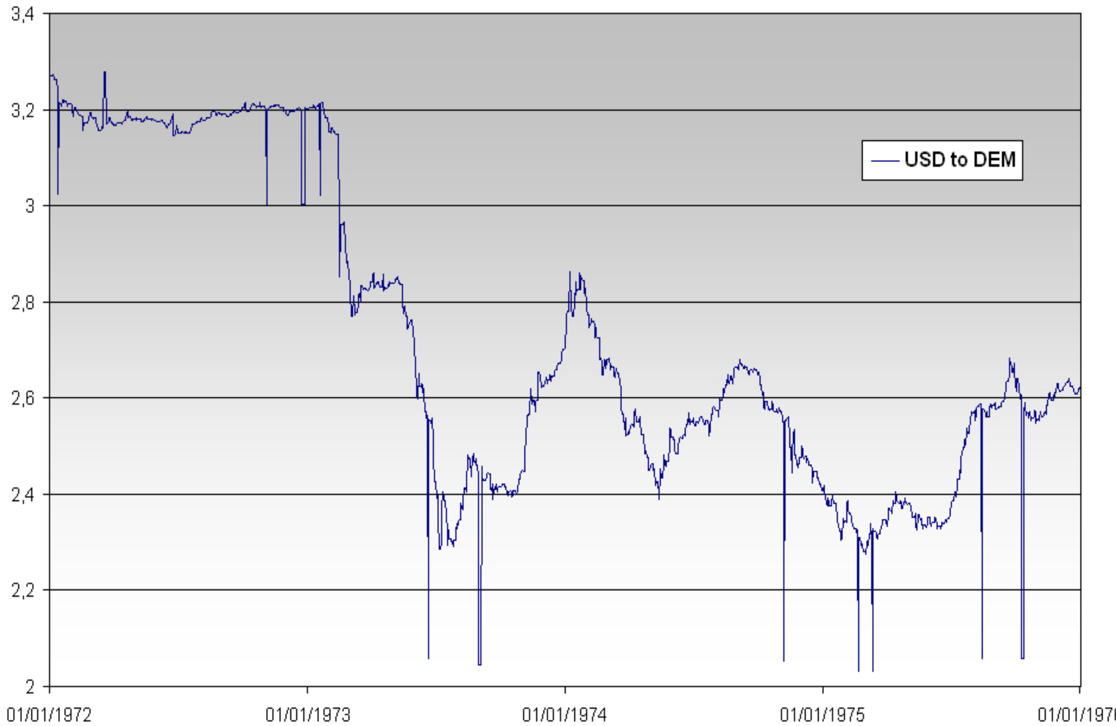
established at the conference. A key feature of this system was that all International Monetary Fund (IMF) member countries agreed on a system of exchange rates that could only be adjusted within predefined parities with the U.S. dollar. In the case of a "fundamental disequilibrium in balance of payments" the IMF could agree to change these parities (Canadian Economy Online , 2006). In the early 1970s, the fixed exchange rate system proved to be unworkable, and it was finally replaced by floating exchange rates in February 1973. Companies operating on an international basis had to encounter an increased currency exchange risk. Figure 1, p. 7, shows the effect of fixed and floating foreign currency exchange rates, considering US-Dollar to Deutsche Mark as an example.

2. As stated by MacAvoy (1982) and El-Mokadem et al (1984), the price of crude oil on the world market had been stable between \$2.50 and \$3.00 from 1948 to 1972. In October 1973, Israel started the Yom Kippur War by attacking Syria and Egypt. As a result of the support given to Israel by the United States and many other western countries, several Arabian oil exporting nations established an embargo on the supporters. The price of oil on the world market rose from around three to over twelve dollars per barrel by the end of 1974.

3. In 1979, the inflation rate in the United States had crossed the 10% barrier and was threatening to rise even higher. Taylor (2005) describes how the Federal Reserve System (FED) with Paul Volcker as its new chairman approved major increases of the discount rate to fight the inflation rate. After drastic changes of

Figure 1

Foreign Currency Exchange Rate US-Dollar to Deutsche Mark 01/01/1972 through 01/01/1976

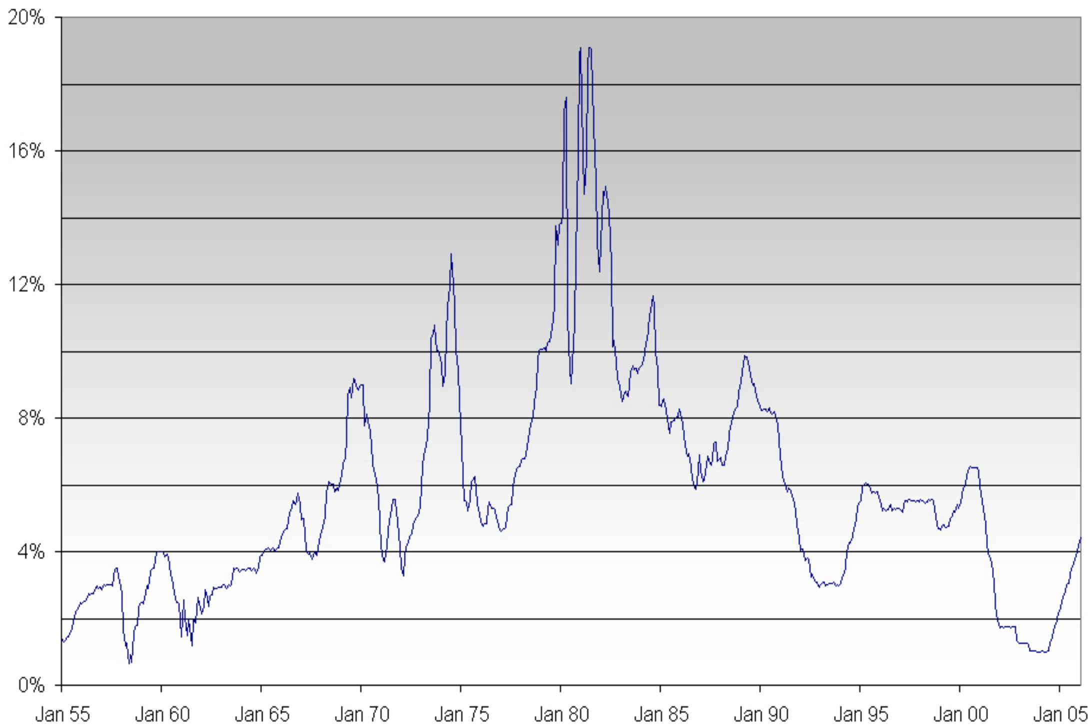


Data-Source: Interbank rate, provided by OANDA (2006)

the discount rate, the inflation finally dropped below 5% again in 1982. Interest rate affects the present value of assets and liabilities of a company that has cash flows in the future - the longer the time until the payment, the bigger the impact of the discount rate. Figure 2, p. 8, displays historical data on monthly federal fund rates. It is easy to see that the changes starting in 1979 were unique in nature up to that date.

4. In 1986 and 1987, the stock market exhibited an excellent performance. The Dow

Figure 2
Federal Fund Effective Rate (monthly) 01/1955 through 01/2006

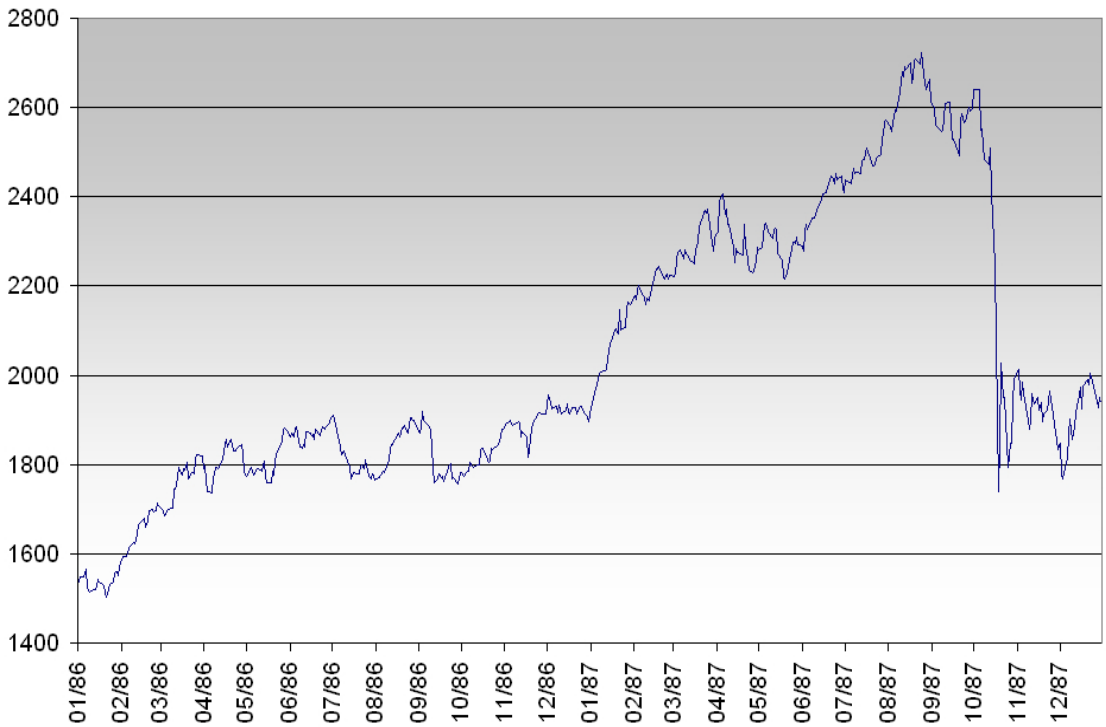


Data-Source: Federal Reserve System (2006)

Jones Industrial Average (DJIA) grew from around 1500 to 2700 points in August 1987. MacKenzie (2004) describes the course of events that were happening on Monday October 19, 1987 when the DJIA saw its largest one-day move in stock prices ever. The index dropped by 22.6%, and other major stock markets all over the world showed significant losses over the course of the week, as well. Bose (1988) comments on the changes in behavior among investors on that day: "Suddenly the money people realized the monster they had created. Risk was spread quickly, dangerously, explosively." He concludes that a lack of understanding of markets was part of the reasons for the crash.

Figure 3, page 9, shows the developing of the Dow Jones Industrial Average from January 1986 through December 1987 and illustrates that investors have to be aware of their equity risks.

Figure 3
Dow Jones Industrial Average Daily Chart (01/1986 through 12/1987)



Source: Dow Jones & Company (2006)

According to Rawls and Smithson (1989), all these environmental changes stimulated the demand for new financial instruments. In fact, forward rate agreements, future contracts, swaps, and options as defined by Panjer et al. (1998) - allowing for the transfer of risk to the financial market - had already long existed. The Chicago Board of Trade (2006) claims to have started trading future contracts in 1865, for example. Therefore, Rawls and Smithson (1989) call these financial instruments "rediscoveries" rather than "innovations" to account for both their long history and the fact that they are taken into consideration for management of financial risks in today's environment much more frequently. They have become an important tool, not only for risk management purposes, but also in investment banking, corporate strategic planning, and trading of derivatives (Panjer et al. , 1998).

"Financial Risk Management"(FRM) is a term that is often used as a matter of course. It is rarely formally distinguished from "risk management". However, FRM is, as the name implies, concerned with risks that arise from fluctuations in interest rates, currency exchange rates, and commodity or equity prices. Schwartz and Smith (1993) provide a complete overview of financial risks and the instruments that FRM can use to mitigate them.

COSO (2004) states a definition of the concept of "Enterprise Risk Management" (ERM):

Definition 1 (ERM, COSO (2004))
"ERM is a process, effected by an entity's board of directors, management and other personnel, applied in strategy setting and across the enterprise, designed to identify potential events that may affect the entity, and manage risks to be within its risk appetite, to provide reasonable assurance regarding the achievement of entity objectives."

Both FRM and ERM have their seeds in risk management. They can be described as a process, not a single action but a series of actions, continuing as long as business is made. The actions involve the identification and management of risks. However, while FRM is concerned with financial markets, ERM involves the whole enterprise, including the personnel and their actions.

The following section on "Incentives in Management" presents the position of Feldblum (2006), who argues that ERM simply, "as the name implies, [...] extends risk from the investor to the enterprise". He states that the behavior of an enterprise is different from the behavior of a stock price which makes ERM more than a simple translation of FRM.

Systematic and Diversifiable Risks

The capital asset pricing model (CAPM) is a prominent part of modern financial economics. It provides a relation between risk and return, both of which are important terms in risk management. According to Bodie, Kane, and Marcus (1999), the expected return of an individual asset can - under certain assumptions - be expressed by its covariance with the market portfolio, a risk-free rate, and the properties of the market portfolio. The market portfolio is defined as the optimal portfolio, efficiently diversified across all stocks. It contains all stocks in an amount proportional to their market value (Bodie, Kane, and Marcus , 1999).

Theorem II.1 (CAPM, risk premium)**.** *Under the following assumptions as stated and discussed by Harrington (1983)*

1. *The investor's objective is to maximize the utility of terminal wealth.*

2. *Investors make choices on the basis of risk and return, measured by variance and expected value of the portfolio's rate of return.*

3. *Investors have homogeneous expectations of risk and return; their estimates are similar.*

4. *Investors have identical single-period time horizons.*

5. *Information is costless and available to everyone.*

6. *There is a risk-free asset. Investors can borrow and lend at the risk free rate.*

7. *There are no taxes or transaction costs.*

8. *The market is limited to a fixed quantity of assets that are all marketable and divisible.*

a following formula of the CAPM is true:

Given an individual asset i and the market portfolio M, with random rates of return r_i and r_M, respectively, the risk premium of the security $\mathbb{E}(r_i) - r_f$ can be expressed by:

$$\mathbb{E}(r_i) - r_f = \beta_i(E(r_M) - r_f) \tag{2.1}$$

where r_f *is a constant denoting the risk-free rate of return and*

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}. \tag{2.2}$$

σ_M^2 *is the variance of the market portfolio.*

Proof. Derived by Bodie, Kane, and Marcus (1999). □

Harrington (1983) states that, although investors require increased returns from an asset to compensate them for tolerating the risk that the expected returns may not be realized, only the covariance of the security's returns with the market portfolio's returns (measured by the standardized factor β (2.2)) affects the size of the risk premium (2.1). By definition of the market portfolio, this part of the risk cannot be eliminated by diversification and is called "systematic risk" of the asset. Risk that can be eliminated is called "non-systematic", or "diversifiable risk", and is unique to the firm issuing the security.

Incentives in Management

The value and return of an enterprise is not as easy to describe as the value and return of a stock. This section shows the importance of incentives in the management of a firm, as well as how they relate to differences in the description.

Principal-Agent Problem

In economics, the principal-agent problem treats the difficulties that arise from different interests of the agent and the principal whom they work for. Although the principal expects the agent to completely preserve his interests and can threaten to fire him, it is difficult for him to control the agent's commitment. If anything, he can only recognize the results of the agent's efforts. The different knowledge of principal and agent on the company's decisions and their background is called asymmetric information. Usually the agent has better information on himself, internal activities, and the employees within his/her field of responsibility. The asymmetric information can be used by the agent to pursue his/her personal intentions without the knowledge of the principal.

In practice, the principal-agent problem is found in most employer/employee relationships. In a situation where shareholders hire a manager for their corporation, we can typically find occurrences of the principal-agent problem:

- Shareholders are interested in higher returns and are willing to assume any diversifiable risk that assists this goal. They avoid systematic risk that is not contributing to higher returns.

- Managers are mainly interested in securing their jobs. Senior managers, especially, might fear the inability to find a new job with a comparable income. They want to retain their job until retirement.

Example 2 (Labor strife)

In an more economically difficult situation, a company needs to anticipate increasing competition along with less demand for its main product. The labor union wants to secure the jobs and threatens to initiate a labor strife otherwise. The responsible manager has to face a conflict of interests:

- As long as the risk of a possible labor strife is unique for this company and not prevalent in the whole market, the risk is diversifiable to the shareholders and can be eliminated from their portfolios.

  To the manager, the labor strife that can be charged back to him/her is an immediate danger. Labor strife does not only cause a prompt loss from a tie-up, it also dissatisfies employees leading to less productivity. The manager can be held accountable for being unable to effectively manage the labor relation and has to fear losing his job.

