GUARANTEES IN EQUITY-INDEXED ANNUITIES

AND VARIABLE ANNUITIES

Denis Toplek

183 Pages August 2002

Equity-indexed annuities and variable annuities, guarantees, design, mathematical models, reserving, asset liability management, cash flow testing and investment policy.

APPROVED:

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Date Krzysztof M. Ostaszewski, Chair

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Equity-indexed annuities and variable annuities have become one of the most popular forms of retirement savings in the United States. Beginning in the mid-1990s, sales figures for these products started to soar with the bull market. In the recent two years, the market increases were slowing down and even turning into downward movements. This development led insurance companies to include guarantees in their variable annuities and to emphasize that equity-indexed annuities are designed to give the customer the upside potential with downside protection.

This thesis examines equity-indexed annuities and describes some guarantees in variable annuities that currently are offered in the market and that are a by-product of equity-indexed annuity designs. The first chapter should give an introduction to annuities, explain general concepts of annuities and familiarize the reader with the basic technical terms used with annuities. In the second chapter, equity-indexed

annuities are analyzed and all the crucial contract features and designs are presented. The third chapter then gives a market overview for equity-indexed annuities and variable annuities. In the fourth chapter, equity indices and bond indices are presented and analyzed. Those indices are used to determine interest for equity-indexed annuities. Chapter five presents mathematical models for two of the most common equity-indexed annuity designs. In chapter six equity-indexed annuities are classified by designs and types of guarantees and variable annuities are classified by types of guarantees offered. Several products currently on the market are presented. Chapter seven discusses the legal framework and issues about reserving for equity-indexed annuities. In chapter eight the risks related to equity-indexed annuities, asset liability management and cash flow testing for equity-indexed annuities are discussed. Chapter nine talks about the investment policy of equity-indexed annuities since some special problems related to equity-indexed annuities have to be considered. In chapter ten disintermediation risk is discussed since this is a special problem for equity-indexed annuities.

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DENIS TOPLEK

A Thesis Submitted in Partial
Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

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THESIS APPROVED:

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Denis Toplek

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CHAPTER I

INTRODUCTION

Equity-indexed annuities and variable annuities have become one of the most popular forms of retirement savings in the United States. Beginning in the mid-1990s, sales figures for these products started to soar with the bull market. In the recent two years, the market increases were slowing down and even turning into downward movements. This development led the insurance companies to include guarantees in their variable annuities and to emphasize that equity-indexed annuities are designed to give the customer the upside potential with downside protection.

This thesis examines equity-indexed annuities and describes some guarantees in variable annuities that are currently offered in the market and that are a by-product of equity-indexed annuity designs. The first chapter gives an introduction to annuities, explain general concepts of annuities and familiarize the reader with the basic technical terms used with annuities. In the second chapter, equity-indexed annuities are analyzed and all the crucial contract features and designs are presented.

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1.1 Description of an Annuity

An annuity is an insurance contract between an insurance company and a customer designed to provide the customer with income in the future. It is usually purchased by the consumer because of a need for income in the future, typically retirement income. The customer pays

a premium or a series of premiums in order to obtain benefits on a predetermined basis over a specified period of time. The company invests the money it receives from the customer and pays him or her back according to the specifications in the contract. The payments the customer receives include the return of his investment in the contract plus interest or other return on the invested capital. This, of course, does not distinguish an annuity from any other investment contract. However, annuities are provided with various forms of guarantees given by the insurance companies. Traditionally, annuities were sold with a guarantee of income for the rest of consumer's life, beginning with some time in the future, e.g., retirement.

The purpose of the first annuities that were developed by life insurance companies was to provide individuals with income during their retirement years [Insurance.com Insurance Agency 2002].

In the United States, an attractive feature of an annuity is that the earnings on an annuity are tax-deferred until the customer begins to receive benefits from the insurance company, which issued the annuity. This is similar to a qualified retirement plan (such as a 401(k) plan, 403(b) plan, or Individual Retirement Account). Because of deferral of taxation, the customer's investment in the annuity can become considerably larger than if the money was invested in a comparable taxable investment. However, similarly as in a qualified retirement plan,

the customer may have to pay a 10 percent tax penalty if he or she starts withdrawing money from an annuity before the age of 59.5.

As mentioned above, an annuity is a contract. Therefore, it is important to know the parties of an annuity. There are four parties to an annuity.

First, there is the annuity issuer, which is the insurance company that issues the annuity. Second, there is the owner of the annuity who is the person that buys the contract from the issuer and pays the contributions. The third party is the annuitant. The annuitant is the person whose life is the measure for the benefits. The annuitant and the owner do not have to be the same person, but usually they are. The fourth party is the beneficiary who is the person that gets a death benefit from the annuity in case of death of the annuitant. Again, the owner and the beneficiary need not be the same person, but commonly they are. An annuity may have more than one beneficiary. For example, in the United States, some annuities pay benefits for as long as at least one of two married spouses is alive. Defined benefit pension plans in the United States are in fact required to provide the following annuity as the default benefit: a life annuity to the plan participant, with at least 50% of the benefit paid after the death of the plan participant to the surviving spouse, as long as such spouse is alive.

There are two separate phases to an annuity. The first phase is the *accumulation (or investment) phase*. This is the time period during which the consumer pays the premium for the contract, and thus the annuity accumulates the funds for the consumer. The annuity either can be purchased by paying a lump sum (this is called a single payment annuity), or by several payments, which can be of equal or variable size. The second phase is the *distribution (or payout) phase*. In this phase, the customer receives payments from his annuity.

The distributions in the payout phase can be paid out in two different forms. The first possibility is that the value of the annuity (principal plus earnings) can be paid out as a lump sum or over a certain time period. The second option is to receive a guaranteed income stream from the annuity. Therefore, this is called the *guaranteed income* (or *annuitization*) option. If this option is chosen the issuer guarantees to pay the annuitant an amount of money periodically. The annuity owner can choose between a fixed annuity payout where the amount for each payment period is fixed and a variable annuity payout where the amount for each payment period is variable. The payout can take place over the entire remaining lifetime of the annuitant or over another specified time period or over the entire lifetime of the annuitant and another individual, which is called a *joint and survivor annuity*.

When a customer purchases an annuity, he or she has two possibilities to define the point of time when the payout phase begins. One can buy an *immediate annuity*, which means that the payout begins within 12 months after the customer purchases the annuity. This type of annuity does not have an accumulation phase; the purchase is made with a single, lump-sum payment. It consists only of the payout phase where the lump sum is converted into an income stream according to the payout option he or she has chosen.

The second possibility is to buy a *deferred annuity*. This is the conventional type of annuity and the predominant type in the market. Deferred annuities usually are funded by a series of premium payments during the accumulation period but the customer also can choose to make just a single lump-sum payment. The reason why those annuities are called deferred is that the payout phase begins at some point of time in the future, typically at retirement.

Usually, annuities allow the owner to withdraw up to 15 percent per year without a penalty [Insurance.com Insurance Agency 2002]. Beyond that, most annuities have surrender charges. Those charges are designed to penalize early withdrawals above the free withdrawal amount and they usually decrease over a period of seven years or longer.

The incentive for buying an annuity with withdrawal penalty is that the insurance company usually offers a 3 to 5 percent bonus added

to the principal amount up-front. Sometimes this bonus is compensated by higher fees and longer surrender periods, usually eight to nine years.

There are annuities without surrender charges, so called no-surrender or level load annuities, for investors that might suddenly need access to their money. These annuities have a somewhat higher liquidity, but therefore they do not offer bonuses and sometimes come with higher fees or lower interest rates.

Regardless of the withdrawal charges, early withdrawals are subject to taxes and an IRS tax penalty of currently 10 percent if withdrawal is made before the age of 59.5 years.

Annuities have several advantages that can serve as an incentive for potential customers to invest his or her money in them.

First, annuity earnings are tax-deferred until the payout phase begins which may be advantageous because the annuity owner might be in a lower tax bracket at that time, which is usually retirement. Second, the invested money compounds tax deferred for many years. Another advantage is that there is no maximum amount that one can invest in annuities as opposed to the limits placed on qualified retirement savings plans. Furthermore, an annuity is a retirement investment and death protection at the same time and it is an excellent retirement savings vehicle once the maximum contributions to traditional retirement plans have been made. Usually, annuities have a better performance than

comparable investment forms, such as Certificates of Deposit [Insurance.com Insurance Agency 2002].

On the other hand, customers should also be aware of some disadvantages that annuities offer.

Annuity contributions are not tax-deductible if they are not made within a qualified retirement savings plan. However, within those plans there are usually better suitable forms of investment which means that it is not advantageous to invest in an annuity within a qualified retirement savings plan. Another issue that one has to be aware of is that annuities are long-term investment vehicles with limited immediate liquidity (except for immediate annuities). In addition, the IRS imposes a 10 percent tax penalty on early withdrawals before age 59.5. Another problem might occur if the beneficiary chooses to receive the payout as a lump sum payment because he or she might be shifted to a higher tax bracket.

1.2 Fixed Annuities

Fixed annuities are annuities that can be deferred or immediate, consist of a single payment premium or a flexible payment premium. At the end of the accumulation phase the beneficiary can choose if he or she wants a lump sum payment, annuitization, or reinvestment. The earnings from the fixed annuity usually are tax-deferred. Fixed annuities

were the first type of annuities on the market. A fixed annuity has a fixed interest rate guarantee for the investment phase, sometimes adjustments for inflation, and a guarantee that the contributions will be paid back. If the annuitization option is chosen, the periodic benefit amount is also guaranteed for the distribution phase, which might be the whole remaining lifetime.

The annuity contributions usually are invested in low-risk fixed-rate assets such as government securities, high-grade corporate bonds, or mortgages. The investment decisions are made solely by the insurance company, the customer has no influence on those decisions.

Traditional fixed annuities do not have a variable element and therefore will not be dealt with in this thesis.

1.3 Variable Annuities

Variable annuities have most of the characteristics of a traditional annuity. However, there are some very important differences. When a customer purchases a variable annuity, he or she makes the investment decisions and therefore the customer usually bears the whole investment risk. Usually a variable annuity comes with no or just a few guarantees. There is no guarantee or projection from historical rates of any rate of return on the underlying investment portfolio. The return depends entirely on the selected investments' performance.

Variable annuities are separate account products. This means that the customers' premium payments are held in an account separated from the insurance company's general account. Separate account balance is effectively customer's property, and is invested in various investment vehicles and managed by professional portfolio managers, in a manner similar to a mutual fund. The money in a separate account is segregated from the insurance company's general account and it is protected from claims of the insurance company's creditors. The separate accounts are established according to specific state statutes [American Academy of Actuaries 1998a]. Their sole purpose is to hold assets allocated to variable investment options in variable annuities and other products with investment character.