- Excessive costs to buy labor peace can lead to the shut down of a factory. This does not happen directly, but over the term of years, and therefore might not effect the plant manager, but rather his successors and the shareholders. The additional costs decrease the company's return and ultimately the shareholder's return. The systematic risk, however, remains unchanged.

In a situation like this, the manager has to be partly held accountable for costs arising from securing the jobs in the long run as well. However, if he has to forecast the costs of labor strife and buying job security, he does not have the incentives to do so accurately. In order to pursue his own interest of retaining his job, he is likely to overestimate the costs of labor strife and underestimate the costs of buying labor peace.

The solutions suggested by Feldblum (2006) involves a re-estimation of costs by an ERM specialist, who also assigns a portion of each risk to the manager, while leaving the remaining part to the company. The idea behind this is to ensure: both proper incentives for the manager, and appropriate business decisions.

Example 3 (Re-estimation, Feldblum (2006))

The manager estimates the costs of a labor strife to be $10 MM and the costs to assure the jobs to be $4 MM. As illustrated in example 2, the first estimate is likely to be too large, while the second is too small. This proportion of assigned costs creates the incentive to promise future benefits (job security) while avoiding the short term risk (tie-up).

An ERM specialist, who must have an objective outlook, trusted by both the manager and the shareholders, re-estimates the costs to be $8 MM and $6 MM. The final decision must be based on these estimates.

Insurance products usually present an alternative way to deal with a company's risks by simply transferring them against their expected value and a loading. Example 4 illustrates that this alternative creates incentives for the manager that are not necessarily in line with the shareholders' ideas.

Example 4 (Insurance and reinsurance, Feldblum (2006))

An insurer sells homeowners insurance as its only line of business. The decision of how to treat Gulf Coast states that face the additional risk of hurricanes is left to the manager. Hurricanes pose a threat that is likely to affect more than just one company. Therefore the returns of the insurer will be correlated with the market, and II.1 implies that the risk of hurricanes can only be partly eliminated from the investors' portfolios.

1. The insurer can sell homeowners insurance in these states and collect an additional risk load on the premium to buy reinsurance. Costs for reinsurance involve expenses for underwriting, taxes, etc.

2. The insurer can avoid making business in areas that involve additional hurricane risks.

3. Lastly, the insurer may write windstorm coverage including the risk load but without buying reinsurance.

In the last case, shareholders have to trade off between the additional return offered by the risk load on premiums with windstorm coverage and the systematic part of hurricane risks that affect their portfolio. Depending on the actual scenario they may reject or prefer to do business in Gulf Coast states without buying reinsurance. They may prefer to do so even though the insurer has a larger risk of going broke.

Managers, rather, have the incentives to protect their firms - including their own jobs. The company cannot mitigate their risk by holding a diversified portfolio like the shareholder does. Therefore, Feldblum (2006) argues that managers have the incentive either to buy reinsurance or to avoid the business in Gulf Coast states.

To escape this dilemma, shareholders can consider paying the manager an additional premium in order that he/she will assume the catastrophe risk. In this case, both parties can gain from the transaction. However, the managers might still want to avoid business in costal areas without admitting it to the shareholders.

General approaches to mitigate the problematic nature of the principal/agent relationship involve compensations dependant on the performance of the agent. Holmstrom (1979) discusses how these compensations can to be specified in the contract. Additionally, the agent's performance needs to be monitored as effective as possible. In ERM Feldblum (2006) advises to use risk analysis to revise decisions such that they are "left with persons having the greatest incentive to reduce cost" or systematic risk, respectively.

Moral Hazard

Moral hazard describes the increased risk of a changed behavior caused by the lapse of consequences. It is imminent whenever a conflict between rational behavior in a collective and individual context occurs. It can be illustrated with some examples that this is a very common problem in insurance.

Example 5 (Car insurance)

Car insurance is bought by an individual to reduce the harm of a possible accident. Since the individual does not pay for the losses but the collective (in this case the insurer and ultimately all policy holders), incentives to drive cautiously are reduced. This results in higher costs borne by the collective than without insurance.

Example 6 (Governmental aid)

The situation is similar whenever the government serves as an insurer. Disincentives to build in areas known for catastrophic events such as hurricanes and earthquakes are removed if the government provides financial aid and rebuilding afterwards.

Example 7 ("Too large to fail")

Some very large companies are considered "too large to fail" and they rely on the government to bail them out in case of an emergency. They do not fear the consequences of failure.

Moral hazard can also be seen on a large scale for the insurance industry. The National Organization of Life & Health Insurance Guaranty Associations (NOLHGA , 2006) lists 52 different life & health insurance guaranty associations in the United States, one for every state, the District of Columbia, and Puerto Rico. The states require all insurance companies that are licensed to write business to be members of their guaranty association. NOLHGA (2006) states that in case of insolvency of one of the member companies, the state guaranty association continues to cover individual policyholders and their beneficiaries within certain limits. State guaranty associations do not only exist for life & health insurance. Property and casualty guaranty funds are gathered in the National Conference of Insurance Guaranty Funds (NCIGF , 2006).

Feldblum (2006) argues that "guaranty funds help fuel the insolvencies, not mitigate them". Before the existence of guaranty funds, the agents issuing policies had a strong disfavor with risky insurers. In case of the insurer failing to pay a claim, the agent could be sued for negligence by the policyholders. In order to find agents to do business for them, insurers had to prove their reliance with a strong balance sheet. Guaranty funds override this incentive for the agent. Risky insurers that are covered by guaranty funds can possibly even offer higher returns to their policyholders, making them more attractive for agents as well.

ERM has to take into account that insurance reduces the incentives to control the risk. The party that is able to control the risk does not bear the consequences if the loss is insured. The conclusion presented by Feldblum (2006) is that "risk should reside with the party most able to reduce it". However, insurers are usually able to better diversify risk as they hold large portfolios of different types of risk.

Interplay of Risk

Large companies often consist of several lines of businesses. These sectors are initially treated separately, but need to be viewed in an overall interrelationship. As an example, the company Siemens (2006) specifies its eight different lines of business as mining, metal production, steel mill operation, pulp and paper, oil and gas, water, commercial shipbuilding, and navy.

In addition companies are managed in several divisions like production, sales, financing, accounting, controlling, human resources, marketing, and more. The separation of responsibilities simplifies decision making at first glance, but clearly also bears the danger of losing the overall picture. Risks are often interrelated and controlling the interrelations gets increasingly difficult in the process of segmentation.

Exogenous and Endogenous Risks

When observing the interplay of risks among different lines of business of a company, we have to differentiate between exogenous and endogenous risks.

Definition 2 (Exogenous, endogenous risk)
Given a system and events that pose a threat to this system. Exogenous risks are incidents generated outside of the system that may have negative effects on the system. By contrast, endogenous risks have their source inside the system.

Example 8 (The Millennium Bridge)
The official induction of the Millennium Bridge crossing the Thames in London on June 10th 2000 had to be aborted due to the wobble that started once passengers trod on the bridge. Danielsson and Song Shin (2002) provide an overview of the subsequent

analysis and relate the results to the concept of endogenous risks.

While the engineers paid very close attention on the bridge's responses to storms and earthquakes when planing the construction, they forgot to think about the effect of a thousand people walking in step on the bridge. Storms and earthquakes have their source within the earth and in weather changes, respectively, clearly outside the system described by the bridge and the passing traffic. They are considered exogenous risks. The thousand of visitors that slowly went on the bridge during the opening ceremony were almost walking in step, everyone creating the same vertical force with their steps. This caused the bridge to wobble. The source of the undesirable movement was the passing traffic and the construction that did not account for it. Therefore it was an endogenous risk that was even amplified with every new passenger stepping on the bridge.

In a company exogenous risks have to be carefully handled. They effect different - if not all - parts of the business. Risks as interest rate or inflation risk are of exogenous nature and may possibly be treated as a factor common to all parts of the business rather than specific (and uncorrelated) risks for financing, sales etc. With the experience of past events it is usually possible to come up with a fairly sophisticated model for exogenous risks.

As illustrated in example 8, the endogenous risk was amplified within the system. The same may be true of endogenous risks of a firm. Managers react to changes in their environment including changes based on the decisions of other managers. How distress can feed on itself can be shown with an example of the financial markets discussed in more detail by Danielsson and Song Shin (2002).

Example 9 (Financial market)

When dealing with securities in the financial market, a lot of investors want to limit their possible losses. Often this is a "stop-loss" rule that triggers to sell the security once its price drops beyond a certain threshold. The depreciation of asset prices in the stock market may cause more investors to sell their stocks as prices get closer to their limits. Trying to sell even more stocks can amplify the downward movement.

Since management systems are in place to deal with exceptional risks that are often hard to model, ERM has to consider how interaction of managers within the company can affect the overall risk of the firm.

Prospects

Can ERM provide value to a firm? Modigliani and Miller (1958) state one of the basic theorems on the relationship of capital structure and the value of a firm. Under the assumptions of

1. absence of taxes
2. no bankruptcy costs

3. no asymmetric information

4. complete and efficient capital market

two companies that are financed differently and are identical otherwise have the same value. If ERM is viewed as a tool of financing it should not affect the value.

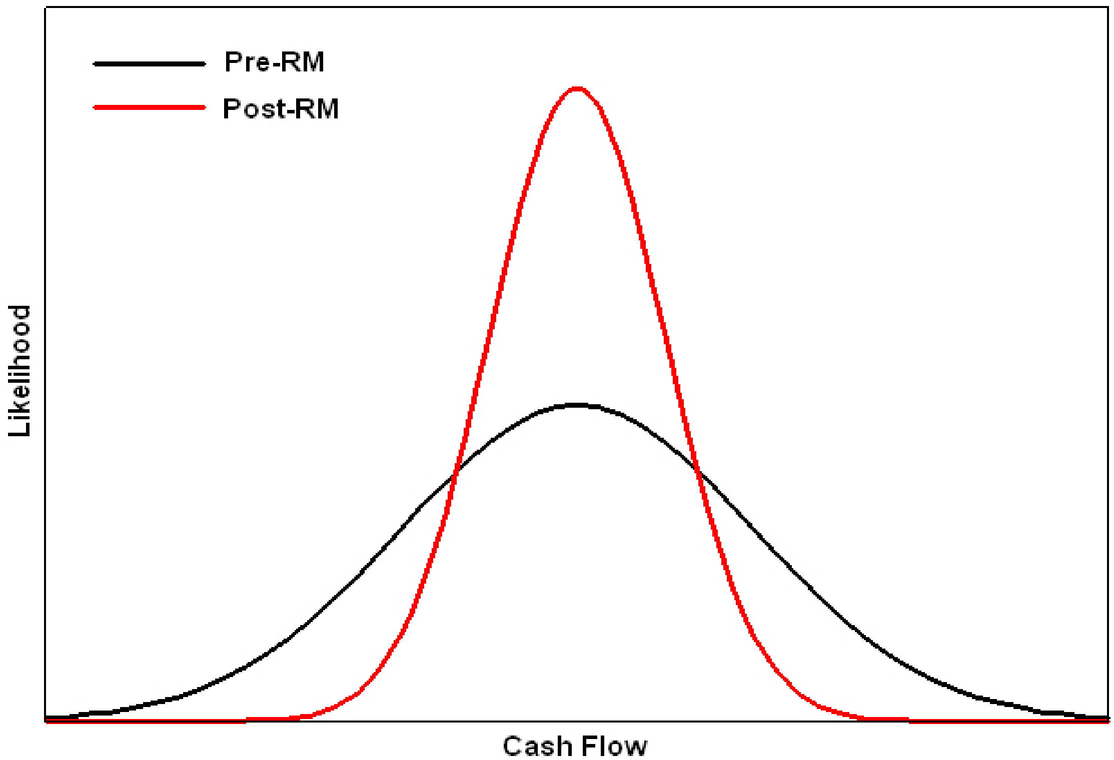
However, since the assumptions for the theorem are usually violated in practice ERM can still have a significant impact. We can illustrate this with an Example.

Example 10 (Creation of value)

Risk management, in general, is a tool used to control and alter the probabilities of financial outcomes. Regulation may require a company to insure against solvency-threatening scenarios or provide a certain amount of capital to support their risks.

By diversification inside the company - e.g. redistributing the responsibilities of decision-making - risk management may achieve a new distribution of outcomes with less probability of extreme losses or gains. Figure 4, p. 25, shows the exemplary impact of risk management on cash flow volatility. The new probability density function looks more compressed with the main probability mass centered around the mean. As a result the requirements of capital decrease and the company may be able to handle the risk without buying (re-)insurance, saving transaction costs. Similarly since income is less likely to be very high, the expected cost of taxes may be smaller in a scenario where higher incomes correspond to higher tax rates. We can view this as an increase

Figure 4
Volatility Improvement Using Risk Management



Source: author's calculation, based on Gorvett and Nambiar (2006)

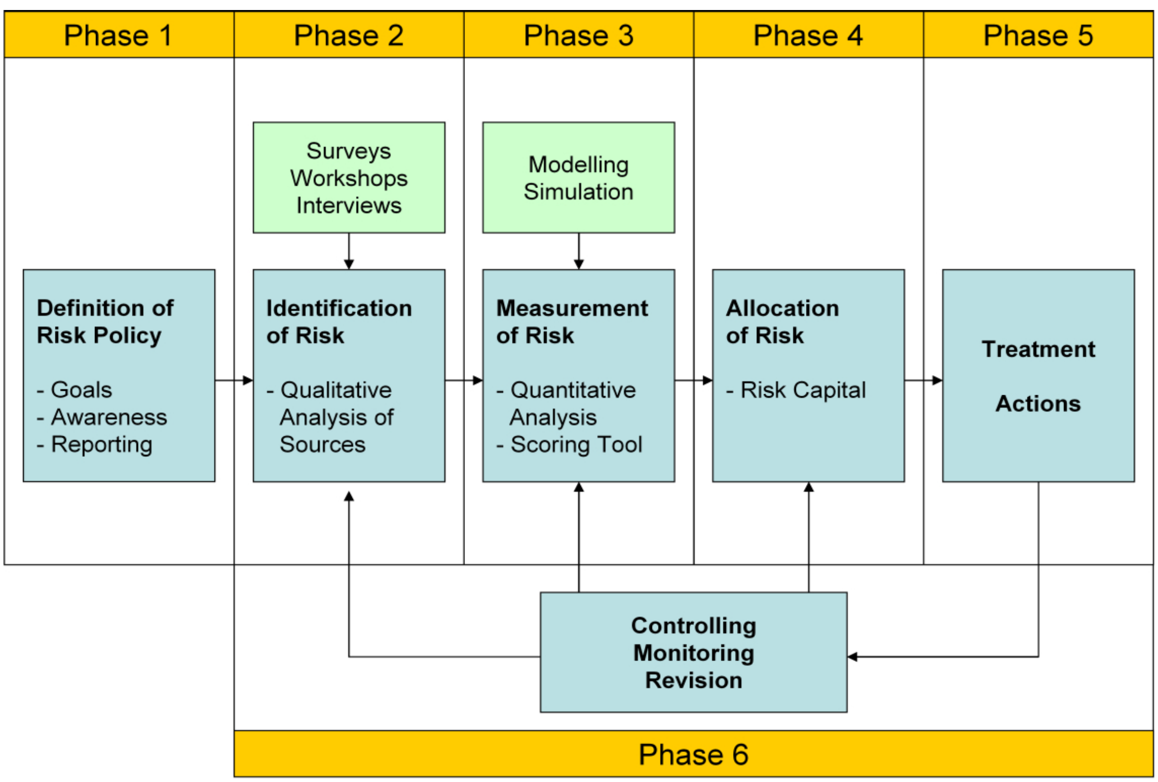
in value to the firm.

CHAPTER III

PHASES OF THE MANAGEMENT PROCESS

Enterprise risk management needs to be addressed on several stages. The process can be divided into phases, starting with organizational preparations and awareness, and leading to treatments that consider the whole firm. Figure 5, page 26, shows the systematic structure of the process. The initial phase is the definition of a risk policy.

Figure 5
Framework of ERM



Source: author's figure based on Aon (2006)

This involves structural decisions about the organization of the firm that is further discussed in the section on "Organizational Structure".

The next important step is the search for causes. Identifying the source of a risk is essential to come up with the correct actions. Identification and classification of risks into different categories is addressed in the section on "Identification of Risk".

Once risks are identified, they need to be quantified ("Measurement of Risk") and allocated ("Risk Allocation") to allow for a proper distribution of capital.