A variable annuity allows the customer to invest his contributions in a selection of investment options, which are called sub-accounts. These sub-accounts are tied to market performance, and are often modelled according to a corresponding managed investment, such as an investment fund. The customer buys units of a sub-account rather than shares of the underlying investment. There is a wide range of possible investments which are offered to the customer ranging from the most conservative, such as government bond funds, and money market, guaranteed fixed accounts, to more aggressive such as growth, small cap, mid cap, large cap, capital appreciation, aggressive growth, and

emerging markets funds. Some variable annuities offer forty or more underlying investment choices with ten or more managers, and allow the customer to switch between them during the accumulation phase.

If annuitization is chosen at the beginning of the payout phase, then there are two payout type options for the payee. One can choose a fixed payout, which means that he or she will receive equal, periodical payments depending on the amount of money in the annuity. Alternatively, one can pick a variable payout which means that the performance of the investment portfolio will determine the amount of each payment, or he or she can pick a combination of the two which guarantees a minimum fixed payment and in addition a variable payment that is based on the performance of the investment portfolio.

Variable annuities are considered securities and therefore must be registered with the Securities Exchange Commission (SEC). This causes the cost of introduction and maintaining of the variable annuity to be much higher than for non-registered products, but it can also be seen as some kind of consumer protection since the SEC supervises the securities market and enforces the Securities Exchange Act of 1934, which prohibits any misrepresentation or manipulation of the markets. Another consequence is that the agent who sells the variable annuity has to be registered with the SEC, too. Note that a typical agent selling insurance is registered with state insurance authorities. In contrast,

salespeople of investment products must be registered with SEC and pass appropriate examinations required for registered representatives. Some sales people, of course, are licensed in insurance and investment products.

1.4 Equity-Indexed Annuities

Equity-indexed annuities (EIA) are a mix between variable annuities and fixed annuities. The purchase of an equity-indexed annuity also means an investment into an account that is tied to a stock market index, most often the Standard & Poor's 500 (S & P 500), just like with a variable annuity. The performance of the stock market index determines the return of the equity-indexed annuity but a big difference to a variable annuity is that the insurance company also guarantees a minimum return over a certain time period in case the index does not perform well enough to cover that minimum percentage which is usually two to three percent.

The most important difference between equity-indexed annuities and variable annuities is that equity-indexed annuities are a general account product whereas variable annuities are a separate account product. This means that the insurance company holds the premiums collected for equity-indexed annuities within the general account of the company. Consequently, an equity-indexed annuity is not a variable

product in legal terms, although it has a rather variable profile. The Illinois Department of Insurance states on its website [State of Illinois Department of Insurance 2001] that equity-indexed annuities are fixed annuities. The consequences of this treatment are that the insurance company has to include equity-indexed annuities in its general account. On the other hand, equity-indexed annuities are not considered securities, like variable annuities are, and therefore they do not have to be registered with the SEC. This lowers the introduction and maintenance cost for this kind of product, which was exactly one of the reasons for its development.

The S & P 500 index is the predominant index underlying equity-indexed annuities. While it is possible to tie these annuities to any published index, or even to create a new index, the majority of the products on the market use the S & P 500 as the underlying index. However, the number of other indices used is growing and as of September of 2001, ten carriers offered indices other than the S & P 500, according to the Advantage Group [The Advantage Group 2002]. The indices used are the Dow Jones Industrial Average Index, The NASDAQ 100, Lehman Brothers Aggregate Bond Index, Lehman Brothers High Yield Bond, Lehman Brothers U.S. Treasury, Russell 2000, and even one international index, that consists of the London FTSE 100, the Tokyo Nikkei 225, the German DAX, and the Paris CAC 40.

The big advantage of the S & P 500 is that options on it are readily available as exchange-traded options and need not be specially designed. This makes hedging of S & P 500 contracts much easier and cheaper than hedging of other underlying indices.

CHAPTER II

DESIGN AND FEATURES OF EQUITY-INDEXED ANNUITIES

The following chapter gives a general description, design choices and product feature descriptions for equity indexed deferred annuities, and equity indexed immediate annuities. The American Academy of Actuaries distinguishes two classes of equity-indexed annuities in its response to the Securities Exchange Commission [American Academy of Actuaries 1998b]: Equity-Indexed Deferred Annuities and Equity-Indexed Immediate Annuities.

2.1 Design and Features of an Equity-Indexed Deferred Annuity

Equity-indexed deferred annuities are a type of deferred annuity that connects all or a part of the payable benefits to the performance of an external index. According to the American Academy of Actuaries [American Academy of Actuaries 1998b] equity-indexed deferred annuities are best defined by a set of parameters:

- the length of the period during which the interest is based on the index,

- the type of index-based interest calculation,
- the index that is underlying the equity-indexed annuity
- the type of averaging of index values,
- the conversion method from the amount of index change into an interest rate,
- the excess interest crediting-method,
- the return guarantee at the end of the term.

Consider, for example: a 12 year, annual ratchet, based on the S&P 500 index, using 6 month index averaging, with70% participation, and a guarantee of 90% accumulating at 3%. Consider, alternatively, a 10 year, point-to-point, based on the NASDAQ index, using year-end index values, with 100% participation minus a 2% spread, and a guarantee of 100% accumulating at 3%. More parameters such as premium payment flexibility, vesting of interest credits, cash value profile, use of a market value adjustment, whether the annuity is embedded in a broader product, etc. can also be used to distinguish equity-indexed deferred annuities.

Equity indexed deferred annuities can appear in many different designs which can be produced out of many different components. The crucial point for comparing different product designs is that no design is inherently financially superior to any other design. Two products will

provide equivalent value and spend the same amount on hedging cost if all their other characteristics are identical, i.e., expenses, fixed investment yield, cash values, lapses, etc., although the participation rates may differ because of the design differences. The benefits under a specific set of circumstances may differ; however, the various possibilities will be priced on the call option market such that equivalent value is available under all designs. Following is a description of the design choices observed on the market:

2.1.1 Index Term Period

The *index term period* is the period over which equity index benefits are calculated and at the end of which a guaranteed return is provided. Typically, the full contract value is available without surrender charges at the end of a term. Commonly, each term is followed by another index term period. The contract value at the beginning of each index term period is set equal to the greater of the equity index benefit and the guaranteed minimum benefit at the end of the previous period. Some contracts offer several index term periods from which to choose and in those cases different terms can be chosen at the end of each term. Usual index term periods are from one to ten years. As of 2002, the trend is towards index term periods longer than 10 years.

2.1.2 Interest Calculation Methods

Another contract design component is the *interest calculation method*. The numerous different interest calculation methods can generally be classified into several families of designs and mixtures of these families:

Point-to-point methods credit interest as a portion of the percentage growth in the underlying index from the beginning of the term to the end of the term.

Ladder (or *Note*) methods are enhanced point-to-point methods. They also credit interest as a portion of the percentage growth in the underlying index from the beginning of the term to the end of the term. In addition, they provide a guarantee that the recognized final index value will not fall below a specified index level if the index reached that level at specified points during the term. This is also called a *cliquet* method or *ratchet* method, although it is not the original ratchet. One or more levels of the recognized index may be specified. The typical measurement points are the contract anniversaries, but it is possible to choose a more frequent basis.

High watermark methods credit interest as a portion of the percentage growth in the underlying index from the beginning of the term to the highest value the index has achieved at specified measurement points up to the end of the term. The measurements are typically done

on contract anniversaries, but a greater frequency is possible. Some averaging technique could be applied to each of these measurement points. The high watermark method also is sometimes called the discrete lookback method, which originates from the name of the type of call option utilized to hedge it.

Low watermark methods credit interest as a portion of the percentage growth in the underlying index from the lowest value the index has achieved at specified measurement points during the term to the index value at the end of the term. The typical measurement points are the contract anniversaries, but again a greater frequency is possible. Each of these measurement points could use some averaging technique. The low watermark method also is sometimes referred to as the discrete lookforward method, in recognition of the type of call option utilized to hedge it.

Ratchet (or ciquet) designs credit index-based interest to the current contract value periodically throughout the term. The following variations of the design are used:

- The *interest accumulation method* used is one distinctive feature. Interest can either be credited just to the premium or to the current contracts value, which might be premium plus previous interest earned and locked in. A compound ratchet applies the index-based interest rate to the current

contract value at the time of the crediting. A simple ratchet applies the index based interest rate to the premium minus cumulative withdrawals at the time of the crediting.

- *Accumulation frequency* refers to the frequency the ratchet "clicks into place". Most ratchet designs lock in the earnings annually; however, it is possible that the frequency is lower.
- The *Length of guarantee of index change recognition* is another characteristic component of ratchet designs. The current participation rate, cap, or spread charge can be guaranteed only for the current interest crediting period, for the entire term, or for some intermediate period. If the guarantee is only for the current interest crediting period, a lesser guarantee commonly is provided for the balance of the term and subsequent terms.
- The *Minimum guaranteed interest* is one of the more important features of a ratchet. For each interest crediting period, the ratchet provides a specified minimum guaranteed interest rate, which is generally constant for all interest crediting periods. Typically, this is 0%, although some companies use a higher interest [American Academy of Actuaries 1998b].

2.1.3 Equity Index Used

The next defining parameter of an equity-indexed deferred annuity is the *used equity index.* An equity-indexed deferred annuity can be tied to any published index, which does not have licensing restrictions. It is also possible for the insurance companies to construct their own indices but the choice of indices is heavily influenced by the availability of hedging instruments. Equity indices generally reflect the movement in the price level of the underlying stocks and do not include value growth due to dividend payments. Most contracts in the U.S. are based upon the S&P 500 Index for several reasons. First, interested customers recognize the S & P 500 and so marketing is easier than with an unknown index. Another reason is that the call options needed to hedge the risk are readily available and liquid. The S & P 500 also does not have complicated licensing requirements and has some advantage in this point when it is compared to other indexes such as the Dow Jones.

2.1.4 Index Averaging Method

The next defining feature of equity-indexed deferred annuities is the *index averaging method.* The simplest index measurement just looks at the index value of a single day; however, using miscellaneous averages of index values can reduce the volatility of the index increase measurement and moderate the value credited to the annuity contract.

The characteristic items of averaging techniques are the length of the averaging period and the frequency of the measurements within the period. Contracts that use averaging techniques are called Asian end or Asian beginning contracts, originating from the names used in option hedges:

Short term averaging is usually used at the end of each contract year, and sometimes at the beginning of the contract, in order to reduce the volatility of the index measurement. Periods of 30 or 60 days might be used for daily averaging.

Long term averaging may be used at the end of a multi-year point-to-point benefit determination, e.g., when the index benefit is determined solely upon the change in the index from the beginning of the index term period to the end of the index term period, which could be up to ten years. Such averaging might be over a period of two to 24 months and commonly might use the average of monthly indices, although daily averaging could be used. This type of average provides some comfort to the purchaser that the benefit determination will not be based upon a relative low-point value of a single day, and it additionally produces a less expensive benefit, which could support a higher participation rate.