As indicated in figure 5, these steps have to be monitored and regularly checked for revision.

Organizational Structure

As seen in Chapter II, ERM has gained a status of growing importance for large companies. Not only its potential ability to add value to firm and shareholders, but also statutory requirements have contributed to this fact. The German Stock Corporation Act (AktG , 2005) requires the management of companies that are quoted on the stock exchange to establish a monitoring system that precociously analyzes the risk of the company. Additionally, the German Commercial Code (HGB , 2005) requires companies to state and evaluate their primal chances and risks as part of the management report.

In order to analyze a company's structural framework of their risk policy, we might raise some initial questions:

1. What risk-strategy does the company act on and what are its goals?

2. How is risk-awareness taught and promoted? Are there courses of instruction for both managers and other employees?

3. When and to whom are risks reported?

Lam (2003) considers some examples on how these questions were realized by companies that revised their risk management system and draws conclusions. In his opinion, an effective risk management system should require individuals or groups of individuals in key positions to provide checks and balances to justify their decisions and disclose the results. This prevents these people from gaining too much power to assume any risk on behalf of the company. Lam (2003) also considers the existence of limits and boundaries as an important part of a company's risk policy. He compares a company without clear limits to the driver of a race car without brakes. Once any business ratio passes its legal limit, it should be subject to intensive investigation. The balanced scorecard, as introduced as a measurement of a company's success by Kaplan and Norton (1992), can also be used for risk control.

Enterprise Risk Management Office

Projects that require the participation and coordination across the entire company become increasingly difficult with growing company size. Rad and Levin (2002) describe how Project Management Offices (PMO) have often contributed to the success of projects on a company-wide level. Since ERM's objective is to manage the overall risk, an Enterprise Risk Management Office (ERMO) is clearly another institution that requires actions on the ”macro-level” of the company. In their approach on how to set up an ERMO, Gorvett and Nambiar (2006) list key principles of an ERMO analogous to their characterization of the PMO.

1. ERMO needs a widely recognized charter, including a mission statement that reflects the individual risk policy of the company.

2. The chief executive officer and the Board of Directors (or equivalent) should be sponsors of the ERMO within the organization.

3. The governance should be embedded into an adequate reporting system. The ERMO also requires an independent evaluation and assurance process.

4. A risk management culture should be embedded into the firm. It is most essential that the awareness of risks and its communication is present throughout the company.

5. An education program should help teach firm management and employees the purpose and benefits of an ERMO and the whole risk management process.

The key principles are completed by a couple of more characteristics, such as a risk monitoring system that is frequently evaluated. Another important aspect includes the transparency of information among the departments that are represented by members in a company-wide communication system for risk management purposes. The list of benefits arising from an effective ERMO specified in Gorvett and Nambiar (2006)’s paper include:

1. The greater efficiency in managing risk and their interrelations can help to lower costs ultimately.

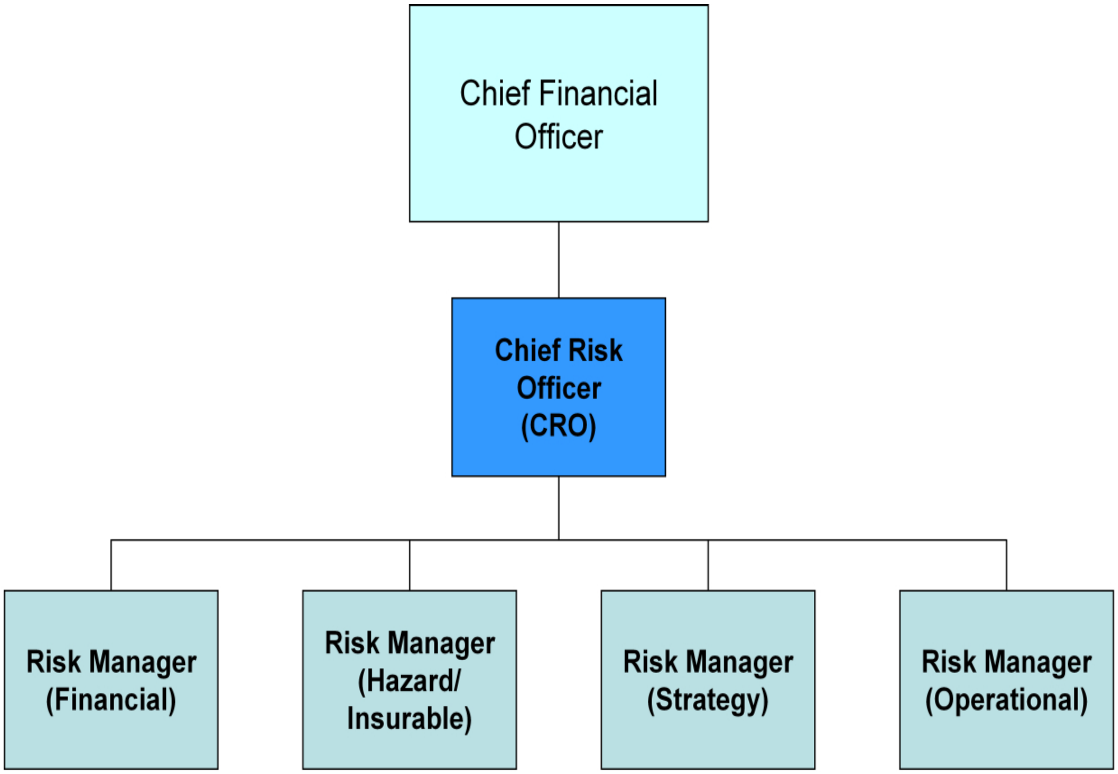
2. A better understanding of the firm's chances and risks makes meeting the firm's objectives more likely.

3. The continuing discussion of risks that are necessary for management on a firm-wide basis reduces boundaries and encourages cross-boundary communication in general.

4. The participation in a process that everyone is exposed to may improve the employee morale.

Figures 6, p. 31, and 7, p. 32, present two different approaches to integrate the ERMO into the company. While figure 6 displays the risk officers as part of the Chief Financial Officer's (CFO) field of responsibility, and with the Chief Risk Officer (CRO) as their direct superior, figure 7 shows four different hierarchical levels. Gorvett and Nambiar (2006) state that the reporting line of the CRO to the Chief Executive Officer (CEO) and the Board of Directors is important to avoid conflicts of interest. In addition, the chief officers are displayed on one level with a dotted connection indicating their interaction to ensure a holistic management.

Figure 6
ERMO Organizational Chart (COSO)



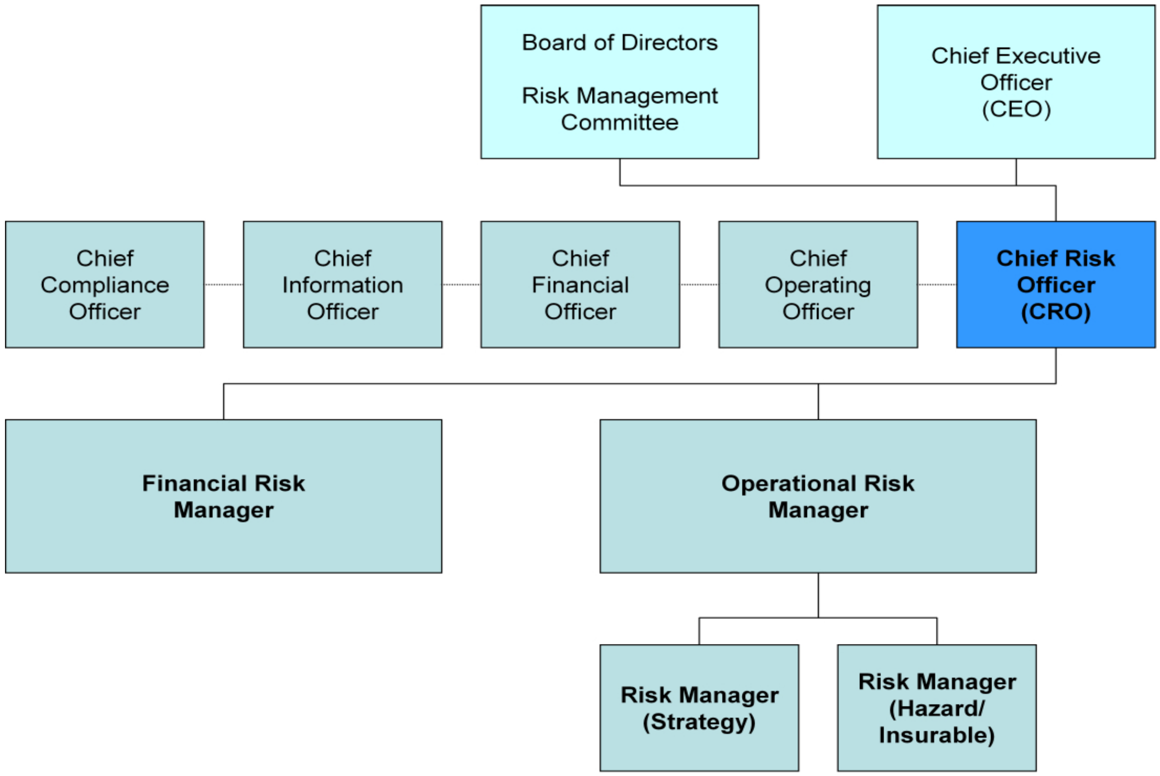
Source: Presented by Gorvett and Nambiar (2006) and based on COSO (2004)

Chief Risk Officer

As seen in figures 6, p. 31, and 7, p. 32, the Chief Risk Officer (CRO) fills an important position in modern enterprise risk management concepts. The role of a chief officer being responsible, "for developing and implementing an ERM strategy including all aspects of risk" (Lam , 2003), has been widely adopted in risk-intensive businesses, not always using CRO as a title. Lam (2003) specifies a list of direct responsibilities of the CRO:

1. As the key player in a company's ERM, the CRO has to provide a company-wide

Figure 7
ERMO Organizational Chart (functional)



Source: Gorvett and Nambiar (2006)

leadership that promotes his visions and ideas on risk awareness and handling of risks.

2. Since ERM deals with all possible risks of a company, the CRO has to establish an integrated risk management framework that takes into account all aspects of risk across the organization.

3. The CRO has to specify risk limits and a measurement of it for decision makers throughout the organization to ensure a measured assumption of risk.

4. The CRO is responsible for a set of risk indicators, and their reporting, to aid giving early warnings and finding the firm's key exposures.

5. The allocation of capital to optimally support different lines of business, depending on their interplay with different parts of the company's business, lies within the CRO's responsibility.

6. The company's risk profile has to be presented to different interested parties like stakeholders, regulators, and business partners. The CRO is responsible for management reports or public statements on the company's risk profile.

7. The risk management has to be constantly supported by data collection and analysis. The CRO has to develop and provide tools to maintain and improve this process.

The CRO's role is not indisputable. Especially the need for another position on the chief officer level has been questioned several times. Since the Chief Executive Officer (CEO) is always the one ultimately responsible for the firms risk performance, and part of the risk management is already done by the Chief Financial Officer's (CFO) organization, it makes the CRO appear to be redundant. However, Lam (2003) argues that the role of the CRO, "represents a core competency that is critical to the success for the company." While the risk management is just an implied part of the CEO's profession, ERM needs a person explicitly in charge. Depending on the size of a company, this person does not always need to pursue ERM as a full-time job.

Identification of Risk

Identifying the correct sources of risk factors is of major importance to a company. Business decisions often do not only involve one kind of risk, but also the interrelation of different risks in several divisions of the company:

Example 11 (New product)

Let us consider a company that wants to launch a new product or business in a foreign country. Lam (2003) identifies four required actions that involve risks for the company.

1. The pricing and the strategy to enter the market has to be developed by the business unit. This involves the risk that sales may be lower than expected due to incorrect rating.

2. The funding for the new product is dependent on interest rate scenarios as well as foreign exchange rates. These are clearly financial risks the firm is exposed to.

3. The new business needs to be supported by the information technology and operations function of the company that can be the source for losses if not working as intended.

4. Regulation may have new requirements that need to be addressed. This involves legal risks that are of special importance if the company has never done business in that country before and may be unfamiliar with the system.

In example 11, we could see that the single risk starting from the launch of a new product/business can be broken down into more individual risks. They may still not be the source, but they can be specified further. Sometimes it is hard to identify the main sources of risk, as can be illustrated by another example.

Example 12 (Catastrophe risk)

A homeowners insurer identifies exposure to hurricanes in the gulf region as a main source of risk to his business. He may not realize that a large portion of this risk is due to the fact that his portfolio of wind coverage is not as diversified as it could be. In fact, the small business volume in a single state caused by mispricing, for example, may have had a huge impact on the risk attributed to hurricanes.

Some helpful tools to recognize a company's risk and identify the sources include surveys and interviews conducted among the employees. If workers are trained properly in workshops, they can have a better appreciation of the risks that they are dealing with than inspectors.

Example 13 (Workers experience)

In a production unit of a company, workers monitor the automated processes every day. Their awareness of an altered risk of machine breakdown of an object that is getting older may be important to update risk forecasts. Workers also need to be aware of their role and responsibilities in managing these risks.

Risk Categories

Risk is usually categorized according to its sources. Depending on the type of industry, or even more specifically the company, categories may differ in importance and content. If a risk report is required by law or regulation, content and nomenclature of the categories is generally defined to provide uniformity and avoid misinterpretation.

The Basel Committee on Banking Supervision (2005) has proposed a framework for measuring capital adequacy of banks depending on their risk profile. Similarly, the COSO (2004) issued a more general framework to help companies evaluate and improve their ERM. An alternative set of standards for risk management is offered by the Standards Australia and Standards New Zealand (2004) committee.

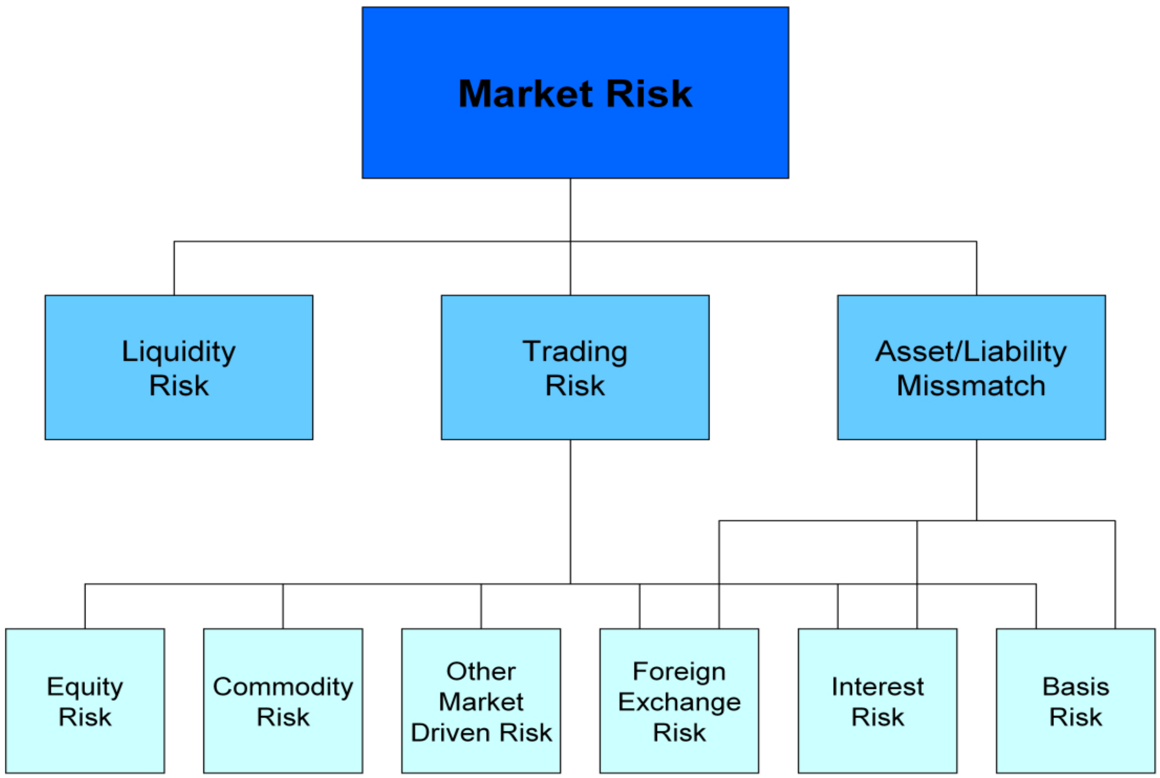
The most common main categories of risk are described by Lam (2003): market risk, credit risk, and operational risk.

Market Risk

Most companies are exposed to some kind of market risk. It involves the exposure to losses resulting from changes in market prices and rates. When mentioning "financial risk" earlier, it often had the same meaning as market risks; referring to changes in interest rate, foreign currency exchange rate, or commodity prices. As an example, we can think of an energy firm that is operating an offshore platform to obtain crude oil in a foreign country. The company's revenue is largely dependent on the current exchange rate as well as commodity prices for crude oil and petroleum or fuel.

Figure 8, p. 37, shows an overview of the different kinds risks that are summarized under the keyword ”market risk”.

Figure 8
Types of Market Risk



Source: Lam (2003)

The three major parts of market risk are given as liquidity risk, trading risk, and asset/liability mismatch. While trading risks usually have a short term character and evolve from changes in the trading portfolio, asset/liability mismatches are often long term risks. Mismatches arise from a different sensitivity to interest rate, typically measured by duration and convexity (Gajek and Ostaszewski , 2004), of assets and liabilities. They are of major concern to banks and insurance companies.

Lam (2003) describes liquidity risk as the risk that a company will not be able to meet its obligations as they come due. It is a common type of risk among all companies. They may incur losses to raise the funds necessary to meet the obligation as they increase their liabilities or convert their assets.

Figure 8, p. 37, also shows that trading risks and asset/liability mismatches can be further broken down into individual risk types. While some were already introduced or are self-explaining, basis risk and other market driven risks might need some illustration. Basis risk originates from relative changes of two rates. The example used by Lam (2003) involves the prime rate charged by banks to borrowers considered most creditworthy and the "London Interbank Offered Rate" (LIBOR) that is used as reference for interest rates. Other market driven risks are explained by Lam (2003) as additional risks (e.g. option risks or exposures to real estate prices).

Credit Risk

While credit risk usually refers to situations in which an institutional or individual borrower is unable to repay his loan due to bankruptcy, Lam (2003) also considers it credit risk if losses occur as a result of the counterparty failing to fulfill its obligations in a timely manner. Obviously, financial institutions such as banks are dealing with a large amount of individual credit risks, and are very concerned to diversify their portfolio of borrowers, however almost any firm has to deal with counterparties involving some kind of contractual obligations, and therefore needs to recognize their credit risk.

Operational Risk

Operational risk is nothing new. Lam (2003) states that although it was not recognized as a risk factor, and has been managed on an informal level for a long time, businesses always had to deal with human failures, processes that did not achieve their objectives, or flaws in technologies that led to malfunctions. There are prominent examples for the huge impact that unnoticed operational risks can have. Probably one of the best known is documented by Leeson and Whitley (1996). Barings Bank collapsed on February 26, 1995, due to Nick Leeson's losses of $1.4 billion. The true outcome of his speculations on the Singapore International Monetary Exchange (SIMEX) was hidden by fictitious gains that his unit was reporting. While this was truly a case of human failure and fraud, the management failed as well because they were unable to get the correct overview of their Singapoore trading operations. Finally, the Nikkei dropped significantly after an earthquake, causing the bankrupcy of England's oldest merchant banking company.

Lam (2003) mentions that today, operational risk management has been widely accepted as a discrete area of risk management. Instead of characterizing it as "not credit or market risk" - which was the initial approach - operational risk has to be clearly defined to allow measurement. The results of a study by BBA, ISDA, PWC, and RMA (1999), suggest that industry sources converged to one common definition:

Definition 3 (Operational risk)

"Operational risk is the risk of direct or indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events."

Although this definition is mostly agreed upon, there is still room for varying interpretations. The Basel Committee on Banking Supervision (2005) explicitly includes legal risk, but excludes strategic and reputational risk from their definition of operational risk.

Regardless of the exact defintion, the goal of operational risk management is probably the same for every company. Lam (2003) identifies three clear benefits that effective operational risk management can have:

1. Operational risk management should minimize both: frequent losses appearing on a day-to-day basis, and at the same time, "reduce the potential for occurences of more costly incidents." More costly incidents even include bankrupcy (as seen in the previous example on page 39).

2. Operational risk management strengthens a company in achieving business goals, such as maximizing their revenue-generating activities rather than managing crisis situations.

3. Accounting for operational risk management helps to get a more complete picture of the risks and rewards in different lines of business. This is necessary for a sophisticated ERM that interrelates all kinds of risk.

Other Risks

Depending on the exact definitions, there can be more risk categories. For example, losses arising from more competition or smaller profit margins are often summarized as business risks. Reputation and the losses associated with disreputability can also be treated as a separate facet of risk (Basel Committee on Banking Supervision , 2005).

Measurement of Risk

In business there is a saying that, ”you cannot manage what you cannot measure”. Measurements provide a fundamental aid in decision making. However, it is very important that any decision maker understands the tools that he is using. Misunderstandings might easily lead to wrong decisions, leading the agent to believe that everything is taken care of. New measurements might also bear the danger of not being generally accepted and appreciated because of their more complex, and therefore, intimidating nature.

In the following two sections I want to discuss the technical and practical quality of different measures and allocation methods using Kaye (2005) as a guideline.

Evaluating Measurements

When evaluating measurements we always have to keep in mind that there is no uniquely best measure. Different stakeholders have different interests. From a regulatory point of view, the downside of the financial outcome is always more important. Regulators are mainly interested in the likelihood of a company going bankrupt and not being able to meet their obligations. A common look at a company's risk is in terms of how much capital is required to support a given exposure. Managers might share this perspective, to some extent, since they are usually interested in retaining their job, but for investors - as discussed in Chapter II - this may be quite different. From their perspective, measures that favor high average returns might be preferred even if this includes an increased danger of total loss.

In addition, measures might vary in the time horizon that is considered. Managers and investors usually share a higher interest in short-term performance, but regulation is also concerned about events that lie far ahead. This is especially true for insurance companies that hold liabilities with possible cash flows within the next several decades.

We can conclude that it is typically best to speak of quality of a measure in terms of how well it fits a certain interest.

Technical Properties

The term ”risk measurement” usually refers to a broad range of different concepts. In order to compare alternatives of measures and to understand their limitations, it is helpful to define risk measures and to identify some basic properties. The concept of coherence is thereby commonly used to represent a set of reasonable principles. The definitions are stated such that they reflect the descriptions by Kaye (2005).

Definition 4 (Set of risks)

Let $(\Omega, \mathfrak{B}, P)$ *be a probability space. A set* $\mathcal{X}$ *of (almost surely) bounded random variables* $X : \Omega \to \mathbb{R}$ *denoted by* $\mathcal{X} = L^\infty(\Omega, \mathfrak{B}, P)$ *is called* ***set of risks****. Positive values of the random variable* X *are treated as gains. Denote the identity*

$I(\omega) = 1 \quad \forall \omega \in \Omega \qquad$ *by* $I \in \mathcal{X}$*.*

Definition 5 (Risk measure)

A mapping $\mu : \mathcal{X} \to \mathbb{R}$ *defined on the set of risks* $\mathcal{X}$ *is called a* ***risk measure****.*

Definition 6 (Coherent risk measure)

A risk measure $\mu : \mathcal{X} \to \mathbb{R}$ *is considered to be coherent if and only if it fulfills to following properties:*

1. *Sub-Additivity*

   Combining two portfolios X *and* Y *should not create additional risk. The combined risk, however, can be less than the summation of the two individual risks due to diversification benefits.*

$$\mu(X + Y) \leq \mu(X) + \mu(Y), \quad X, Y \in \mathcal{X} \tag{3.1}$$

2. *Monotonicity*

   If the financial outcomes of a portfolio X *are never less than the outcomes of another portfolio* Y*, then the portfolio* X *cannot be riskier than portfolio* Y*.*

$$X(\omega) \geq Y(\omega) \quad \forall \omega \in \Omega \quad \Rightarrow \mu(X) \leq \mu(Y), \quad X, Y \in \mathcal{X} \tag{3.2}$$

3. *Positive Homogeneity*

   Scaling a portfolio X with a constant factor λ will result in the same scaling of the risk. This especially means that adding another identical portfolio to a portfolio X will double the risk.

$$\mu(\lambda X) = \lambda \cdot \mu(X), \quad X \in \mathcal{X}, \lambda \in \mathbb{R}_+ \tag{3.3}$$

4. *Translation Invariance*

   Adding a risk-free portfolio $a \cdot I$ to a portfolio X reduces the risk of the portfolio by the amount a.

$$\mu(X + aI) = \mu(X) - a, \quad X \in \mathcal{X}, a \in \mathbb{R} \tag{3.4}$$

Note that risk measures are also often required to return non-negative results that are not greater than the maximum loss, if applied to a loss-only situation.

Calculation Background

For illustration of the risk measures we can use a fictional company "Utopia" that consists of three different lines of business. These lines are considered portfolios of the company and their returns are denoted by the random variables X_1, X_2, and X_3.
$X_i \quad i = 1, 2, 3$ are identically distributed with

$$X_i = 240 \cdot (1 - 0.3) - Y_i \qquad i = 1, 2, 3 \tag{3.5}$$

where Y_i follow a lognormal distribution (Klugman, Panjer and Willmot , 2004).

$$Y_i \sim \text{lognormal}\ (\mu = 5,\ \sigma = 0.25) \qquad i = 1, 2, 3 \tag{3.6}$$

Y_1 and Y_2 are assumed to be perfectly correlated while, Y_3 is uncorrelated with both Y_1 and Y_2. Their pairwise coefficients of correlation are

$$\varrho_{12} = 1, \quad \varrho_{13} = 0, \quad \varrho_{23} = 0. \tag{3.7}$$

We can think of Utopia as an insurance company collecting a certain premium of 240 in every line of business. For simplification we just consider expenses that are charged as a percentage of premium. The expense ratio used is 30%. The random variables, Y_i, represent the aggregate claim sizes during the period (assumed to be a year) that is considered. To model the overall financial result, we make use of the knowledge on correlations.

Theorem III.1. *Let $a, b \in \mathbb{R}$ be real numbers, and X_1, X_2 random variables with $0 < Var(X_1), Var(X_2) < \infty$. It is valid*

$$\varrho(X_1, X_2) = 1 \Leftrightarrow X_2 = aX_1 + b \tag{3.8}$$

where

$$a = \varrho(X_1, X_2)\frac{\sqrt{Var(X_2)}}{\sqrt{Var(X_1)}}, \quad b = \mathbb{E}(X_2) - a\mathbb{E}(X_1) \tag{3.9}$$

Proof. Available in Tucker (1962). □

Using the results in (3.8) on our example with (3.7) we get

$$\begin{aligned} \varrho_{12} = 1 \quad &\Longleftrightarrow \quad Y_2 = aY_1 + b \\ &\overset{(3.6),(3.9)}{\Longleftrightarrow} \quad Y_2 = Y_1 \\ &\Longleftrightarrow \quad Y_1 + Y_2 = 2Y_1 \end{aligned}$$

and since

$$\ln(2Y_1) = \ln 2 + \ln(Y_1) \sim \mathcal{N}\left(\mu + \ln 2,\, \sigma\right)$$

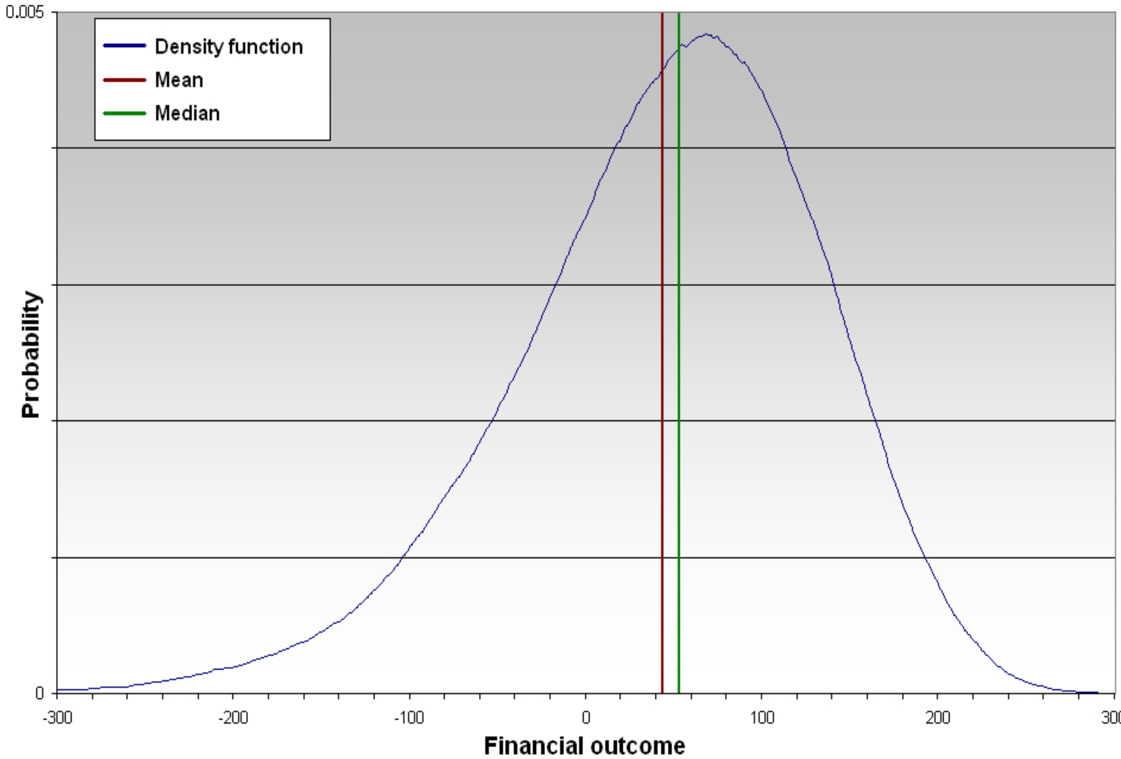
we conclude

$$Y_1 + Y_2 \sim \text{lognormal}\ (5 + \ln 2,\, 0.25). \tag{3.10}$$

Figure 9, p. 47, shows the estimated probability density function of Utopia's financial outcome $X = X_1 + X_2 + X_3$. The estimation is based on the simulation of total losses $\sum_{i=1}^{3} Y_i$ by 50000 pairs of uncorrelated random variables using (3.6) and (3.10). For density estimation, a uniform kernel with bandwidth 25 was used. Information on kernel smoothed density estimation is provided by Broverman (2005).

In our calculations we assumed to know the exact distribution of the individual losses without considering risk that evolves from the fact that these assumptions may be wrong. In comparing measurements this is a reasonable simplification. In practice however, the challenge to create a sophisticated model and take into account the risk of uncertainty still remains in place.

Figure 9
Utopia: Probability Density Function of Fiancial Outcomes



author's calculation

Point Measures

Point measures summarize a class of techniques that utilize the value of the distribution of outcomes at a single point.

Definition 7 (Quantiles)

Given $X \in \mathcal{X}$ *and a significance level* $\alpha \in (0, 1)$*:*

$$q_\alpha(X) = \inf\{x \in \mathbb{R} : P(X \leq x) \geq \alpha\} \text{ is the } \textbf{lower } \alpha\textbf{-quantile} \text{ of } X. \tag{3.11}$$

$$q^\alpha(X) = \inf\{x \in \mathbb{R} : P(X \leq x) > \alpha\} \text{ is the } \textbf{upper } \alpha\textbf{-quantile} \text{ of } X. \tag{3.12}$$

Using quantiles, the answer to the question, "what is the maximum potential loss that a portfolio can suffer in the best $(1-\alpha)$ best cases within a time period t", can be stated using

Definition 8 (Value at risk)

Given the length of a period $t > 0$ and a significance level $\alpha \in (0, 1)$. The monetary amount VaR is defined such that

$$VaR_\alpha(X_t) = -q^\alpha(X_t) \tag{3.13}$$

*where $X_t \in \mathcal{X}$ denotes the the random variable at time t. VaR is called **value at risk** or **probable maximum loss** (PML) at significance level α.*

The negative value of the quantiles and the VaR, as a special case of them, are typical examples for point measures. But the shortfall probability, the probability that losses exceed a specified threshold, can also be used as a measure.