Using *annual averaging* of index values within each year for ratchet designs can reduce the volatility in the interest credited to the contract. A side effect is that a nominally higher portion of the calculated index

increase rate is reflected in the interest rate. Daily averaging, monthly averaging, and quarterly averaging are used. These methods contain on average half to slightly more than half of the annual index increase percentage; however, this share will vary considerably from year to year with the profile of the index volatility during the year.

2.1.5 Participation Adjustment Methods

Another feature that defines equity-indexed deferred annuities is the *method of adjusting the index increase percentage.* The index-based interest crediting rate is some part of the increase in the index and it is adjusted by using a participation rate, a spread deduction, a cap, or a combination of the methods. All these methods not only reflect current market developments but they also are possible sources of profit for the insurance company.

The *Participation Rate* is a multiplier applied to the percentage increase in the index in order to determine the index-based interest rate. Participation rates are dependent upon interest rates and call option costs and, consequently, are determined separately at the beginning of each period during which they are guaranteed. The highest participation rates are for point-to-point products and lowest for ratchet products. Participation rates usually are in the range of 70% to 100%. According to

Leavey [Leavey 1999], some states prescribe a lower bound on participation rates of 40%.

Spread Deduction is a deduction from the percentage increase in the index in the calculation of index-based interest. The use of this deduction is to finance the downside risk protection.

The *Benefit Cap* is a maximum applied to either the annual or the cumulative index-based interest rate.

The participation rate, spread deduction, and cap are generally guaranteed at their current level either annually or for each index term period.

2.1.6 Minimum Return Guarantees

Equity-indexed deferred annuities contain a *minimum return guarantee*. The annuities guarantee to return at least a portion of the premium at the end of the index term period and an additional amount in form of interest. The amount of guarantee is generally a percentage of the consideration applied at the beginning of the period with accumulation at a specified rate of interest. The minimum is the Standard Nonforfeiture Law minimum, i.e., 90% of premium accumulated at 3% for single premium contracts and 65% of first year premium and 87.5% of subsequent premium for flexible premium contracts. Most common are 90% accumulated at 3% and 100%

accumulated at 3% or a higher rate. Generally, the sum of premium and index-based interest are compared against the fixed return guarantee, which serves as a minimum guarantee. A variation is to add the index-based interest to the guarantee. The minimum guarantee can be transferred to subsequent index term periods in three different ways. The first possibility is to compound the initial guarantee at 3 percent all the time. This provides the lowest guarantee value. A higher value generally is provided if each index term resets the guaranteed value at the maximum of the previous term guarantee and 90 percent of the amount of the contract value at the end of that term. The highest value is provided if the maximum of the greater of the guarantee at the end of the previous term and the contract value at the end of that term period minus 10 percent of the initial premium paid is used.

Index-based interest can be credited to the contract value either when it is calculated or at the end of the term. Interest in point-to-point contracts invariably is credited at the end of the term because its amount is unknown until then. Interest in other types of interest calculation methods is credited to the contract value at the time it is determined, generally annually, if the cash surrender value is a percentage of the contract value. However, it is credited either annually or at the end of the term if the cash surrender values are determined as a percentage of the

guaranteed return. Usually interest is credited before the deduction of fees [The Advantage Group 2002].

If index-based interest is credited before the end of a term it may be subject to *vesting*. This is the percentage of the interest used to calculate the cash surrender value. The vested percentage usually increases yearly and reaches 100% at the end of the term.

According to the American Academy of Actuaries [American Academy of Actuaries 1998b], there are several *cash surrender value* designs:

- The first existing design in the market subtracts a percentage surrender charge from the contract value. The percentage surrender charge can be subtracted from the current contract value or from the premium. The vesting percentage is applied to the contract value. At the beginning of each index term period, this methodology usually is repeated.
- The option is to subtract the percentage surrender charge from the guaranteed value. If the guaranteed value is greater than the minimum required by the Standard Nonforfeiture Law, the cash surrender value might be calculated based on the guaranteed value.
- Another possibility is to use the guaranteed value. If the guaranteed value is equal to the minimum required by the

Standard Nonforfeiture Law, the cash surrender value might be used as the guaranteed value.

- Imputed Ultimate Annual Returns sometimes are the basis of the cash values' calculation. In this approach the cumulative index-based interest return is distributed along the number of years in the full term and so translated into an imputed annual return. Then a spread deduction is used to reduce this understated annual return and the result is then accumulated for the number of actually elapsed years.
- No cash surrender value can only be used within group contracts. Nonforfeiture values are required at all times under individual contracts if they are available at any time.

Under various circumstances, partial withdrawals or surrender without surrender charges or otherwise reduced values is available:

- The full contract value can usually be withdrawn in a 30 to 45 day window at the end of each index term period. The window either begins or ends with the end of the term.
- Many contracts allow the policyholder to withdraw a specified percentage annually, for example up to 15%, of the contract value or premium, which can either be the full value or the vested value without surrender charges. The free withdrawal is

often not available in the first contract year and there may be other restrictions, such as one withdrawal per contract year or one per each calendar year. If the contract credits interest only at the end of the term, the amount withdrawn might not be entitled to index-based interest credits.

- No surrender charges are often assessed for withdrawals required to satisfy laws and regulations on tax-qualified plans.
- Nursing home waivers, which permit free withdrawals in the event of confinement in a nursing home, and terminal illness waivers, which permit free withdrawals when death is diagnosed as being imminent, are frequently included in the contracts.
- Since the withdrawal options usually provide for a lot of flexibility, policy loans are usually not offered. Sometimes policy loans are available because of the requirements for 403(b) plans.

The *minimum cash surrender value* is determined as the amount specified under the Standard Nonforfeiture Law. This is 90% of the premium accumulated at 3% for single premium contracts and 65% of first year premium and 87.5% of subsequent premium for flexible premium contracts.

Several death benefit designs are possible:

- Full Contract Value is the most common death benefit. For contracts with annual index based interest crediting, this is the most recent anniversary's contract value. For contracts in which interest is credited only at the end of the term, the most recent anniversary before the date of death is assumed to be the end of the term and an interim interest is credited. Generally, for death benefit calculation purposes, vesting is recognized at 100%. A variation of these designs could use the actual date of death instead of the most recent anniversary to determine the index-based benefit.
- Guaranteed Value could be the death benefit. This is not common but could occur in contracts with a cash surrender value equal to the guaranteed value minus a percentage surrender charge.
- Specified Percentage of Premium could be the death benefit. This could occur if the cash surrender value is the Standard Nonforfeiture Law minimum or if there is no cash surrender value.

Equity-indexed deferred annuity contracts are available both as single premium annuities and flexible premium annuities. Each flexible premium payment is generally treated in as a single premium, which means that it establishes the beginning of an index term period.

Nevertheless, it is possible to accrue premiums in a daily interest account during a contribution window until an adequately large amount has been collected or until the window closes. The longest possible period that a premium has to remain in a daily interest account before it starts participating in index development is a contribution window. The contribution window could be a month, a quarter, a year, or possibly even longer. At the end of the contribution window, the total accrued premium in the daily interest account is transformed into one single payment, which is transferred into an equity-indexed account, which is called “bucket” in the American Academy of Actuaries response to the Securities Exchange commission [American Academy of Actuaries 1998b]. The number of equity-indexed buckets depends on the existence and use of contribution windows in a contract, the length of the contribution window, and the length of the index term period. The number of buckets decreases with increasing length of the contribution window and it decreases with decreasing length of the index term period. Premiums received during a contribution window accrue interest in the daily interest account. The contractual guaranteed minimum interest rate is the minimum interest rate credited in this account. Insurance companies may credit higher interest rates, which may be based on their current credited rates on fixed products.

Almost all contracts are supported by assets carried in the general account of the insurer. Some contracts make use of a separate account. The reasons for using a separate account are not related to the equity-index characteristic. One possible reason might be the use of a market value adjustment formula.

Most contracts offer several different choices at the end of each index term, although some automatically continue either another index term or shift to a fixed annuity. Generally, the following choices are available:

- Renew for Another Term. The length of the renewal term can be chosen from among the term lengths offered in the contract. The amount used at the beginning of the renewal term is the amount of the contract value at the end of the currently ending term. The adjustment factors like participation rate, spread deduction, or cap are reset for the renewal term. The surrender charge schedule generally starts over for the renewal term.
- Continue as a Fixed Annuity. The initial amount is the amount of the contract value at the end of the currently ending term.
- Make Withdrawals. At the end of the term, a portion or all of the contract value can generally be withdrawn without the assessment of a surrender charge.

- Most contracts offer only the standard options available with fixed annuities. However, equity index-based annuitization options may be available.

The equity indexed annuity feature is available in a variety of combinations with other annuity alternatives:

The equity-indexed annuity might be a stand-alone contract. There might be several choices of index term period offered.

The equity-indexed annuity might be offered in combination with fixed annuities. The contract might allow allocations and switching between equity-indexed and fixed alternatives at the end of each term.

The equity-indexed annuity might be an alternative within a variable annuity contract.

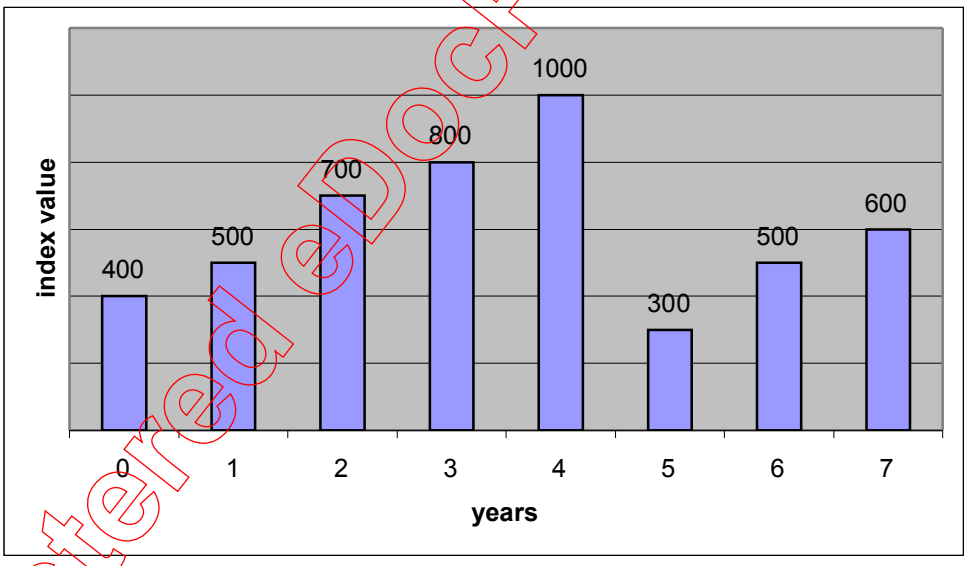
The equity-indexed annuity is basically a fixed annuity with a different way of determining the credited interest rate. Therefore, equity indexed annuities can contain any feature which might be found in a traditional fixed annuity. Current designs offer bonus interest rates, two-tier structures, and market value adjustments.