Definition 9 (Shortfall probability)

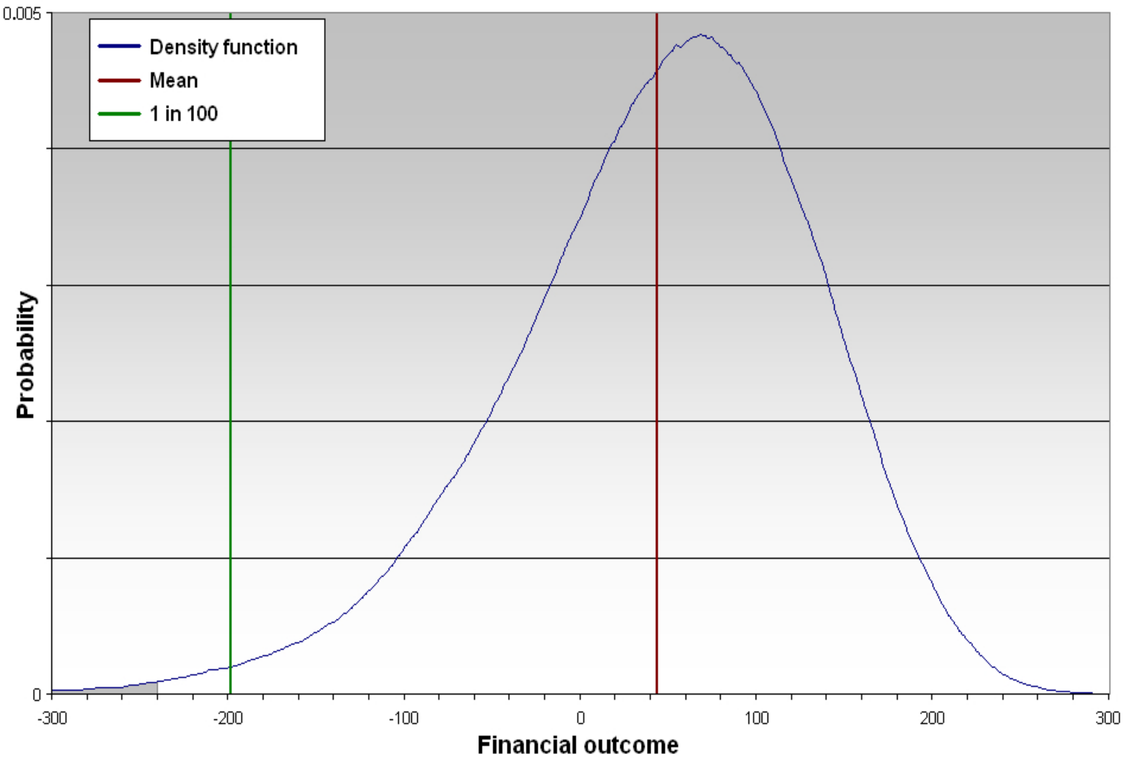
*Given a threshold level $x \in \mathbb{R}$. The **shortfall probability** p_{sf} is given by*

$$p_{sf} = P(X < x) = 1 - P(X \geq x). \tag{3.14}$$

*p_{sf} is also called **risk of ruin** or **probability of ruin**.*

Note that $p_{sf} \in [0, 1]$. It is not a measure of how much extra cash is needed to support a portfolio and therefore cannot satisfy the translation invariance.

Figure 10
Utopia: PDF, "1 in 100" and Threshold



author's calculation

Figure 10, p. 49, shows the VaR at a 1% significance level to be 197.44. With $t = 1$ year, Utopia expects to face losses larger than 197.44 once every 100 years ("1 in 100" result). Similarly, the shaded area under the density function indicates the probability of facing a loss larger than a threshold of 240 to be 0.396%. This is Utopia's shortfall probability.

Technically, point measures are very narrow. They focus on a single point, neither taking the information available on the tail, nor on other parts of the distribution into account. Kaye (2005) suggests not to use them without assistance of further metrics, as their sole use implies a serious danger of not catching the whole picture and drawing wrong conclusions about the unknown parts of the distribution.

Theorem III.2 (VaR, properties)**.** *VaR fulfills monotonicity, positive homogeneity, translation invariance, but not sub-additivity.*

Proof. Given $X_1, X_2 \in \mathcal{X}$.

$$\begin{aligned} & X_1 \geq X_2 \\ \Rightarrow \quad & \{x : P(X_1 \leq x) > \alpha\} \subset \{x : P(X_2 \leq x) > \alpha\} \\ \Rightarrow \quad & \inf\{x : P(X_1 \leq x) > \alpha\} \geq \inf\{x : P(X_2 \leq x) > \alpha\} \\ \Rightarrow \quad & -\inf\{x : P(X_1 \leq x) > \alpha\} \leq -\inf\{x : P(X_2 \leq x) > \alpha\} \\ \Rightarrow \quad & VaR_\alpha(X_1) \leq VaR_\alpha(X_2) \end{aligned} \tag{3.15}$$

And with $\lambda > 0$ ($\lambda = 0$ is the trivial case) we have

$$\begin{aligned} VaR_\alpha(\lambda X_1) &= -\inf\{x : P(\lambda X_1 \leq x) > \alpha\} \\ &= -\inf\{x : P(X_1 \leq \left(\frac{x}{\lambda}\right)) > \alpha\} \\ &= \lambda \cdot -\inf\{\left(\frac{x}{\lambda}\right) : P(X_1 \leq \left(\frac{x}{\lambda}\right)) > \alpha\} \\ &= \lambda \cdot VaR_\alpha(X_1). \end{aligned} \tag{3.16}$$

For any $a \in \mathbb{R}$ we get

$$\begin{aligned} VaR_\alpha(X_1 + aI) &= -\inf\{x : P(X_1 + aI \leq x) > \alpha\} \\ &= -\inf\{x : P(X_1 + a \leq x) > \alpha\} \\ &= -\inf\{x : P(X_1 \leq (x - a)) > \alpha\} \\ &= -(\inf\{(x - a) : P(X_1 \leq (x - a)) > \alpha\} + a) \\ &= VaR_\alpha(X_1) - a. \end{aligned} \tag{3.17}$$

To disprove sub-additivity we use a counter-example. Let $X_1, X_2 \in \mathcal{X}$ be two random variables.

$$\begin{aligned} X_1 = X_2 &= \begin{cases} 10 & \text{, with prob. } 99.1\% \\ -100 & \text{, with prob. } 0.9\% \end{cases} \\ X_1 + X_2 &= \begin{cases} 20 & \text{, with prob. } 98.2081\% \\ -90 & \text{, with prob. } 1.7838\% \\ -200 & \text{, with prob. } 0.0081\% \end{cases} \end{aligned}$$

And therefore

$$\underbrace{VaR_{0.01}(X_1 + X_2)}_{=90} \nleq \underbrace{VaR_{0.01}(X_1)}_{=-10} + \underbrace{VaR_{0.01}(X_2)}_{=-10} \tag{3.18}$$

□

Not being sub-additive is the major flaw of VaR.

Kaye (2005) states that point measures are most widely used among all risk measures. His two key reasons for this are the simplicity that makes them easy to understand and the little knowledge about the distribution of outcomes that is needed to calculate them. As long as the upper α-quantile is known, VaR can be applied to any kind of a portfolio. Regulators, for example, require their measures to be used industry-wide, and it is, therefore, important for them that every company can provide comparable measurements of their risk. The Basel Committee on Banking Supervision (2005) supports the use of VaR for several calculations.

Conway and McCluskey (2006) point out that VaR is most commonly used in the banking sector. The time horizon that is looked at is usually very short (often between 1 and 10 days), and the availability of historic data on asset prices is very good due to the existence of long term, active markets. It is questionable if VaR can establish similarly in settings with longer time periods and less knowledge on the tails of the distribution.

Standard Deviation and Higher Moments

Variance and standard deviation, its square root, describe the spread of a distribution of outcomes. Spread is a simple measure of uncertainty - how far results can deviate from the mean.

Definition 10 (Standard deviation, skewness and kurtosis)

Standard deviation σ_X *of* $X \in \mathcal{X}$ *is the square root of the second central moment of* X.

$$\sigma_X = \sqrt{E((X - E(X))^2)} \tag{3.19}$$

While

$$E(X^n) \qquad \text{is the n-th moment.} \tag{3.20}$$

$$E((X - E(X))^n) \qquad \text{is the n-th central moment.} \tag{3.21}$$

Skewness *and* ***kurtosis*** *are the are the third and fourth standardized central moment.*

$$\frac{E((X-E(X))^3)}{\sigma_X^3} \qquad \text{is is called } \textbf{skewness} \text{ of } X. \tag{3.22}$$

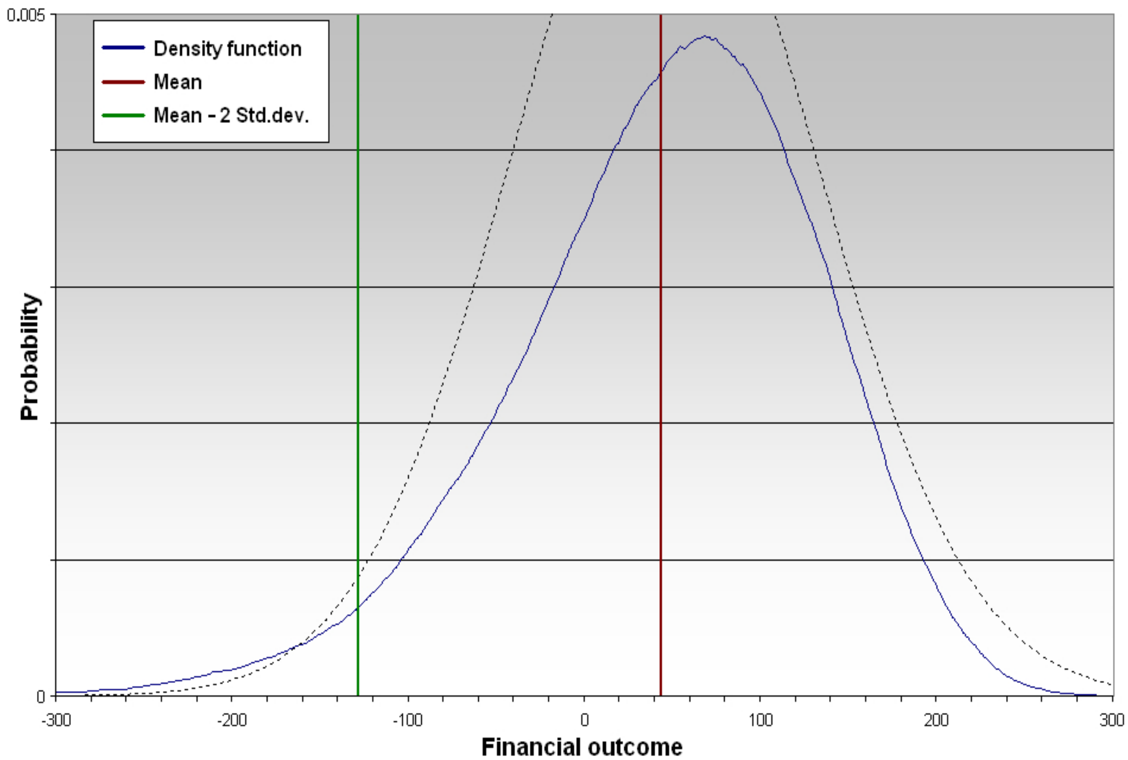
$$\frac{E((X-E(X))^4)}{\sigma_X^4} \qquad \text{is is called } \textbf{kurtosis} \text{ of } X. \tag{3.23}$$

To describe the shape of a distribution, standard deviation is of limited use. Kaye (2005) argues that distributions with the same standard deviation can be very different, one with a long tail, the other one symmetrical for example. To give a better description of the shape, several moments can be used; however the elegance of a single metric is lost in the process.

Figure 11, p. 54, shows mean and standard deviation of Utopia's financial outcomes. Simulated mean and standard deviation are 44.66 and 86.45, respectively. The figure also shows a combination of mean and standard deviation as a possible risk measure and a shaded graph with identical mean and standard deviation. The shaded density function is symmetrical and is generated by the normal distribution. It is significantly different than the density function of Utopia's financial outcomes.

Theorem III.3 (Standard deviation, properties)**.** *The standard deviation is a sub-additive and positive homogeneous risk measure, but does not fulfill monotonicity*

Figure 11
Utopia: PDF, Standard Deviation



author's calculation

and translation invariance.

Proof. Given $X, Y \in \mathcal{X}$.

$$\begin{aligned}
\sigma^2_{X+Y} &= E((X + Y - E(X + Y))^2) \\
&= E((\ (X - E(X)) + (Y - E(Y))\ )^2) \\
&= \sigma^2_X + \sigma^2_Y + 2 \cdot Cov(X, Y) \\
&\leq \sigma^2_X + \sigma^2_Y + 2\sigma_X\sigma_Y \\
&= (\sigma_X + \sigma_Y)^2
\end{aligned}$$

Since standard deviation is non-negative we conclude

$$\sigma_{X+Y} \leq \sigma_X + \sigma_Y. \tag{3.24}$$

And with $\lambda \geq 0$ we have

$$\begin{aligned}
\sigma_{\lambda X} &= \sqrt{E((\lambda X - E(\lambda X))^2)} \\
&= \sqrt{E(\lambda^2 \cdot (X - E(X))^2)} \\
&= \lambda\sqrt{E((X - E(X))^2)} \\
&= \lambda\sigma_X.
\end{aligned} \tag{3.25}$$

Standard deviation is not translation invariant since

$$\begin{aligned}
\sigma_{X+aI} &= \sqrt{E((X + aI - E(X + aI))^2)} \\
&= \sqrt{E((\ (X - E(X)) + (aI - a)\ )^2)} \\
&= \sqrt{\sigma_X^2 + Var(aI) + 2a \cdot Cov(X, I)} \\
&= \sigma_X \\
&\overset{a\neq 0}{\neq} \sigma_X - a.
\end{aligned} \tag{3.26}$$

To disprove monotonicity we use a counter-example. Choose $\tilde{X}, \tilde{Y} \in \mathcal{X}$ with $\tilde{X} \geq \tilde{Y}$

$$\begin{aligned}
\tilde{X} &= \begin{cases} -100 & \text{, with prob. } 50\% \\ 100 & \text{, with prob. } 50\% \end{cases} \\
\tilde{Y} &= -100 \cdot I = -100.
\end{aligned}$$

Their standard deviations are

$$\begin{aligned}
\sigma_{\tilde{X}} &= \sqrt{E((\tilde{X} - E(\tilde{X}))^2)} \\
&= \sqrt{(100 - 0)^2 \cdot 0.5 + (-100 - 0)^2 \cdot 0.5} \\
&\approx 141.42 \\
\sigma_{\tilde{Y}} &= \sqrt{E((\tilde{Y} - E(\tilde{Y}))^2)} \\
&= \sqrt{(-100 - (-100))^2 \cdot 1} \\
&= 0.
\end{aligned}$$

Especially

$$\sigma_{\tilde{X}} \not\leq \sigma_{\tilde{Y}} \tag{3.27}$$

□

Since the standard deviation is not a monotone risk measure, it is an instrument that has to be used carefully. When it comes to determine the amount of capital needed to support the business, even a combination of mean and standard deviation cannot assure monotonicity. Using the counterexample from the proof of theorem III.3 and a valuation method $X = \sigma_X - E(X)$ will still result in a higher amount for $\tilde{X}$ than $\tilde{Y}$.

Kaye (2005) identifies the fact that standard deviation takes into account the whole distribution - unlike point measures discussed earlier - as a possible advantage. However, it is dependent on the situation if this is desirable or not.

Practically, standard deviation has a clear advantage over most measures. Its concept is already known to most decision makers and it is quite easy to calculate using spreadsheets. Variance, the probability weighted sum of squared deviations from the mean, is taught in school, often connected to the normal distribution with its two parameters mean and standard deviation (Johnson and Wichern , 2002); yet the standard deviation's association with the normal distribution also bears a danger. Kaye (2005) points out that it can lead to false confidence. The normal distribution is completely described by mean and standard deviation. Implicitly assuming normal distribution of the outcomes which is commonly done in statistics to simplify the results using the central limit theorem (Johnson and Wichern , 2002), may lead to using quantiles that do not match the quantiles of the true distribution of outcomes.