Contracts generally are issued on a weekly or bi-weekly basis in order to be able to combine larger amounts of premium for the efficient purchase of hedging options.

2.2 Examples of Equity-Indexed Deferred Annuity Designs

In the following paragraph, some of the different policy designs are presented and sample calculations for those designs should point out the differences between the designs. For the sample calculations, a fictitious index development will be assumed, depicted in the following figure. The annuity term will be assumed to be seven years. These examples are following the concepts of Bodmayr [Bodmayr 1998].

Figure 1: Assumed Index Values



Source: Bodmayr 1998

First, the above figure should be used to point out, which index values would be used for which calculation method.

The point-to-point design simply looks at the index value at the beginning of the annuity term, which would be 400 in the above figure. Then, the index value at the end of the term is taken, which is 600 and the ratio is formed.

The high watermark design uses the highest anniversary value in the policy term, in this case 1000. This value is compared to the value at the beginning of the term, which would be 400.

The low watermark design compares the lowest anniversary value, 300 in this case, to the value at the end of the term, 600 in this case.

The ladder design would assume several index determining points. For example, in the above figure there could be a three-year index period first and a subsequent four-year index period. This means that the index value after three years would be determined, in figure 1 this would be 800, and at the end of the second period the index value would also be determined, in figure 1 this is 600. Then the maximum of those two index values would be compared to the beginning index value and interest would be credited according to this ratio.

The annual ratchet design is a little more sophisticated. It determines the index value at the end of every year and compares it to the value at the beginning of the particular year. For example, in figure 1

interest for the first period would be credited using the value at the beginning of the term (400) and at the end of the term (500). This is done every year for the whole policy term. According to figure 1, in the fifth policy year the index crashed from 1000 to 300. This would imply that for policy year five no index-based interest is credited. However, in the sixth year, the market recovered and index-based interest would be credited again.

In order to clarify the following calculations, which are following the concepts of Bodmayr [Bodmayr 1998], some contract features and some specific index values should be noted separately. The annuity is an equity-indexed deferred single premium annuity. In addition to the index values from figure 1, it is assumed that the average index value over the last year of the index term equals 550. The minimum guaranteed values are the Standard Nonforfeiture Law minimum guarantees.

Table 1: Contract Features

Contract feature	*Notation*	*Value*
Index term	T	7 years
Premium in $	P	10000
Participation rate	α	75%
Index value at issue	I_0	400
Average index value over last year of index term	$I_{7,avg}$	550
Maximum index value over index term	$I_{\max}$	1000
Minimum index value over index term	$I_{\min}$	300
Index value at the end of index term	I_7	600
Minimum interest rate	i_{grt}	3%
Annuitization value at issue	A_0	10000
Guaranteed annuitization value at end of index term	A_7	12298.74

Source: Bodmayr 1998

- The point-to-point method with assumed averaging at the end of the index term would then involve the following calculations:

  First, the index increase is applied to the annuitization value at issue, the resulting value is denoted as A_{index}:

  $$A_0 \cdot \frac{I_{7,avg}}{I_0} = 10000 \cdot \frac{550}{400} = 13750 =: A_{index}$$

  Then, the exceeding increase S_{index} over the guaranteed minimum is calculated:

  $$A_{index} - A_7 = 13750 - 12298.74 = 1451.26 = S_{index}$$

  The participation rate is applied to the exceeding increase and the adjusted increase is denoted as S_{adj}:

  $$\alpha \cdot S_{index} = 0.75 \cdot 1451.24 = 1088.45 =: S_{adj}$$

  The final value of the equity-indexed annuity is then:

  $$A_7 + S_{adj} = 12298.74 + 1088.45 = 13387.19$$

  The effective annual interest rate is then 4.26%

- The high watermark method calculates the annuity value as follows:

First, the index increase is applied to the annuitization value at issue, the resulting value is denoted as A_{index}:

$$A_0 \cdot \frac{I_{\max}}{I_0} = 10000 \cdot \frac{1000}{400} = 25000 =: A_{index}$$

Then, the exceeding increase S_{index} over the guaranteed minimum is calculated:

$$A_{index} - A_7 = 25000 - 12298.74 = 12701.26 = S_{index}$$

The participation rate is applied to the exceeding increase and the adjusted increase is denoted as S_{adj}:

$$\alpha \cdot S_{index} = 0.75 \cdot 12701.26 = 9525.95 =: S_{adj}$$

The final value of the equity-indexed annuity is then:

$$A_7 + S_{adj} = 12298.74 + 9525.95 = 21824.69$$

The effective annual interest rate is then 11.79%

- The low watermark method calculates the annuity value as follows:

First, the index increase is applied to the annuitization value at issue, the resulting value is denoted as A_{index}:

$$A_0 \cdot \frac{I_7}{\mathrm{I}_{\min}} = 10000 \cdot \frac{600}{300} = 20000 =: A_{index}$$

Then, the exceeding increase S_{index} over the guaranteed minimum is calculated:

$$A_{index} - A_7 = 25000 - 12298.74 = 7701.26 = S_{index}$$

The participation rate is applied to the exceeding increase and the adjusted increase is denoted as S_{adj}:

$$\alpha \cdot S_{index} = 0.75 \cdot 7701.26 = 5775.95 =: S_{adj}$$

The final value of the equity-indexed annuity is then:

$$A_7 + S_{adj} = 12298.74 + 5775.95 = 18074.69$$

The effective annual interest rate is then 8.82%

- The annual ratchet method calculates the annuity value as follows:

  First, the index increase is applied to the annuitization value at issue, the resulting value is denoted as A_{index}:

  $$A_0 \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot k_5 \cdot k_6 \cdot k_7 =$$
  $$= 10000 \cdot 1.25 \cdot 1.4 \cdot 1.14 \cdot 1.2 \cdot 1.0 \cdot 1.67 \cdot 1.2 = 47975.76 =: A_{index}$$

  with the $k_i, i \in 1,..,7$ being the respective annual index changes. For example, k_1 is calculated as $k_1 = \frac{500}{400} = 1.25$.

  Then, the exceeding increase S_{index} over the guaranteed minimum is calculated:

  $$A_{index} - A_7 = 47975.76 - 12298.74 = 35677.02 = S_{index}$$

The participation rate is applied to the exceeding increase and the adjusted increase is denoted as S_{adj}:

$$\alpha \cdot S_{index} = 0.75 \cdot 35677.02 = 26757.77 =: S_{adj}$$

The final value of the equity-indexed annuity is then:

$$A_7 + S_{adj} = 12298.74 + 26757.77 = 39056.51$$

The effective annual interest rate is then 21.49%

The different effective annual interest rates can be used to rank the different index crediting methods. However, these rankings will always depend on the index development. Therefore, one should be careful to generalize statements about one design outperforming the other. This can change significantly with a changing index pattern. Often, in very volatile markets the ratchet design shows some advantages over the point-to-point design since it locks in possible gains.

2.3 Design and Features of an Equity-Indexed Immediate Annuity

According to the American Academy of Actuaries [American Academy of Actuaries 1998b], equity-indexed immediate annuities are immediate annuities that tie all or a portion of the benefits payable to the performance of an external index. These annuities can include any features, which can be found in fixed immediate annuities. Equity-indexed immediate annuities are new to the market and currently appear

only in limited designs, while equity-indexed deferred annuities are offered by many companies and reflect many different designs. This description is primarily based on currently available products and does not claim to be complete, particularly in view of future developments. An equity-indexed immediate annuity can be described in terms of the type of annuity payout, assumed interest rate, minimum payment guarantees, usage of averaging of index values, index used, participation rate guaranteed, and length of participation rate guarantee. For example, one could buy a life annuity based on a 3.25% assumed interest rate with payments never below the initial payment, based on the S&P 500 using annual index values, with 80% participation guaranteed for 5 years. Another possible equity-indexed immediate annuity design may be a 10 year certain annuity based on a 4% assumed interest rate with payments never below the previous payment, based on the S&P 500 using annual index values, with 90% participation guaranteed for 7 years.

Equity-indexed immediate annuities are composed of many separate design components, which can be put together in many different ways. It is crucial when evaluating the different product designs that no design is inherently financially superior to any other design. If all other features of two products are identical, for example expenses, mortality, fixed investment yield, assumed interest rate, or profit margin, then the two products will have the same hedging cost and will have

equivalent value. However, they may have different participation rates because of the design differences. Therefore, the benefits of different products under a certain set of conditions will differ. However, the call option market will have priced the various possibilities such that equivalent value is available under all designs. Defining design elements and some of the possible design choices of equity-indexed immediate annuities are described below:

2.3.1 Assumed Interest Rate

The initial annuity benefit is determined by an assumed interest rate, which the insurer may allow to be selected by the annuitant. In the calculation of equity index adjusted annuity payments, the assumed interest rate is used as the required interest. Equity index based interest in above the assumed interest rate increases the annuity payment and interest below the rate decreases the annuity payment if there are no guaranteed payment levels.

2.3.2 Minimum Payment Guarantees

There are several payment level guarantees, which can be offered with the annuity payments:

Initial Payment Amount guarantees make sure that no payment will be less than the first annuity payment. This is analogous to a point-to-point benefit in an equity-indexed deferred annuity.

Previous Payment Amount guarantees make sure that no payment will be less than the previous annuity payment. This is analogous to a high watermark benefit in an equity-indexed deferred annuity.

Ratchet Payment guarantees give an increase over the most recent annuity payment if equity index based interest exceeds the assumed interest rate. This is analogous to a ratchet benefit in an equity-indexed deferred annuity.

The annuity amount could be changed as often as the payments are made. Nevertheless, annual adjustments may be the most practical frequency, regardless of the frequency of the annuity payments.

2.3.3 Equity Index Used

Any index can be used as an underlying index to determine index-based interest, as long as it is published and there are no licensing restrictions. In addition, insurers can construct their own indices. The availability of hedging instruments is a crucial factor for the choice of indices. Equity indices generally reflect the movement in the price level of the underlying stocks and do not include value growth due to dividend payments. Most equity-indexed annuities in the U.S. use the S&P 500

Index as the underlying index for two reasons. First, it is one of the indices most easily recognized by potential customers and second the call options needed to hedge the risk are readily available and liquid. Hedging will be discussed in-depth in chapter IX.

2.3.4 Averaging

The simplest form of index measurement uses the index value of a single day, usually the last day of the term. However, averaging of several index values could be used in order to reduce the volatility of the index increase measurement or to moderate the change in the annuity payment.