Expected Exceedence Measures

One point of criticism of VaR was that the shape of the tail beyond the quantile had no effect on the measure. Expected exceedence measures specifically focus on this tail. They involve the expected outcome under the condition of being beyond a threshold. We will see in theorem III.5 that expected exceedence measures can overcome the flaw of VaR of not being sub-additive.

Definition 11 (Tail conditional expectation (TCE))

Given a random variable $X_t \in \mathcal{X}$ at the end of a period with length t and a significance *level* $\alpha \in (0,1)$. *The conditional expectation*

$$TCE_\alpha(X_t) = -E(X_t | X_t \le q^\alpha(X_t)) \tag{3.28}$$
$$= -\frac{E(X_t \cdot \mathbb{I}_{\{X_t \le q^\alpha(X_t)\}})}{P(X_t \le q^\alpha(X_t))} \tag{3.29}$$

where

$$\mathbb{I}_{\{relation\}} = \begin{cases} 1 & , \textit{if relation is true} \\ 0 & , \textit{if relation is true} \end{cases}$$

is called ***tail conditional expectation****, or* ***tail value at risk (TailVaR) at significance level*** α.

While $VaR_{0.01}(X_t)$, with time period $t = 1$ year, describes the value that losses exceed once every 100 years, $TCE_{0.01}(X_t)$ describes the expected losses once every 100 years. A measure very similar to TCE is the excess tail value at risk (XTVaR).

Definition 12 (Excess tail value at risk (XTVaR))

Under the settings used in definition 11 the conditional expectation

$$XTVaR_\alpha(X_t) = -E(X_t - E(X_t) | X_t \le q^\alpha(X_t)) \tag{3.30}$$
$$= -\frac{E((X_t - E(X_t)) \cdot \mathbb{I}_{\{X_t \le q^\alpha(X_t)\}})}{P(X_t \le q^\alpha(X_t))} \tag{3.31}$$

is called ***excess tail value at risk at significance level*** α.

While XTVaR is basically just a shift of the TCE, it has a practical difference when used for business units with different expected outcomes $\mu_i = E(X_t^{(i)})$. The expected shortfall (ES) as defined by Acerbi and Tasche (2002) is another exceedence measure just slightly different from TCE.

Definition 13 (Expected shortfall (ES))

Under the settings used in definition 11

$$ES_\alpha(X_t) = -\frac{1}{\alpha}\left(E(X_t \cdot \mathbb{I}_{\{X_t \leq q^\alpha(X_t)\}}) - q^\alpha(X_t)(P(X_t \leq q^\alpha(X_t)) - \alpha)\right) \tag{3.32}$$

is called ***expected shortfall at significance level*** α*.*

Note that ES sometimes has another meaning in literature that is not equivalent to definition 13. Although the definition for ES does not look similar to TCE at first glance, the only difference is in the adjustment that is made if $P(X_t \leq q^\alpha(X_t)) \neq \alpha$. The adjustment is important to assure sub-additivity of the measure (theorem III.4).

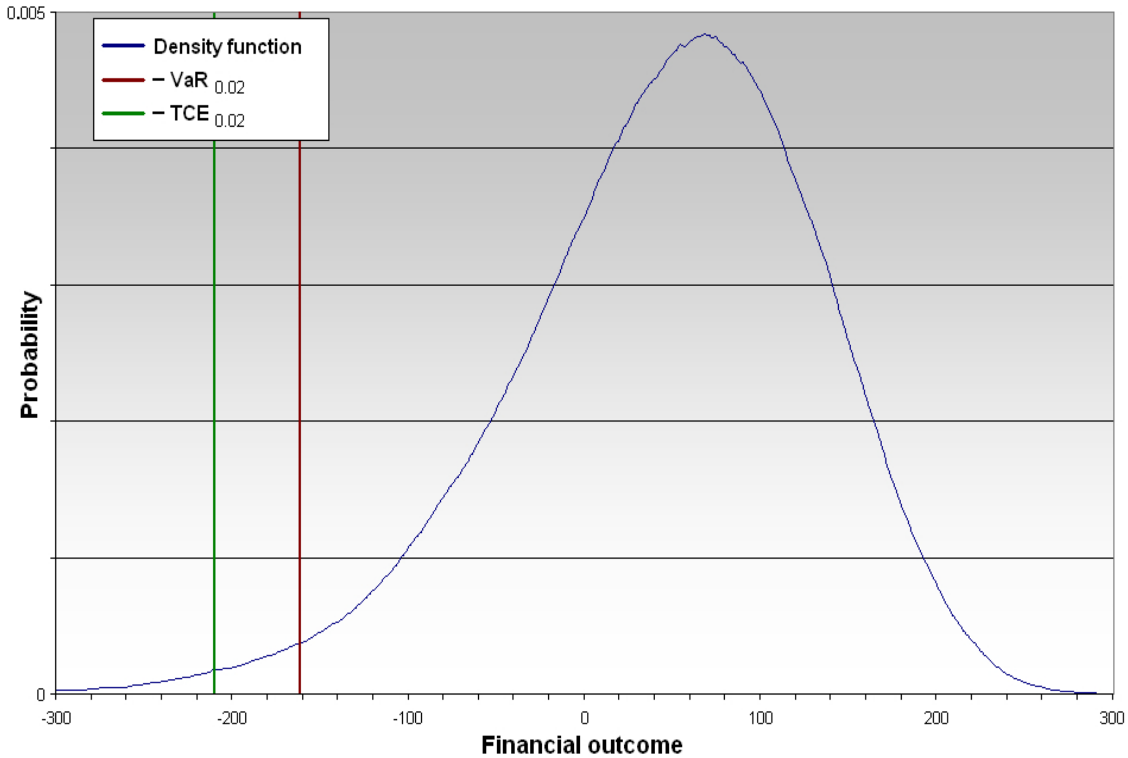
Figure 14, p. 72, shows Utopia's TCE at significance level 2% to be 210.12. The TCE equals the expected losses under the condition that losses exceed the value at risk at the same significance level of 161.68. Recall that the distribution mean is 44.66. With 3.30 the XTVaR can be calculated as $XTVaR_{0.02} = -(-210.12 - 44.66) = 254.78$.

Theorem III.4 (Expected shortfall, properties)**.** *Expected shortfall as defined in definition 13 satisfies the sub-additivity.*

Proof. Shown by Acerbi and Tasche (2002). □

Note that Acerbi and Tasche (2002) also states that ES satisfies 3.2, 3.3 and 3.4. We will only prove these properties for a special case of TCE in

Figure 12
Utopia: PDF, $\mathbf{VaR}_{0.02}$ and $\mathbf{TCE}_{0.02}$



author's calculation

Theorem III.5 (Tail conditional expectation, properties)**.** *Given a random variable $X_t \in \mathcal{X}$ with continuous distribution function, the length of a period t and a significance level $\alpha \in (0, 1)$.*

The tail conditional expectation $TCE_\alpha(X_t)$ is a coherent measure of risk.

Proof. A risk measure that satisfies 3.1, 3.2, 3.3, and 3.4 is called a coherent risk measure. It is therefore sufficient to proof all four properties.

Since X_t has a continuous distribution function we simplify

$$P(X_t \leq q^\alpha(X_t)) = \alpha \tag{3.33}$$

and get

$$\begin{aligned} TCE_\alpha(X_t) &= -E(X_t|X_t \leq q^\alpha(X_t)) \\ &= -\frac{E(X_t \cdot \mathbb{I}_{\{X_t \leq q^\alpha(X_t)\}})}{\alpha} \\ &= ES_\alpha(X_t). \end{aligned} \tag{3.34}$$

With theorem III.4 we conclude that $TCE_\alpha(X_t)$ is sub-additive. Given another random variable $Y_t \in \mathcal{X}$ with continuous distribution function:

$$\begin{aligned} X_t(\omega) \geq Y_t(\omega) \forall \omega \in \Omega &\Rightarrow TCE_\alpha(X_t) \\ &= -E(X_t(\omega)|\omega : X_t(\omega) \leq q^\alpha(X_t)) \\ &\leq -E(Y_t(\omega)|\omega : X_t(\omega) \leq q^\alpha(X_t)) \\ &\leq -E(Y_t(\omega)|\omega : Y_t(\omega) \leq q^\alpha(Y_t)) \\ &= TCE_\alpha(Y_t) \end{aligned} \tag{3.35}$$

For the remaining properties we do not need the continuity of the distribution function. The proof holds in the general case. We can use 3.13 and theorem III.2 to deduce properties for $q^\alpha(X_t)$. For any $\lambda \geq 0$ and $a \in \mathbb{R}$ we have

$$\begin{aligned} TCE_\alpha(\lambda X_t) &= -E(\lambda X_t|\lambda X_t \leq q^\alpha(\lambda X_t)) \\ &\overset{3.16}{=} -E(\lambda X_t|\lambda X_t \leq \lambda \cdot q^\alpha(X_t)) \\ &= -E(\lambda X_t|X_t \leq q^\alpha(X_t)) \\ &= \lambda \cdot -E(X_t|X_t \leq q^\alpha(X_t)) \\ &= \lambda \cdot TCE_\alpha(X_t) \end{aligned} \tag{3.36}$$

and

$$\begin{aligned} TCE_\alpha(X_t + aI) &= -E(X_t + aI | X_t + aI \leq q^\alpha(X_t + aI)) \\ &\stackrel{3.17}{=} -E(X_t + aI | X_t + aI \leq q^\alpha(X_t) + a) \\ &= -E(X_t | X_t \leq q^\alpha(X_t)) - E(aI | X_t \leq q^\alpha(X_t)) \\ &= TCE_\alpha(X_t) - a. \end{aligned} \tag{3.37}$$

□

TCE is not sub-additive in general, which can be illustrated by

Example 14 (TCE and sub-additivity in a discontinuous case)

Consider two discontinuous random variables $X, Y \in \mathcal{X}$. There is a discrete set of equally likely states of the world $|\Omega| = |\{\omega_1, ..., \omega_{10}\}| = 10$. Define

$$\begin{aligned} X(\omega) &= \begin{cases} -100 & , \omega = \omega_1 \\ 0 & , \text{otherwise} \end{cases} \\ Y(\omega) &= \begin{cases} -100 & , \omega = \omega_1, \omega_2 \\ 0 & , \text{otherwise} \end{cases} \\ (X+Y)(\omega) &= \begin{cases} -200 & , \omega = \omega_1 \\ -100 & , \omega = \omega_2 \\ 0 & , \text{otherwise} \end{cases} \end{aligned}$$

For an $\alpha = 0.2$, the problem that leads to the violation of sub-additivity lies with X:

$$P(X \leq q^{0.2}(X)) = 1 > 0.2$$

The TCEs are

$$TCE_{0.2}(X+Y) = 150 \nleq 10 + 100 = TCE_{0.2}(X) + TCE_{0.2}(Y).$$

Technically, expected exceedence measures can be very strong and being a logical extension of the popular VaR makes them, despite their technically more advanced nature, still easy to understand for most decision makers.

Kaye (2005) mentions two issues that remain in place, however. First, the focus on just the tail of the distribution. Two distributions with identical tails but different upside results will be assigned the same measure. While this is desirable in some situations, it is not in others. Second, TCE is dependent on the knowledge of the least $100\alpha\%$ of the distribution of outcomes. If this knowledge is missing, either because of less sophisticated modeling capabilities, or simply not enough information on the tail to come up with a sophisticated model, the TCE will not produce reliable results.

Transform Measures

Transform measures summarize a class of measures that apply transformations, also called distortions, to the original distribution of outcomes. The new expected value under the distorted probability is used as a measure. Kaye (2005) also mentions the use of the difference of original and distorted mean as measurement.

Definition 14 (Distortion risk measure)

Given a random loss variable $Y : \Omega \to [0, \infty)$ *and a non-decreasing distortion function* $g : [0, 1] \to [0, 1]$ *with* $g(0) = 0$ *and* $g(1) = 1$*. The* ***distortion risk measure*** μ_g *is* *defined by*

$$\mu_g(Y) = \int_0^\infty g(1 - F_Y(t))dt \tag{3.38}$$

where F_Y is the cumulative distribution function of Y. $1 - F_Y$ is also called survival function of Y.

Note that we can identify the distortion risk measure μ_g with the regular expected value $E(Y)$ if g is the identity (Tucker , 1962). Specific choices of the distortion function g have been suggested by Wang (1996) and Wang (2000).

Definition 15 (Proportional hazard transformation)
A distortion risk measure μ_g with distortion function

$$g(x) = x^{1-\lambda} \quad , \lambda \in [0, 1) \tag{3.39}$$

*is called a **proportional hazard transformation**.*

Definition 16 (Wang transformation)
A distortion risk measure μ_g with distortion function

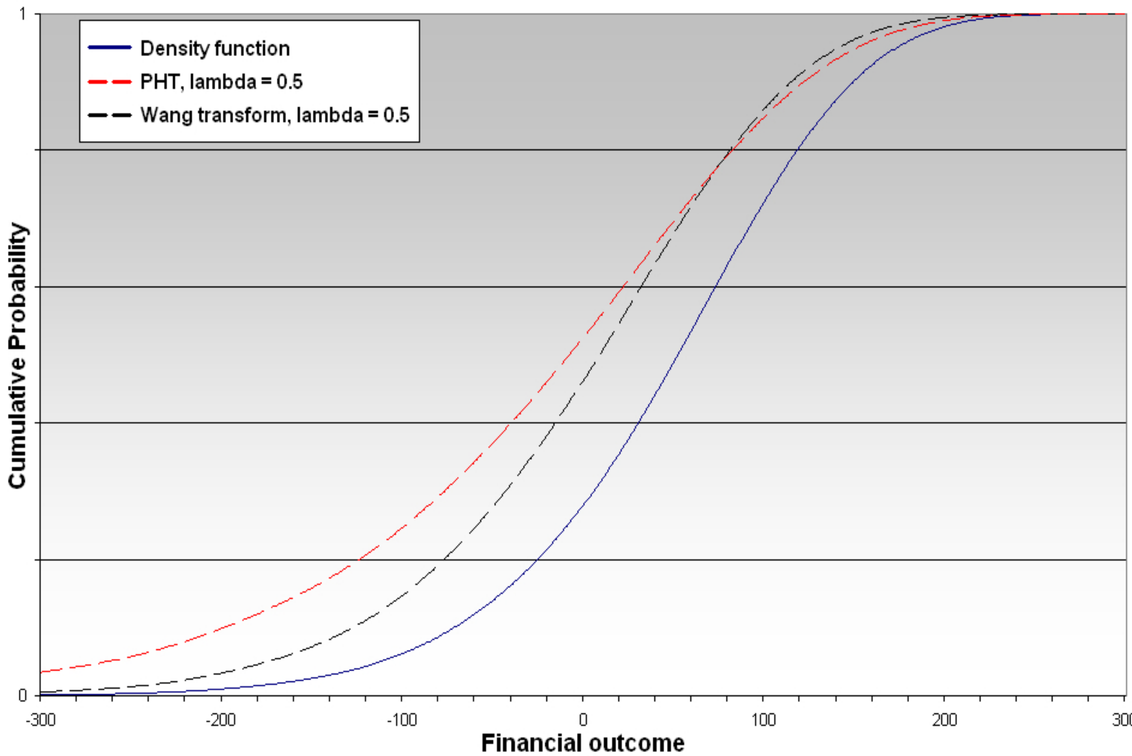
$$g(x) = \Phi(\Phi^{-1}(x) + \lambda) \tag{3.40}$$

*where Φ denotes the cumulative distribution function of the standard normal distribution and Φ^{-1} its inverse, is called **Wang transformation**. $\lambda \in \mathbb{R}$ is sometimes called price of risk and influences the magnitude of the distortion.*

Note that the Wang transformation with $\lambda = 0$ is the identity.

Figure 13

Utopia: CDF, Proportional Hazard and Wang Transformation



author's calculation

Figure 14, p. 72, illustrates how the transformations of the survival function of the losses $Y_1 + Y_2 + Y_3$ effect Utopia's cumulative distribution of financial outcomes. Since the transformed CDFs are shifted to the left of the original CDF, the transformed means will be less than the original mean.

We can also identify VaR and ES as special cases of distortion risk measures.

Example 15 (VaR and ES, distortion risk measure)

Given a random loss variable $Y : \Omega \to [0, \infty)$. In order to keep our expressions consistent with definitions 8 and 13 we are interested in VaR and ES of $(-Y)$ rather than of Y.

$$\begin{aligned} VaR_\alpha(-Y) &= \mu_{\tilde{g}}(Y) \\ ES_\alpha(-Y) &= \mu_{\bar{g}}(Y) \end{aligned}$$

where

$$\begin{aligned} \tilde{g}(x) &= \begin{cases} 0 & , x < \alpha \\ 1 & , x \geq \alpha \end{cases} \\ \bar{g}(x) &= \begin{cases} \frac{x}{\alpha} & , x < \alpha \\ 1 & , x \geq \alpha. \end{cases} \end{aligned}$$

We already know that VaR is not a coherent measure of risk while ES is. It is desirable to find properties of distortions that make them coherent measures of risk.

Definition 17 (Concave function)

*A function $f : [a, b] \to \mathbb{R}$ is said to be **concave** on the interval $[a, b]$ if it satisfies*

$$f(tx + (1-t)y) \geq tf(x) + (1-t)f(y) \quad , \forall t \in [0, 1], \forall x, y \in [a, b]. \tag{3.41}$$

Theorem III.6 (Distortion risk measure, sub-additivity)**.** *A distortion risk measure μ_g is sub-additive if and only if g is a concave distortion function.*

Proof. Proven by Wang and Dhaene (1998). □

Theorem III.6 coincides with results of example 15. While $\tilde{g}$ is a step function, and is therefore not concave, VaR has been proven not to be sub-additive in theorem III.2. Similarly, $\bar{g}$ is a concave distortion function which matches with the result of theorem III.4.

While distortion functions modify the distribution function, or more precisely the survival function $1 - F_X(t)$ directly, transformations can also be applied to ranges in value of the outcome. Mango (1998) discusses the use of concentration charges for measurement.

Definition 18 (Concentration charge)

*Given a random variable $X \in \mathcal{X}$. A **concentration charge** is a set $\mathcal{C} = \{CC_i\},\ CC_i \in \mathbb{R},\ i = 1, ..., n$ of values and corresponding set of non-overlapping intervals $\mathcal{I} = \{I_i\},\ i = 1, ..., n$ that defines a modified mean*

$$\mu_{(\mathcal{C},\mathcal{I})}(X) = E(\sum_{i=1}^{n} CC_i X \cdot \mathbb{I}_{X \in I_i})$$

We can express familiar measures such as the VaR and TCE using concentration charges.

Example 16 (VaR and TCE, concentration charge)

Let $\mathcal{C} = \{-1\}$ and $\mathcal{I} = \{[-q^\alpha(X), -q^\alpha(X)]\}$. The value at risk can be expressed using a concentration charge

$$VaR_\alpha(X) = \mu_{(\mathcal{C},\mathcal{I})}(X).$$

Similarly we define $\tilde{\mathcal{C}} = \{-1\}$, $\tilde{\mathcal{I}} = \{(-\infty, -q^\alpha(X)]\}$ to express the tail conditional expectation as

$$TCE_\alpha(X) = \mu_{(\tilde{\mathcal{C}},\tilde{\mathcal{I}})}(X). \tag{3.42}$$

We can conclude from example 16 that concentration charges can define coherent measures of risk; however their properties are very dependent on the individual settings.

As distortion measures are a generalization of the VaR and ES concept, they have the potential of becoming more important in the industry. They can be adapted to any situation applying weights or distortions to certain parts of the distribution. While this flexibility is certainly a strength, it comes with the challenge to come up with an appropriate weighting/distortion. Kaye (2005) points out that, "thinking through what risk really means in a given circumstance," is important and should not be overlooked. The possibility that any part of the distribution can make its contribution to the risk measure - unlike VaR and TCE/ES - is a key benefit from his viewpoint.

Performance Measures

Performance measures, as introduced by Kaye (2005), do not generally qualify as risk measures under definition 5 and can, therefore, not be tested for coherence. They do not measure the amount of capital required to support a portfolio like most of the presented measures did. However, they relate the upside and downside performance of a portfolio and may be of good use to compare alternative portfolios. The initial idea to relate return and standard deviation has been reviewed by Sharpe (1994).

Definition 19 (Sharpe ratio (SR))

Let the random variable R_i *be the return on an investment within a specified period. Let* R_b *be the return on a benchmark portfolio. If* $P(R_b = r_f) = 1$*, the benchmark portfolio is called risk-free, and* r_f *denotes the risk-free rate.* r_f *is the highest rate of return that can be obtained in the market without assuming any risk. The sharpe ratio of the investment* i *is defined as*

$$SR_{i,b} = \frac{E(R_i - R_b)}{\sigma_{R_i - R_b}}. \tag{3.43}$$

$\sigma_{R_i - R_b}$ *denotes the standard deviation of* $R_i - R_b$*.*

Note that Sharpe (1994) distinguishes between the sharpe ratio "ex ante" and "ex post" depending on the period analyzed. Considering the standard deviation as a measurement of risk (and if the benchmark is risk-free), the SR describes how much return in excess of the risk-free rate is provided by the portfolio per amount of risk.

Example 17 (Sharpe ratio)

Portfolio A has an expected rate of return $\mu_A = 6\%$ and standard deviation $\sigma_A = 10\%$. The benchmark used is risk-free and has a return of $r_f = 4\%$. The SR of the portfolio is calculated to be

$$SR_{A,f} = \frac{\mu_A - r_f}{\sigma_A} = 0.2.$$

We can compare A to another portfolio B with $\mu_B = 8\%$ and $\sigma_B = 16\%$. Its SR is

$$SR_{B,f} = \frac{\mu_B - r_f}{\sigma_B} = 0.25.$$

Comparing the sharpe ratios makes B a more attractive portfolio than A.

The same idea to relate return and risk, up- and downside in a ratio is proposed by Ruhm (2001). While the numerator of the "risk coverage ratio" (RCR) is basically the same as the numerator of the SR, its denominator is different.

Definition 20 (Risk coverage ratio (RCR), Ruhm (2001))

Given the same settings and notations used in definition 19. The benchmark portfolio is assumed to be risk-free. The ***risk coverage ratio*** *is defined as*

$$RCR_i = \frac{E(R_i) - r_f}{E(\max\{0, r_f - R_i\})} \tag{3.44}$$

The denominator of the RCR can be interpreted as expected losses, treating any return lower than the risk-free rate as a loss and higher returns simply as zero losses. Therefore we can think of RCR as the value how often the expected return "covers" the expected losses. Note that the expected value in the numerator is neither cut-off at zero as the expected losses are, nor a conditional expectation.

Keating and Shadwick (2006) utilize conditional expectations of up- and downside in their concept of the "omega function".

Definition 21 (Omega function)

*Given a random variable $X \in \mathcal{X}$ with cumulative distribution function F_X and a threshold $L \in \mathbb{R}$. L separates gains from losses. The **omega function** is defined by the quotient*

$$\Omega_X(L) = \frac{E(X-L|X>L)\cdot P(X>L)}{E(L-X|X<L)\cdot P(X<L)} \tag{3.45}$$

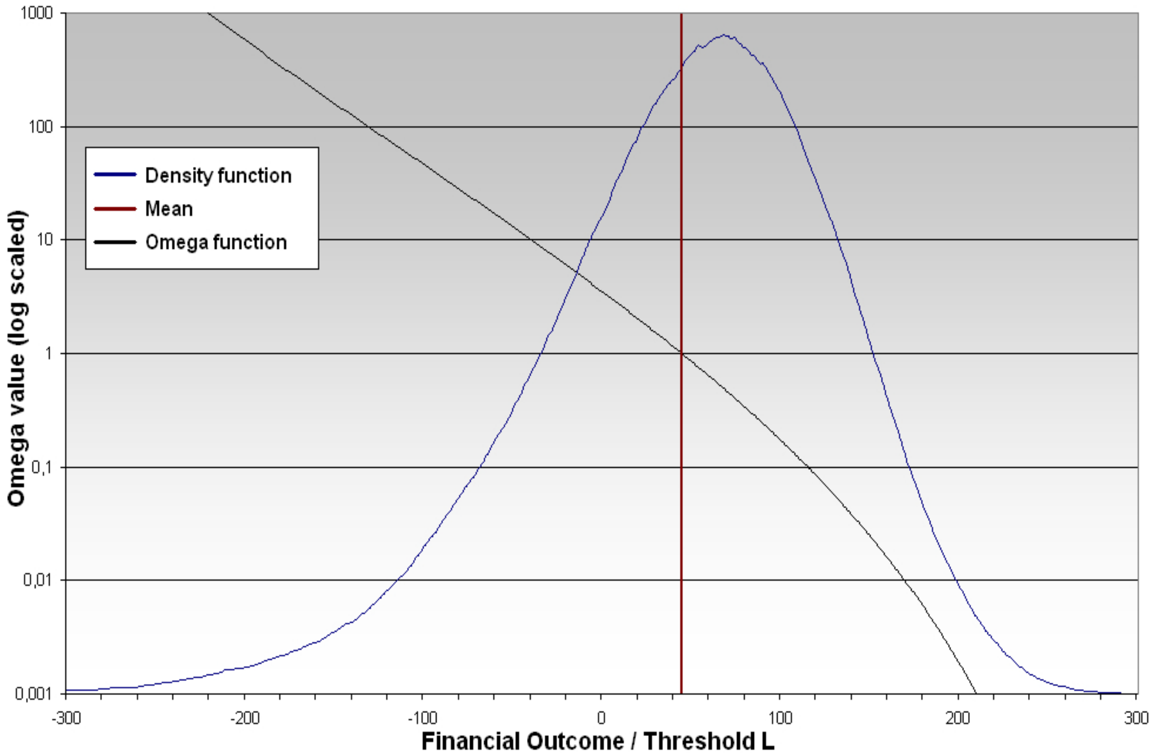
$$\overset{cont.}{=} \frac{\int_L^{\infty}(1-F_X(t))dt}{\int_{-\infty}^{L} F_X(t)dt}. \tag{3.46}$$

The second equation is the alternative representation for a continuous distribution of X.

Note that $\Omega_{X,L}$ is not dependent on the units of X used to measure losses and gains in excess of L, as they cancel each other out in the ratio. Also note that while Keating and Shadwick (2006) use a and b instead of $-\infty$ and ∞, $X \in \mathcal{X}$ is almost surely bounded by definition.

A general advantage of the omega function over other performance measures mentioned by both Keating and Shadwick (2006) and Kaye (2005) is its dependance on all moments of the distribution, rather than just expected value and standard deviation in the case of the SR for example. Since the value of the omega function is largely dependent on the choice of the threshold L, Kaye (2005) advises to plot the omega function for the whole range of L. While values of the omega function are difficult to interpret on their own, they are easy to handle when comparing portfolios. A larger value will indicate a better coverage of downside results by upside results. Figure 14, p. 72, shows the omega function corresponding to the financial outcomes of

Figure 14
Utopia: PDF, Omega Function



author's calculation

Utopia. The omega function takes values in $\mathbb{R}_+$ and equals 1 when the threshold L is equal to the mean of the distribution.

Utility Based Measures

Rather than looking at absolute gain or loss amounts of a company, one might consider its usefulness for the company instead. Acerbi (2002) introduces the concept of utility using a weight function applied to the quantiles of the distribution.

Definition 22 (Admissible risk spectrum)

Let $\phi : [0,1] \to \mathbb{R}$ *be non-negative, normalized, and decreasing.*

$$\int_I \phi(p)dp \geq 0 \quad \forall I \subseteq [0,1] \tag{3.47}$$

$$\int_0^1 \phi(p)dp = 1 \tag{3.48}$$

$$\int_{q-\epsilon}^{q} \phi(p)dp \geq \int_q^{q+\epsilon} \phi(p)dp \quad \forall q \in (0,1), \epsilon > 0 \tag{3.49}$$

$$\text{such that } [q-\epsilon, q+\epsilon] \subseteq [0,1]$$

ϕ *is called* ***admissible risk spectrum*** *or* ***risk aversion function***.

Definition 23 (Spectral risk measure)

Given a random variable $X \in \mathcal{X}$ *and an admissible risk spectrum* ϕ*, the weighted average* $M_\phi(X)$ *of the quantiles of* X

$$M_\phi(X) = -\int_0^1 \phi(p) q_p(X) dp \tag{3.50}$$

is called a ***spectral risk measure*** *generated by* ϕ*.*

Spectral risk measures are very similar to concentration charges and analogous their properties will depend on the weight function ϕ. The properties in definition 22 assure the coherence of M_ϕ.

Theorem III.7 (Spectral risk measure, properties)**.** *The spectral risk measure M_ϕ is a coherent measure if and only if ϕ is an admissible risk spectrum.*

Proof. Proven by Acerbi (2002). □

The advantage of spectral risk measures over concentration charges is that they can be directly related to an agents utility function (Fishburn , 1970).

Other Measures

Other measures that are used in practice are often a combination of the existing ones. Using multiple measures, or combining them, seems to be reasonable if it is possible to use all their advantages while at the same time mitigate the flaws that the measures have on their own.

Novosyolov (2003) proposes the combination of the expected utility and distortion risk measures. The idea behind his recommendation lies with the positive homogeneity. While positive homogeneity is usually a desired property, and part of the definition of a coherent risk measure (definition 6), Novosyolov (2003) points out that it can be undesirable as well.

Example 18 (Undesirable positive homogeneity, Novosyolov (2003))

Consider a lottery that only has two different outcomes: a gain of the amount a with probability p, or no gain with probability $(1-p)$. The price of the lottery is assumed to be b. A person can participate in the same lottery once, or alternatively multiple times, investing $\lambda \cdot b$ to get a possible gain of $\lambda \cdot a$, where λ is a non-negative whole number. A risk measure that satisfies the positive homogeneity will indicate that the risk of the alternative lottery is λ times as much as the original one. A player, however, might consider the usefulness of winning the first amount a larger than the usefulness of any additional amount a that can be won. He might not be willing to pay the complete price $\lambda \cdot b$ of the lottery. In utility theory this is called risk aversion. Similarly, an insurance company might not consider the amount of $\lambda \cdot b$ to be sufficient to cover a possible loss of $\lambda \cdot a$, although b is sufficient to cover a.

The combined risk measure proposed by Novosyolov (2003) does not satisfy the sub-additivity, but retains other properties of distortion risk measures.

Risk Allocation

One purpose of the risk measures discussed earlier was to provide a measurement of a company's exposure to risk, often directly identified as a monetary amount necessary to support the whole business. Risk allocation - synonymously used with capital allocation - goes one step further and tries to allocate part of the total risk to every individual portfolio (or line of business). Why are companies interested in this allocation? A company cannot go bankrupt line by line.

Risk allocation enables a company to measure the performance of a business unit in terms of their income and required capital as part of the system. "Risk-adjusted return on capital" (RAROC) and "economic value added" (EVA) are two performance measures presented by Cummins (2000). It is important for risk allocation to take interdependencies of a firm's portfolios into account as the value of a portfolio is not only described by its stand-alone characteristics, but also by the diversifying effect that is has on the business.

Technical Properties

Denault (2001) extends the concept of coherence as a set of desirable properties to risk allocations.

Definition 24 (Set of risk allocation problems)

*N is the set of all n sub-portfolios of a company and μ is a risk measure. A **set of risk allocation problems** A is defined as the set of all pairs (N, μ).*

Definition 25 (Allocation principle)

Given a set of risk allocation problems A and random variables X_i, $X = \sum_{i \in N} X_i \in \mathcal{X} \;\forall i$ that describe the outcome of sub-portfolios and a company's *total. An **allocation principle*** $\Pi : A \to \mathbb{R}^n$ *is defined as*

$$\Pi(N,\mu) = \begin{pmatrix} \Pi_1(N,\mu) \\ \Pi_2(N,\mu) \\ \vdots \\ \Pi_n(N,\mu) \end{pmatrix} \text{ where } \sum_{i=1}^{n} \Pi_i(N,\mu) = \mu(X). \tag{3.51}$$

The last constraint ensures that the risk of the company will be completely allocated to the portfolios.