2.3.5 Participation Rate

The index-based interest rate credited in annuity payments is some portion of the increase in the index over the period being measured and it is called the participation rate. Interest rates and call option costs are two determining factors for participation rates. Since those two factors change constantly, participation rates are determined separately at the beginning of each period during which they are guaranteed. The highest participation rates are credited for initial payment amount guarantees and the lowest participation rates are credited for ratchet guarantees.

2.3.6 Participation Rate Guarantee

The participation rate can be guaranteed for any period. However, generally it is guaranteed for a certain number of years, after which it would be redetermined and guaranteed for another period. The subsequent periods may have a minimum participation rate guarantee. The assets supporting the equity-indexed immediate annuity are held in the insurance company's general account if there is no design feature, other than the equity index feature, which would result in the need of using a separate account.

The equity indexed immediate annuity contract features occur in different types. The equity-indexed immediate annuity can be a stand-alone contract. The contract may be combined with fixed alternatives and might allow allocations between equity indexed and fixed alternatives. The equity indexed immediate annuity might be a settlement option in form of a payout alternative within an annuity which itself may or may not have equity index features.

The equity indexed immediate annuity is basically a fixed immediate annuity with a different way of determining the annuity payments. Therefore, equity indexed immediate annuities can contain any features that a traditional fixed immediate annuity contains.

CHAPTER III

MARKET OVERVIEW FOR EQUITY-INDEXED ANNUITIES AND VARIABLE ANNUITIES

This chapter provides an overview over the markets for equity-indexed annuities and variable annuities in the U.S. First, we will look at the sales figures of equity-indexed annuities and variable annuities to get an idea why these types of products should be considered important parts of the annuity market and the retirement system in the U.S. Then we will classify annuities by the point in time when their payout period starts. Next, we will distinguish between variable annuities, fixed annuities as a benchmark and another group that contains equity-indexed annuities.

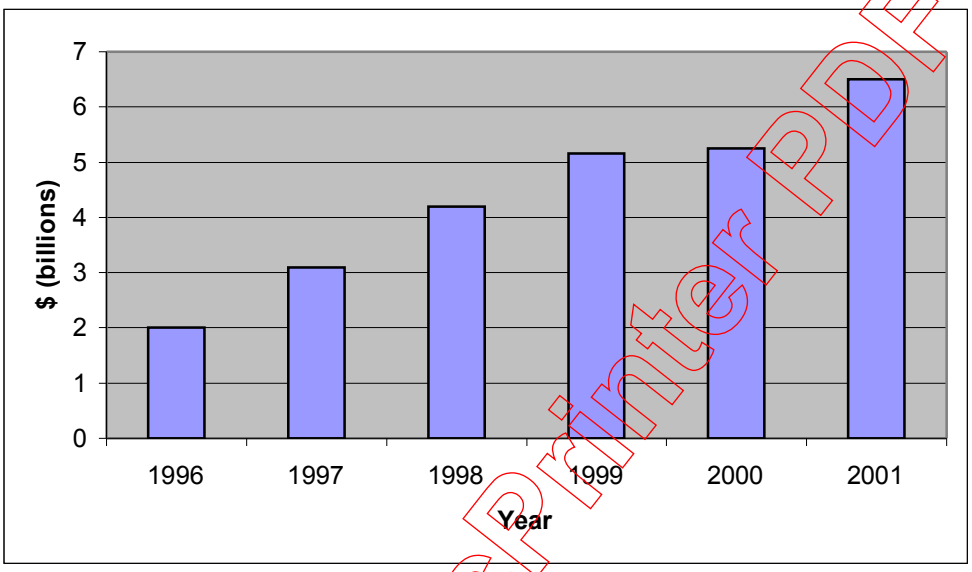
The idea of equity-indexed annuity (EIA) has its roots outside the U.S., mainly in the United Kingdom. Guaranteed equity life and annuity products have a market share of around 25% of all products sold in the UK. In the US, several banks offered equity-indexed certificates of deposit (CDs). In the 1980s, Fidelity Benefit, a subsidiary of the First Capital Holdings Corporation, offered an EIA contract similar to today's

products. However, this product was not very successful because of Fidelity Benefit's insolvency, which was triggered by the financial difficulties of First Capital caused by reductions in assets and premiums. The equity-indexed annuity did not contribute to this insolvency since it was a relatively new a not very widespread product.

Keyport Life Insurance Company introduced the first successful equity-indexed annuity in the US in 1995. In the following year, approximately 35 carriers entered the market.

Equity-indexed annuities have been growing very fast ever since they were introduced. Since they are only in the market for seven years now one cannot really compare their sales figures to variable annuities. Right now, equity-indexed annuities are profiting from a bear market that favors investment guarantees. The following chart was compiled out of data from the Advantage Group's website [The Advantage Group 2002]. It shows the development in sales almost since the introduction of equity-indexed annuities.

Figure 2: Equity-Indexed Annuity Sales



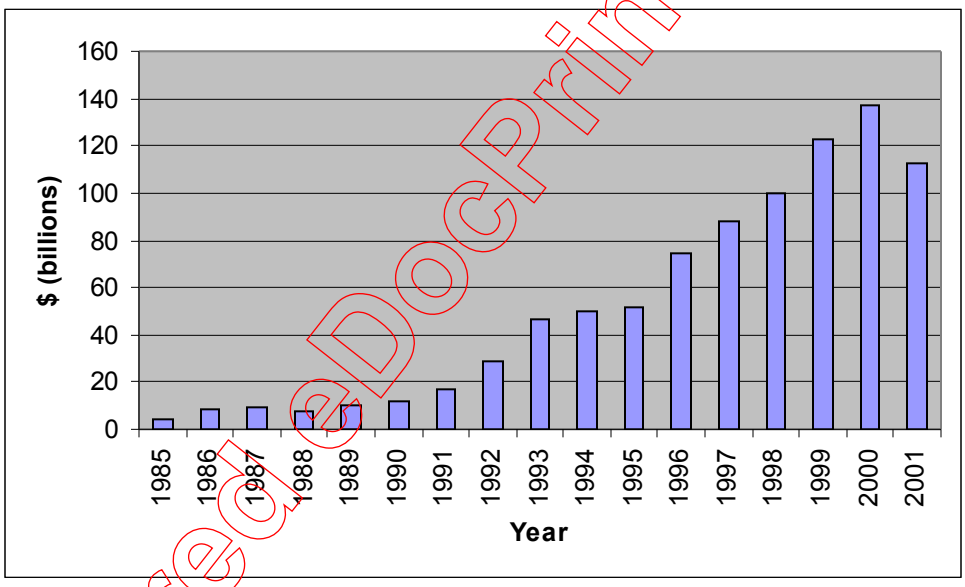
Source: The Advantage Group 2002

Variable annuities exist for a much longer time than equity indexed annuities. In 1952, the College Retirement Equities Fund (CREF) was established as the first variable annuity fund. Two years later, the Participating Annuity Life Insurance Company offered the first variable annuity contracts to the public.

In the 1990s, variable annuities profited largely from a bull equity market. Variable annuity sales soared because everybody wanted to participate in the positive development of the stock market. The question

of guarantees was a negligible one to the investors since the markets were only going in one direction. Since that trend stopped by the end of the year 2000 and the markets actually started to lose money people have become more aware of the volatile character of variable annuities.

Figure 3: Variable Annuity Sales



Source: Info-One 2001

The data for the following charts were obtained from the Life Insurance Fact Book published by the American Council of Life Insurers [American Council of Life Insurers 2001].

One possible classification of annuities is whether they are *group annuities* or *individual annuities*. Group annuities got their name from the fact that employer-sponsored retirement plans insure groups of people. In the year 2000, total contributions to group annuities reached $163.6 billion. The other group of annuities is called individual annuities since individual can also purchase annuities from life insurers. The demand for this type of annuity is growing since people want to save individually for the future besides their employer-sponsored retirement plans. From 1990 to 2000 individual annuity contributions were growing from $54 billion to $140 billion whereas group annuity contributions were growing from $75 billion to $164 billion. This shows that people are more concerned about their financial status in the future, particularly retirement.

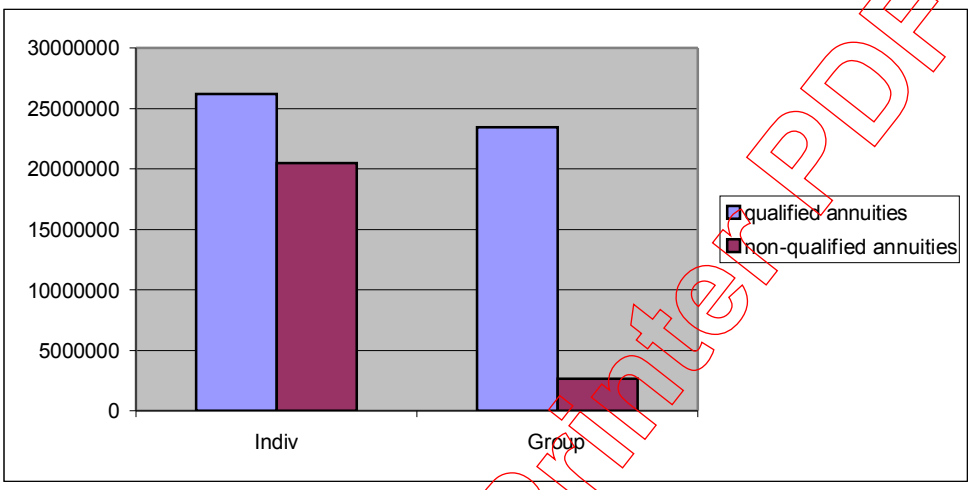
Retirement plans can be classified into *defined benefit plans* and *defined contribution plans*. Defined benefit plans provide a specified benefit for retirement whereas defined contribution plans specify the contributions to the retirement plan and the retirement income is dependent on the contributions and the performance of the investments. In the first case, the benefit amount is usually dependent on the individual's pre-retirement income and his or her duration of service. Profit-sharing, 401(k), 403(b), and 457 plans are defined contribution

plans. They are named after the sections of the specific paragraph in the retirement law.

Annuities can also be classified according to their tax treatment. Contributions to *qualified annuities* are made with pre-tax dollars and receive preferential tax treatment, which means that an individual has to pay income tax on the benefit payments he or she receives from the annuity, but the contributions are not taxed. Employer-sponsored retirement plans usually offer qualified annuities, which include defined benefitplans, 401(k), 403(b), 457, and other similar retirement savings plans.

Non-qualified annuities cannot claim this treatment. Nevertheless, investment income is tax-deferred until withdrawals are made. Since those annuities are still an insurance product, they receive this special treatment of investment income.

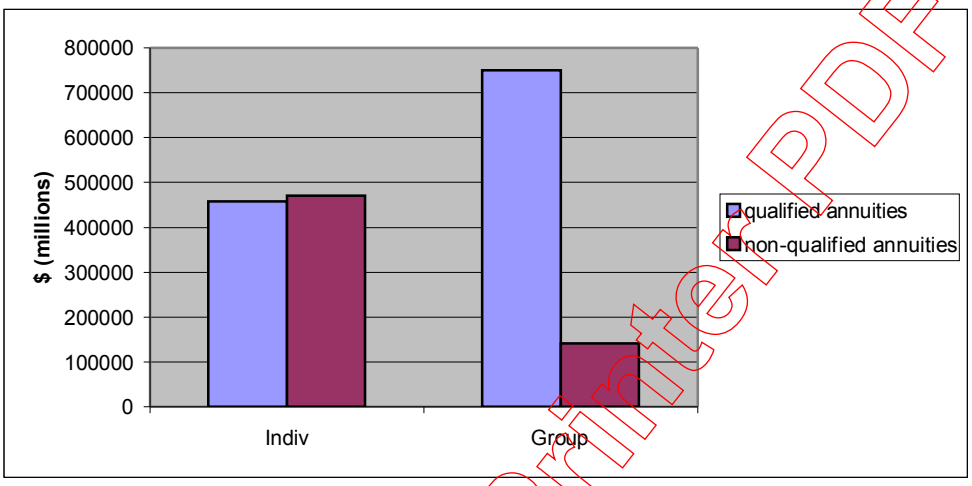
Figure 4: Number of Contracts, 2000 (Qualified vs. Non-qualified)



Source: American Council of Life Insurers 2001

The contracts in the group section are counted per group member. Most of the reserves for group annuities come from qualified annuities since most employers choose qualified annuities for their employer-sponsored retirement plans. 84.1% of the group annuity reserves are for qualified annuities.

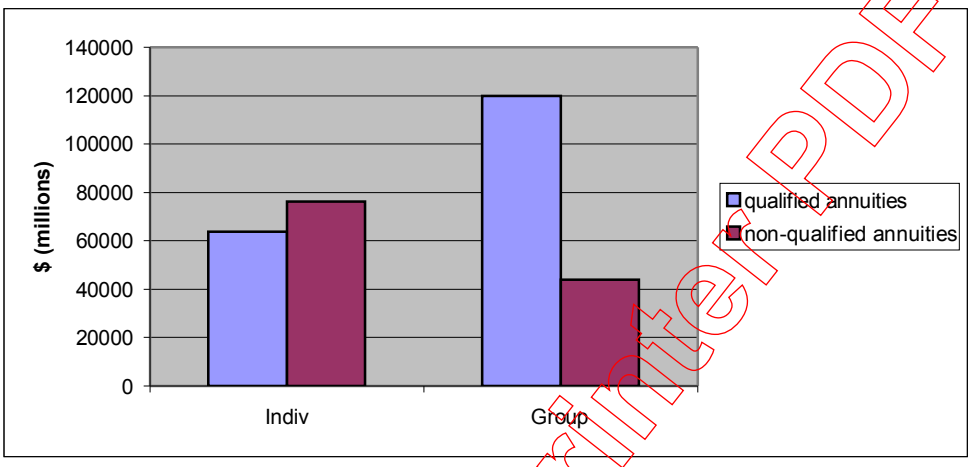
Figure 5: Reserves, 2000 (Qualified vs. Non-qualified)



Source: American Council of Life Insurers 2001

Considerations, which are all payments made into an annuity, also show most of the premium payments for group annuities go to qualified annuities, as much as 73.2%.

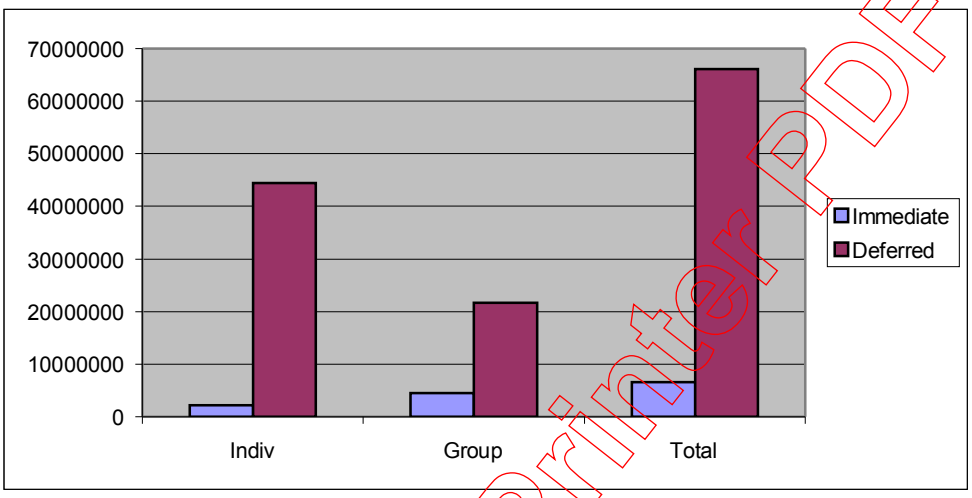
Figure 6: Considerations, 2000 (Qualified vs. Non-qualified)



Source: American Council of Life Insurers 2001

In general, one can classify annuities into *immediate* and *deferred* annuities. The difference between those two is that the payout for an immediate annuity starts soon after the premium is paid, usually one month after the payment, whereas the payout for a deferred annuity starts at some point of time in the future. An analysis of annuity sales, considerations, and reserves shows that deferred annuities dominate the market by far. This should not be surprising since annuities are mainly used as a retirement investment vehicle and to save for the future, so that the policy owner does not outlive his or her assets.

Figure 7: Number of Policies, 2000 (Immediate vs. Deferred)

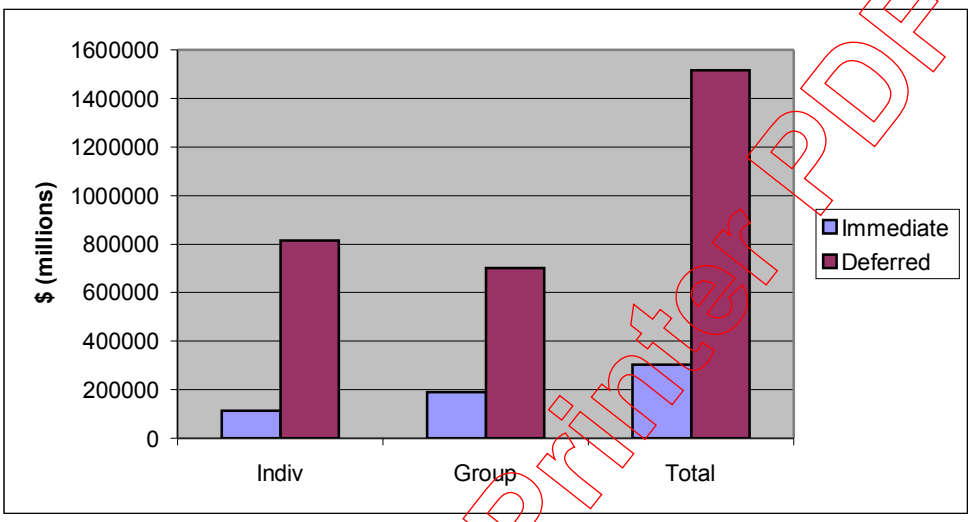


Source: American Council of Life Insurers 2001

In the group of individual annuities, deferred annuities account for 95.3% of the contracts. For group contracts, this figure is slightly smaller with 82.9%.

The reserves back up the impression that deferred annuities dominate the market. 87.8% of the reserves for individual annuities are reserves for deferred contracts.

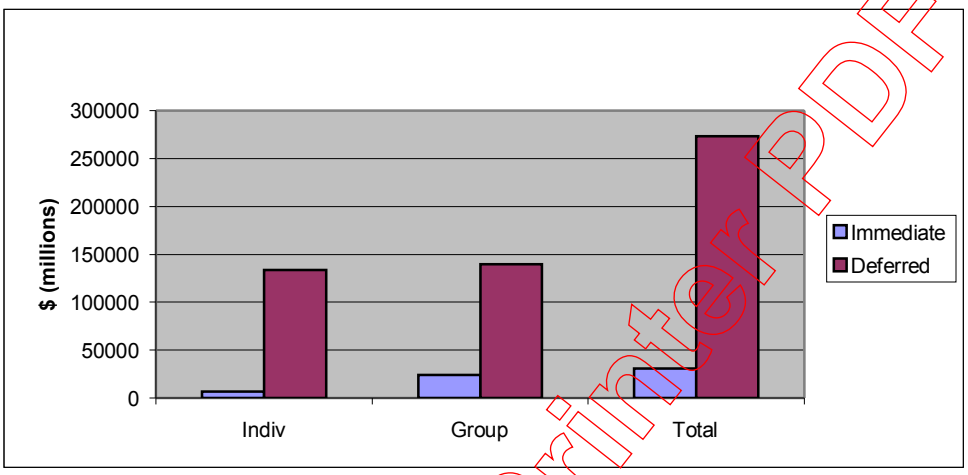
Figure 8: Reserves, 2000 (Immediate vs. Deferred)



Source: American Council of Life Insurers 2001

Another important element of annuities is *considerations,* which was defined above as all the payments into annuities. Looking at this number, deferred annuities again are dominating. They account for 95.2% of all individual annuity considerations and for 85.5% of all group annuity considerations.

Figure 9: Considerations, 2000 (Immediate vs. Deferred)

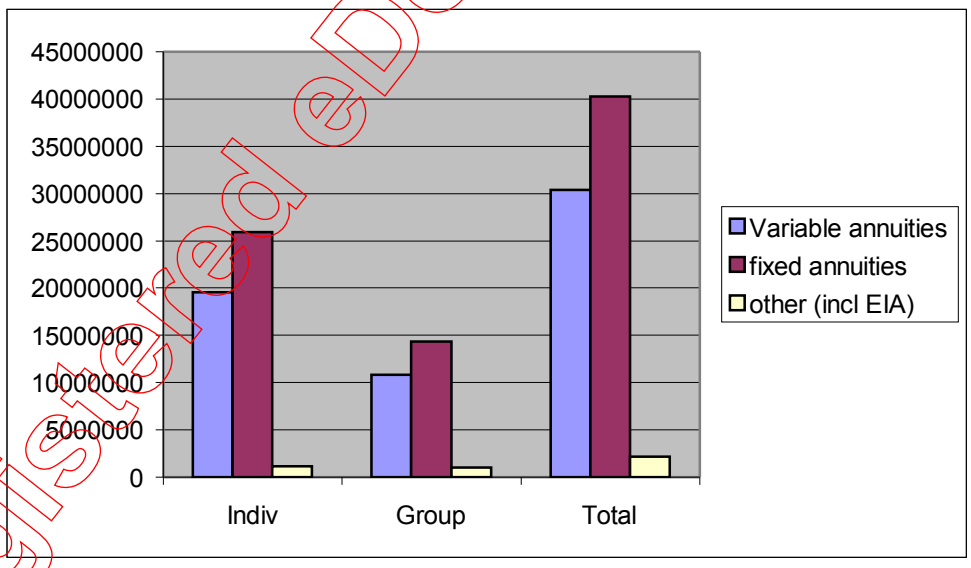


Source: American Council of Life Insurers 2001

Another possible classification of annuities is whether they are *fixed* or *variable*. Equity-indexed annuities legally are considered fixed annuities, but they have partly a variable character. Because of this and since they are such a new product the American Council of Life Insurers [American Council of Life Insurers 2001] included them in another group that collects all types of annuities that are not variable and not fixed. Looking at the number of policies, one can see that fixed annuities still are leading the market. This fact might originate from the 1970s when interest rates were soaring and people started to buy that kind of annuities. However, since the last decade variable annuities are catching

up. In the second half of the 1990s the stock market exploded and people wanted to participate in that growth. Therefore, instead of buying fixed annuities to save for retirement, they bought variable annuities. There is a trend of an increasing market share of variable annuities. Since equity-indexed annuities are relatively new to the market, their market share seems dwarfed compared to the other two types. Nevertheless, as it can be seen above, their market share is constantly growing ever since they were introduced.