Definition 26 (Coherent allocation principle)

*An **allocation principle*** $\Pi : A \to \mathbb{R}^n$ *is called **coherent** if it satisfies the following properties for any pair* (N, μ) *where* μ *is a coherent measure of risk:*

1. *No Undercut*

   The allocation to a set of sub-portfolios should not be greater than its allocation if it is was considered separately.

$$\sum_{i\in M} \Pi_i(N,\mu) \le \mu\left(\sum_{i\in M} X_i\right) \quad ,\forall M \subseteq N. \tag{3.52}$$

2. *Symmetry*

   If two portfolios have the exact same additional contribution to the overall risk when combined with any other set of sub-portfolios, their allocation should be the same.

$$\begin{aligned} \mu\left(X_i + \sum_{k\in M} X_k\right) &= \mu\left(X_j + \sum_{k\in M} X_k\right) \quad ,\forall M \subseteq N\backslash\{i,j\} \\ \implies \quad \Pi_i(N,\mu) &= \Pi_j(N,\mu). \end{aligned} \tag{3.53}$$

3. Riskless Allocation

Adding the cash $a \cdot I$ to a sub-portfolio X_i will reduces its allocation by the amount a.

$$\Pi_i(\tilde{N}, \mu) = \Pi_i(N, \mu) - a \tag{3.54}$$

where $\tilde{N}$ is the modified set of sub-portfolios with $\tilde{X}_i = X_i + aI$ and identical to N otherwise.

Note that definitions may differ. Kaye (2005) allows allocation principles to allocate less than the overall risk of the company to the sub-portfolios but includes another constraint in the no undercut property that assures that a portfolio cannot be better off without any of its sub-portfolios.

It is theoretically possible that a sub-portfolio (such as a portfolio just consisting of cash) can be allocated a negative value. Depending on the circumstance and the measure used, we can interpret this as an amount that can safely be withdrawn from the sub-portfolio. Denault (2001) points out that negative allocations do not pose a conceptual problem but can be very nasty in the application. RAROC, for example, involves the division by the allocation, leading to possibly large negative values that are difficult to interpret.

Independent "First-In"

The "First-In" allocation method is simply based on the sub-portfolios' stand-alone measures of risk.

Definition 27 (Independent "First-In")

Given a set of risk allocation problems A. The allocation principle $FI : A \to \mathbb{R}^n$ defined by

$$FI(N,\mu) = \begin{pmatrix} FI_1(N,\mu) \\ FI_2(N,\mu) \\ \vdots \\ FI_n(N,\mu) \end{pmatrix} \quad \textit{where } FI_i = \frac{\mu(X_i)}{\sum_{k=1}^{n} \mu(X_k)} \cdot \mu(X) \tag{3.55}$$

*is called **independent first-in**.*

By simply scaling the stand-alone risks, the overall diversification benefit of the portfolio is assumed to be equally distributed over all sub-portfolios. We can illustrate with an example that this is a violation of the no undercut property.

Example 19 (Independent first-in, no undercut)

Given three sub-portfolios of a company with standard normally distributed cash flows X_1, X_2, X_3. X_1 and X_2 are perfectly correlated, while X_3 is uncorrelated with the other two. $X_1 + X_2 + X_3$ is therefore normally distributed with mean μ and variance σ^2

(Johnson and Wichern , 2002)

$$\begin{aligned}\mu &= \mu_1 + \mu_2 + \mu_3 = 0\\ \sigma^2 &= \sum_{i=1}^{3}\sum_{j=1}^{3} Cov(X_i, X_j) = 5.\end{aligned}$$

The TCE is a coherent measure of risk since the normal distribution has a continuous cumulative distribution function, and its values at significance level $\alpha = 5\%$ can be computed using the density function of the normal distribution.

$$\begin{aligned}TCE_{0.05}(X_i) &= 2.0622 \quad i = 1, 2, 3\\ TCE_{0.05}(X_1 + X_2) &= 4.1244\\ TCE_{0.05}(X_1 + X_3) &= 2.9164\\ TCE_{0.05}(X_2 + X_3) &= 2.9164\\ TCE_{0.05}(X_1 + X_2 + X_3) &= 4.6113\end{aligned}$$

The independent first-in method allocates $FI_i(N, TCE_{0.05}) = \frac{4.6113}{3} = 1.5371$ to each sub-portfolio. Since $1.5371 + 1.5371 = 3.0742 > 2.9164$ the allocation principle violates the no undercut property and is, therefore, not coherent.

Example 20 (Utopia, capital allocation using independent first-in)

Since the underlying distribution of Utopia's financial outcomes is identical for each sub-portfolio (equation 3.5), we expect them to be allocated the same amount of capital by the independent first-in method. However, since Utopia's knowledge is based on the empirical (simulated) results their allocations may slightly vary. Table 1 summarizes

Table 1
Utopia, Capital Allocation Using Independent First-In

Portfolio	$ES_{0.02}$	%	Allocation
1	102.95	33.17%	69.69
2	102.95	33.17%	69.69
3	104.48	33.66%	70.73
Total	310.38	100%	210.12
$1 + 2 + 3$	210.12	-	-

author's calculation

the results of the capital allocated to Utopia's sub-portfolios based on the independent first-in method. Since the expected shortfall is sub-additive, the summation of the individual sub-portfolio risks will always be greater or equal to the total portfolio's risk. The allocation as a part of the total risk will, therefore, never be greater than the individual risks.

Since the independent first-in neither penalizes highly correlated portfolios, nor rewards portfolios that offer diversification benefits, Kaye (2005) does not recommend using it.

Marginal "Last-In"

The "Last-In" allocation method has a similar underlying idea as the independent first-in. Instead of using the sub-portfolios stand-alone measure of risk, it uses its marginal impact when added to the overwise complete portfolio.

Definition 28 (Marginal "Last-In")

Given a set of risk allocation problems A. The allocation principle $LI : A \to \mathbb{R}^n$ defined by

$$LI(N,\mu) = \begin{pmatrix} LI_1(N,\mu) \\ LI_2(N,\mu) \\ \vdots \\ LI_n(N,\mu) \end{pmatrix} \text{ where } LI_i = \frac{\mu(X) - \mu(X - X_i)}{n\mu(X) - \sum_{k=1}^{n} \mu(X - X_k)} \cdot \mu(X) \qquad (3.56)$$

is called ***marginal last-in****.*

Allocating the value $\mu(X) - \mu(X - X_i)$ to sub-portfolio i completely rewards it for any diversification benefits instead of splitting them up among the uncorrelated portfolios. Kaye (2005) treats this unscaled marginal impact as a lower border for the allocation. The marginal last-in can be viewed as the opposing extreme to the first-in principle.

Example 21 (Utopia, capital allocation using marginal last-in)

Unlike the results of example 20, the correlations of the sub-portfolios will matter when using the marginal last-in method. Table 2 shows the marginal impact of the sub-portfolios on the total portfolio's risk with $ES_{0.02}$ as the risk measure. Using these marginal values, the last-in sub-portfolio will receive full credit for any diversifying effects that it is involved in. Yet, since every sub-portfolio receives the full credit when its marginal impact is calculated, the marginal impact will never be greater than the final allocation.

Table 2
Utopia, Capital Allocation Using Marginal Last-In

Portfolio i	$ES_{0.02}$ $(X - X_i)$	marginal impact	%	Allocation
1	128.28	81.84	48.75%	102.43
2	128.28	81.84	48.75%	102.43
3	205.91	4.21	2.51%	5.27
Total	-	167.89	100%	210.12

author's calculation

Kaye (2005) states that the marginal last-in method does not satisfy the no undercut property, and hence does not recommend its use either.

Shapley Value

When allocating capital to sub-portfolios, the key question is on how to handle diversification benefits. Adding a new sub-portfolio to an existing portfolio will not only create additional diversification to the existing portfolio, but the new sub-portfolio will also benefit from being part of a larger diversified portfolio. The difficulty is to find an appropriate way to share the overall diversification of a portfolio across its parts.

Denault (2001) relates this problem to the concepts and results of game theory. Shapley (1953) introduced the "Shapley Value" as a solution to the problem of how much each player should get in return for participating in a coalition that benefits from everyone's participation. This approach fits very well to the setting in which every portfolio contributes to diversification and a resulting lower capital requirement; however, it is limited to a whole number of players - portfolios are treated as if they were indivisible.

Definition 29 (Shapley Value)

Given a set of risk allocation problems A. The allocation principle $SV : A \to \mathbb{R}^n$ defined by

$$SV(N,\mu) = \begin{pmatrix} SV_1(N,\mu) \\ SV_2(N,\mu) \\ \vdots \\ SV_n(N,\mu) \end{pmatrix} \tag{3.57}$$

where

$$SV_i(N,\mu) = \sum_{M \subseteq N} \frac{(|M|-1)!(n-|M|)!}{n!} \left(\mu\left(\sum_{k \in M} X_k\right) - \mu\left(\sum_{k \in M \setminus \{i\}} X_k\right)\right) \tag{3.58}$$

is called the ***Shapley value****.*

Example 22 (Utopia, capital allocation using Shapley value)

Consider the ES at significance level 2% as a measure of risk. To calculate

$$\mu\left(\sum_{k \in M} X_k\right) - \mu\left(\sum_{k \in M \setminus \{i\}} X_k\right) \quad \forall i, M \subseteq N$$

we need to know the expected shortfall of all possible sub-portfolios. For a company with n sub-portfolios, there are 2^n different total combinations (while one is the empty portfolio) to pick a subset. In the case of Utopia this is $2^3 = 8$. Note that while we know the exact distribution of X_1, X_2, X_3 and $X_1 + X_2$, the distribution of the other subsets is only estimated using simulation. To be consistent, we will use the empirical results of the simulation to find the ES for all sub-portfolios, analogous to example 20. Table 3 includes all values that are needed for the calculation of the allocation to the

Table 3
Utopia, Expected Shortfall of Sub-Portfolios

Portfolio	1	2	3
$ES_{0.02}$	102.95	102.95	104.48
Portfolio	$1+2$	$1+3$	$2+3$
$ES_{0.02}$	205.91	128.28	128.28
Portfolio	$1+2+3$	empty	
$ES_{0.02}$	210.12	0	

author's calculation

sub-portfolios:

$$\begin{aligned}
SV_1(N, ES_{0.02}) &= \sum_{M \subseteq N} \left[\frac{(|M|-1)!(3-|M|)!}{3!} \right. \\
&\quad \left. \cdot \left(ES_{0.02}\left(\sum_{k \in M} Y_k\right) - ES_{0.02}\left(\sum_{k \in M \setminus \{i\}} Y_k\right)\right)\right] \\
&= \underbrace{\frac{1}{3} \cdot (102.95 - 0)}_{M=\{1\}} + \underbrace{\frac{1}{6} \cdot (205.91 - 102.95)}_{M=\{1,2\}} \\
&\quad + \underbrace{\frac{1}{6} \cdot (128.28 - 104.48)}_{M=\{1,3\}} + \underbrace{\frac{1}{3} \cdot (210.12 - 128.28)}_{M=\{1,2,3\}} \\
&= 82.72
\end{aligned}$$

$$SV_2(N, ES_{0.02}) = 82.72$$

$$\begin{aligned}
SV_3(N, ES_{0.02}) &= \frac{1}{3} \cdot \underbrace{(104.48 - 0)}_{\text{Average 1st-in}} \\
&\quad + \frac{1}{3} \cdot \underbrace{\frac{(128.28 - 102.95) + (128.28 - 102.95)}{2}}_{\text{Average 2nd-in}} \\
&\quad + \frac{1}{3} \cdot \underbrace{(210.12 - 205.91)}_{\text{Average 3rd-in}} \\
&= 44.68.
\end{aligned}$$

Note that the allocation does not require any scaling.

$$\sum_{i=1}^{3} SV_i(N, ES_{0.02}) = 210.12 = ES_{0.02}(X_1 + X_2 + X_3)$$

Theorem III.8 (Shapley value, properties)**.** *Given a coherent measure of risk μ. The Shapley value satisfies the symmetry and riskless allocation properties. Coherence of the risk measure μ is not strong enough to ensure the no undercut property of the allocation in general.*

Proof. Proven by Denault (2001). □

Denault (2001) shows that, under additional assumptions about the cost function used in the game theory context, the Shapley value also satisfies the no undercut property. However, this also implies additional properties of the measure μ such as the linearity.

The main difficulties of the Shapley value beside the problem of the no undercut property are also discussed by Kaye (2005). As seen in example 22, the calculation of the Shapley value requires the measurement of the risk for 2^n subsets of the overall portfolio of a company that consists of n atomic sub-portfolios. The calculations quickly become impractical. Even more important is the fact that in splitting up one part of the portfolio into two identical halves, one would expect the allocation to the other parts of the portfolio to remain unchanged. However, this is not true since the approach by Shapley (1953) was only designed for n non-divisible players.

Aumann-Shapley value

To make the concept of Shapley values fit to more general situations, Aumann and Shapley (1974) extended the concept of the Shapley value from n participants in the game to non-atomic games that allow any friction of a player to be present in a coalition. Denault (2001) borrows this idea and adjusts it to capital allocation problems.

A company's portfolio N still consists of n different parts, but we allow each of these sub-portfolios to be present at a floating (positive) level. Denault (2001) denotes their activity levels by $\frac{\lambda_i}{\Lambda_i}$, where the vector $\Lambda \in \mathbb{R}^n_+$ is a reference unit of the portfolio (e.g. representing the business volumes, and $\lambda \in \mathbb{R}^n_+$ is the absolute participation).

Definition 30 (Risk measure with fractional sub-portfolios)

Given $\lambda, \Lambda \in \mathbb{R}^n_+$ and a measure of risk μ. $r : \mathbb{R}^n_+ \to \mathbb{R}$ denotes the to μ corresponding ***measure of risk for fractional sub-portfolios*** *defined by*

$$r(\lambda) = \mu\left(\sum_{i \in N} \frac{\lambda_i}{\Lambda_i} X_i\right) \tag{3.59}$$

The coherence of r defined by Denault (2001) also allows for fractions of portfolios and is analog to definition 6. The positive homogeneity is translated into homogeneity of degree one: $r(\gamma\lambda) = \gamma \cdot r(\lambda) \quad \forall \gamma \in \mathbb{R}_+$.

Definition 31 (Aumann-Shapley Value)

Given a set N of all sub-portfolios of a portfolio, a corresponding unit vector $\Lambda \in \mathbb{R}^n_+$, and a coherent and measure of risk r allowing for fractional participation. Assuming that the partial derivative of r exists, the Aumann-Shapley value ASV for sub-portfolio i is defined by

$$ASV_i(N, \Lambda, r) = \frac{\partial r(\Lambda)}{\partial \lambda_i}. \tag{3.60}$$

Denault (2001) extends the definition 26 of a coherent allocation principle to fit to non-atomic portfolios keeping the ideas of coherence unchanged.

Theorem III.9 (Aumann-Shapley value, properties)**.** *Given a portfolio (N, Λ) and coherent measure of risk r that is differentiable at Λ. The Aumann-Shapley value is a coherent allocation principle allowing for fractional sub-portfolios as defined by Denault (2001).*

Proof. Proven by Denault (2001). □

Kaye (2005) states that while Shapley values are difficult to calculate, Aumann-Shapley values are significantly easier when using simulation techniques that are formalized by algorithms. The Ruhm-Mango-Kreps algorithm (Ruhm and Mango , 2003) can be used to calculate the Aumann-Shapley values.

Treatment

The final decision of how to handle the risks of the company has to be based on the company's measurement and their ability to mitigate them. If the required capital to support a single part of the business seems to be too high, the company may consider to partly or fully insure this part.

A company will be required to monitor the impact of the identified risk factors, and, if possible, update the information with new/more data. New data allows for a reevaluation of the risks, and therefore, a possible revision of the treatment.

CHAPTER IV

CONCLUSION

The preceding chapters have shown how effective ERM can help the management to achieve the company's objectives. While chapter II points out that ERM has to address the incentives prevalent among the management and decision makers, chapter III provides more tools to come to a correct decision based on the company's exposure to risk. Most of the techniques used for measurement and allocation of risk are still subject to research. Originally flawed techniques have been revised to assure the proper treatment of risks and improve the effectiveness of ERM.

However ERM is no panacea. Improving the likelihood of success does not ensure success. Human decisions are not perfect, and even simple errors can sometimes lead to the breakdown of a process. A poor manager might use ERM as a crutch in order to keep from falling, but he will still struggle when it comes to a race.

In addition ERM is subject to the same limitations as the management (COSO , 2004). Governmental changes of policies and programs, a competitor's action that effects the company, or general changes of the economic environment, can be beyond the control of the management. ERM can, at best, try to help by providing forecasts that indicate critical changes.

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