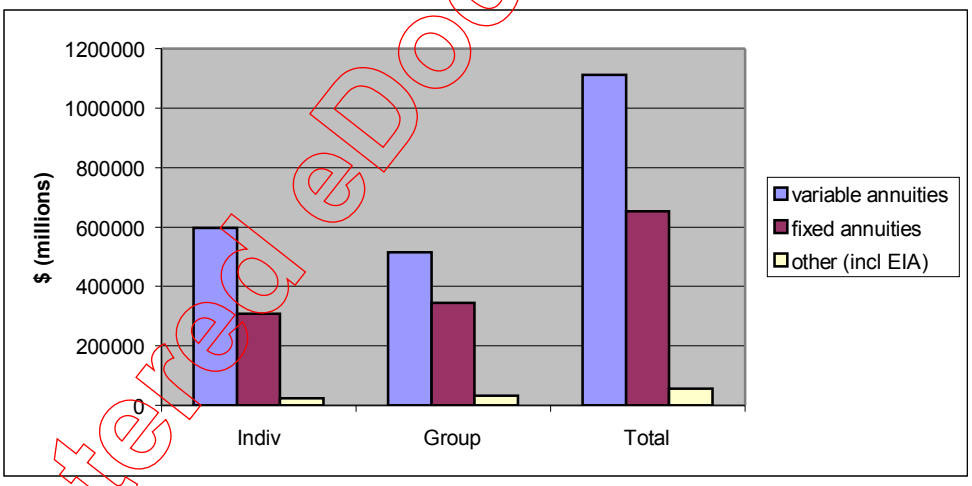
Figure 10: Number of Policies, 2000 (Variable vs. Fixed vs. Other)



Source: American Council of Life Insurers 2001

The reserves back up the above statements. For variable annuities one has to keep in mind that 70 to 80% of the reserves are kept in separate accounts, which means that their value can also decrease since they are exposed to the securities market. Yet, reserves for variable annuities account for 64.2% of individual annuities reserves and 57.8% of group annuities reserves.

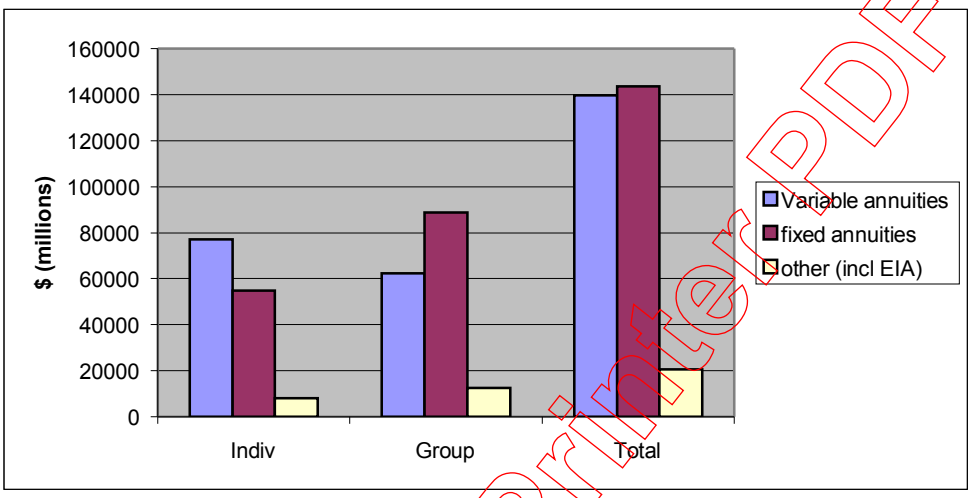
Figure 11: Reserves, 2000 (Variable vs. Fixed vs. Other)



Source: American Council of Life Insurers 2001

The considerations show a slightly different picture. For group annuities, most premiums were deposited in fixed annuities.

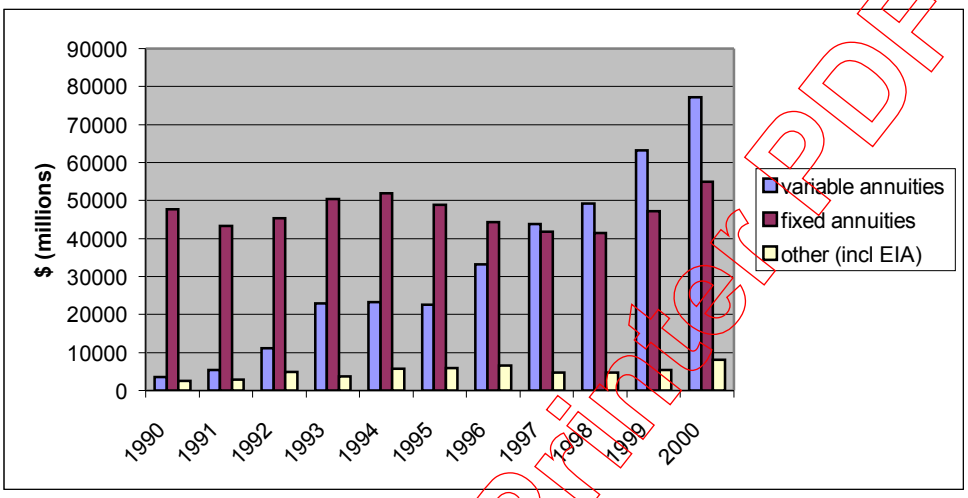
Figure 12: Considerations, 2000 (Variable vs. Fixed vs. Other)



Source: American Council of Life Insurers 2001

Looking at the historical development of the considerations for individual annuities one can clearly identify the trend towards variable annuities. Equity-indexed annuities have yet to prove in the market that they are a very good retirement savings vehicle. Fixed annuities were basically following a wave pattern, probably induced by the interest rate movements, whereas variable annuities considerations, driven by the bullish securities market, were always increasing at dramatic rates of up to 100% in the beginning of the 1990s.

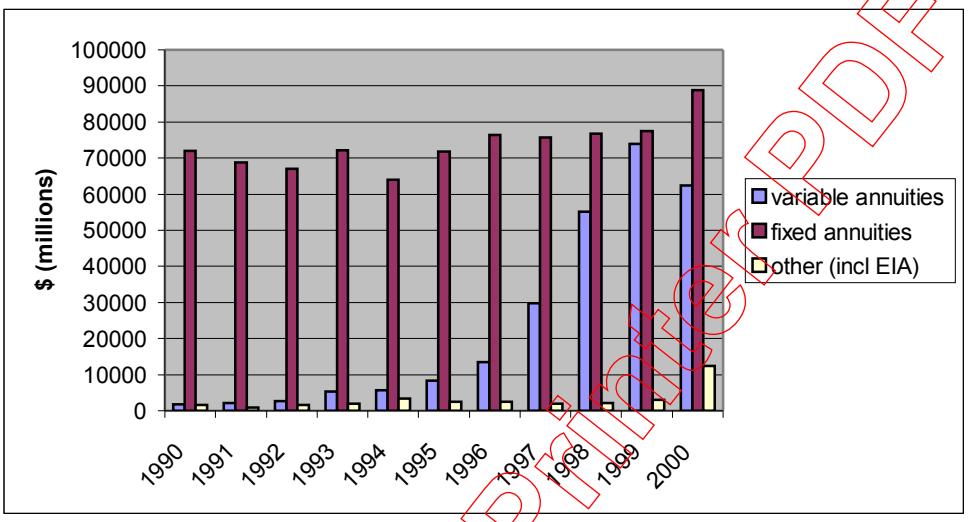
Figure 13: Considerations for Individual Annuities by Type, 2000



Source: American Council of Life Insurers 2001

This same trend can partly also be identified for considerations for group annuities. Note that in 2000 for the first time, the considerations for variable annuities were declining and the group including equity-indexed annuities was multiplying the market share.

Figure 14: Considerations for Group Annuities by Type, 2000

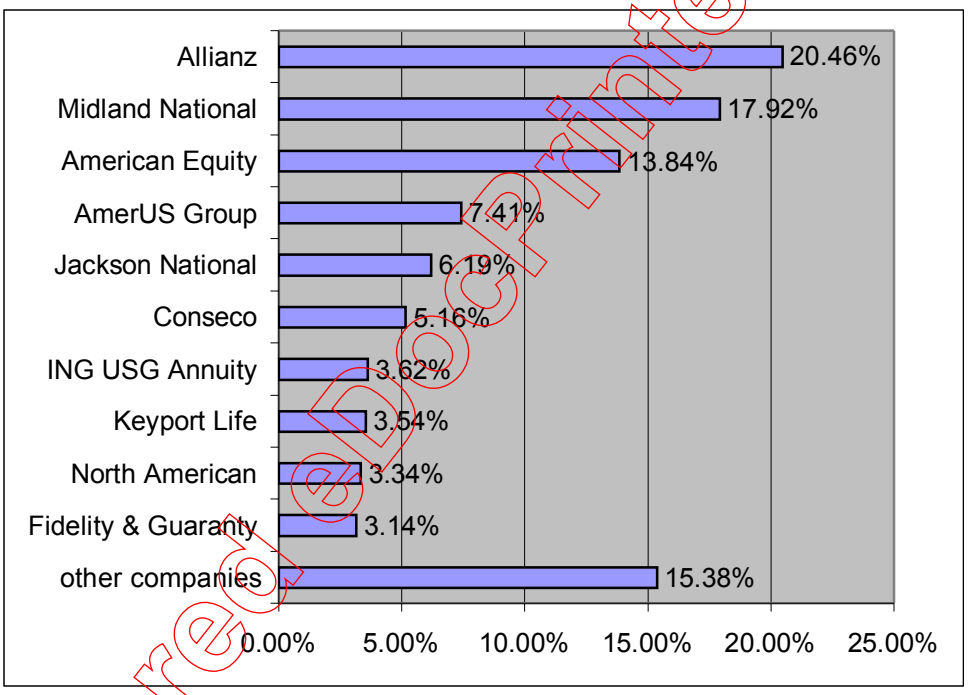


Source: American Council of Life Insurers 2001

The equity-indexed annuity market in 2001 was distributed amongst 37 companies. This is the lowest number in recent years. The top ten companies accounted for 85% of equity-indexed annuity sales and the top five had 66% of the market. The data for the following chart were obtained from the Advantage Group which publishes equity-indexed annuity data on its website [The Advantage Group 2002]. Allianz Life insurance company led the market in 2001 for the second consecutive year, which is somehow surprising since the equity-indexed annuities market is still very young and before Allianz's lead, changes at the top

were quite frequent almost every month. Midland National is a player that entered the market only recently and ranked number 2 in 2002 with an upward trend that could become dangerous for Allianz.

Figure 15: EIA Market Shares of the Top Ten Companies 2001



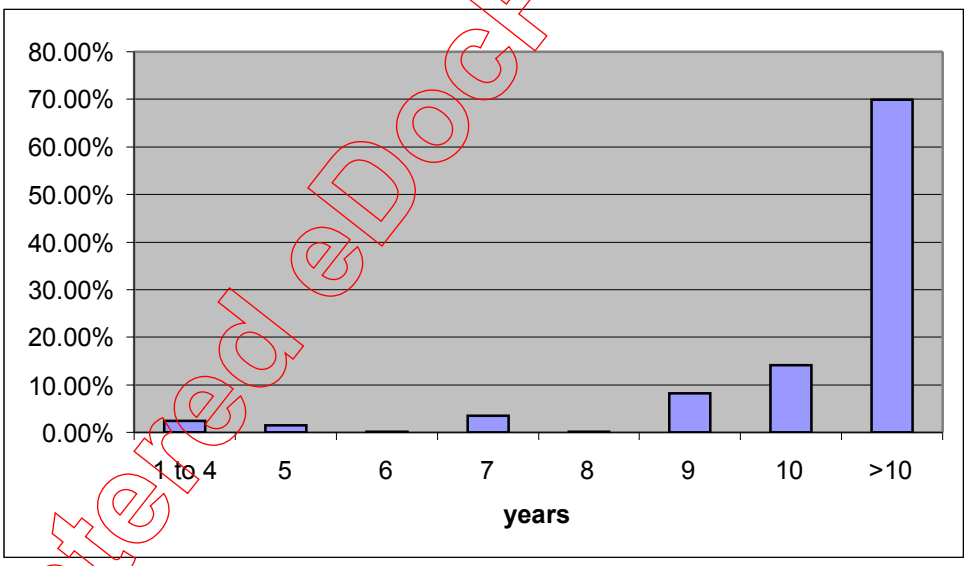
Source: The Advantage Group 2002

Annual reset structures and crediting structures using averaging dominated the market in 2001. Products using some degree of averaging

dropped from over 90% of sales to 83% of sales; annual reset designs represented over 92% of total sales.

Products with surrender periods of ten years or longer accounted for four out of five sales; the weighted surrender period based on product sold was 11.9 years. Index annuities with agent commissions of 9% or more represented 82% of index sales.

Figure 16: Surrender Periods in 4th Quarter 2001 Sales

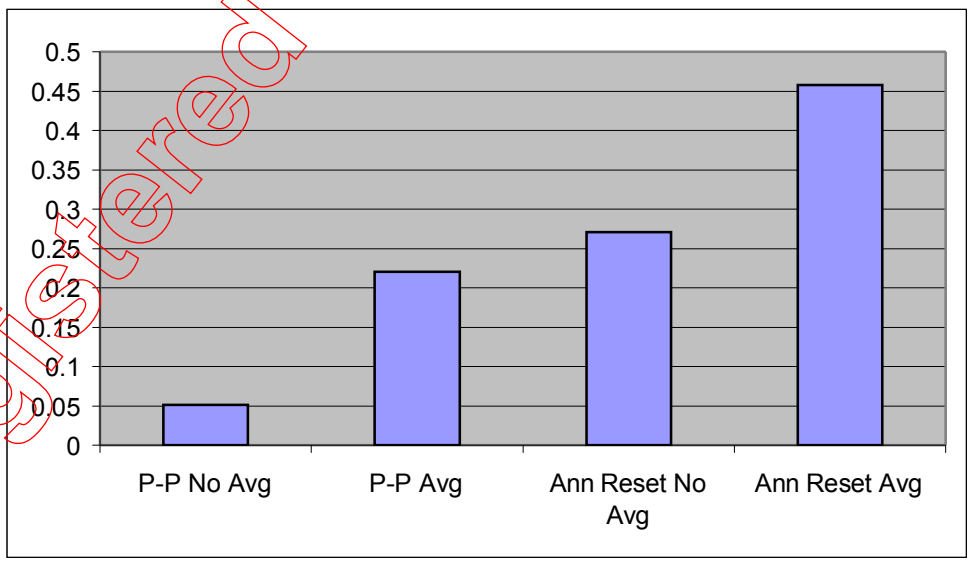


Source: The Advantage Group 2002

Insurance agents sold 95% of equity-indexed annuities. Banks, brokers, and dealers still avoided the market since they prefer to sell registered products. That might change in 2002 since the first registered equity-indexed annuity issued by ING entered the market.

The products sold in the market can be distinguished by the interest crediting methodologies used. In the 4th quarter of 2000 according to the Advantage Group [The Advantage Group 2002], the existing methodologies in the market were a point-to-point design and an annual reset. Both designs were used either with or without some form of averaging.

Figure 17: Methodologies, 4th Quarter 2000



Source: The Advantage Group 2002

Following is a list of companies that offer equity-indexed annuities as of May 2002. This list does not claim to be complete:

Table 2: Insurance Companies.

Allstate	Farmers New World	Midland National Life
American Equity	Fidelity & Guaranty	National Western
Allianz	Jackson National Life	North American
American Express	Great American	Northern Life
Americo	ING USG	Oxford Life
Americom Life	Jefferson-Pilot	Std Life of Indiana
AmerUS Group	Keyport	SunAmerica
BMA	Lafayette Life	Union Central
Clarica	Lincoln Benefit	Western United
Conseco	LSW	

Source: The Advantage Group 2002

It might also be interesting to know the reasons why consumers are buying variable annuities. Eric Sondergeld did a study [Sondergeld 2001] about this issue and found that 41% of all consumers buy variable annuities because it is an easy way of saving for retirement. 25% said they wanted to provide for a guaranteed monthly income in retirement, whereas for only 13% the main reason for buying variable annuities was, because variable annuities are an investment with growth with growth potential. Tax-deferral played an important role for only 17% of all consumers. This shows that consumers want to have some kind of secure retirement income. That is a good growth potential for equity-indexed annuities and for variable annuities with guarantees because both offer upside potential with downside protection. It is also important to know whether the consumers feel they understand the product or not. Sondergeld [Sondergeld 2001] asked annuity owners and non-owners if they understood how annuities work. 55% of annuity owners and 35% of non-owners answered they knew how annuities work. This is a rather small figure and it shows that insurance companies still have to do a lot of work to make people understand this concept and insurance companies should have an interest in educating their potential customers because people might be more inclined to buy annuities if they felt they understood them. This problem becomes even more complicated with equity-indexed annuities since agents have to explain

several concepts like index-based interest crediting method and averaging to the customers. In fact, the wide variety of possible designs of equity-indexed annuities and the possible different variations might be a problem for the growth of equity-indexed annuities. A customer has to understand the different concepts and in general, he or she cannot put two policies side by side and compare them because they will differ in several points, which make them not directly comparable. It is also very interesting that Sondergeld [Sondergeld 2001] found that there is a correlation between age, income, education and the question whether the consumer understands how annuities work. The higher income and education, the higher are the percentages of consumers who understand annuities. Also, the percentage of consumers who understand annuities increases with their age. This might come from the fact that people care more about their retirement when it is actually approaching. The typical annuity buyer also knows several thing about annuities. 73% know that they can choose different investment funds. 60% know that earnings from variable annuities are tax-deferred and 55% know that some variable annuities allow additional contributions.

In November 2000, the Gallup organization once again carried out its annual survey of owners of non-qualified annuity contracts. The survey showed that the average age of owners of non-qualified annuity contracts is 65 and that there are 52% male annuity owners. It also

revealed that 63% of annuity owners are married, while 20% are widowed and only 6% are divorced. 56% of annuity owners are retired, while 38% are employed either full-time or part-time. The survey also showed that a majority of 55% of annuity owners have annual household incomes between $20,000 and $74,999. 85% of annuity owners purchased their annuity before age 65 and the average age when the first annuity was purchased is 50. Eighty-one percent of surveyed annuity owners believe that people in the United States do not save enough money for retirement and 74% believe that the government should give tax incentives to encourage people to save. 88% of the surveyed people believe that they have done a very good job of saving for retirement. However, 47% are concerned that the costs of a serious illness or nursing home care might ruin them in retirement, and 36% fear that they might run out of money during retirement. 83% of the surveyed annuity owners say that they will use their annuity savings as a financial cushion in case they or their spouse live longer than their life expectancy, to avoid being a financial burden on their children, and for retirement income. 70% purchased an annuity to cover the potential expense of unpredictable events such as a catastrophic illness or the need for nursing home care, while slightly fewer purchased an annuity as financial protection against high inflation and bad performance of other investments. 91% agree that annuities are an effective way to save for retirement, that annuities are a good way to

ensure their surviving spouse has a continuing income, and that keeping the current tax treatment of annuities is a good way to encourage long-term savings.

CHAPTER IV

STOCK INDICES AND BOND INDICES

Equity-indexed annuities are annuities whose investment income is determined by an index. This automatically raises the question what indices are used. As mentioned in chapter II, any index can be used as long as it is published and the insurance company gets a license for using the index. Therefore, this chapter presents some stock market indices and some bond market indices that are currently used as underlying performance indicators for equity-indexed annuities. In addition, this chapter should answer the question how such indices are created.

As of September 2001, according to the Advantage Group [The Advantage Group 2002] more than 100 equity-indexed products were using the Standard & Poor's 500 (S & P 500) as the underlying index. The Dow Jones Industrial Average Index (DJIA) was used for 15 products, and the NASDAQ 100 was underlying ten products. Other stock indices used were the Russell 2000 for 9 equity-indexed annuities, the S & P 400 also for 9 products, and there was even one product with

an underlying mix of international indices, such as the London FTSE 100, the Paris CAC 40, the Frankfurt DAX 30, and the Tokyo Nikkei 225.

Bond indices used were the Lehman Brothers Aggregate Bond index for 2 equity-indexed annuities, the Lehman Brothers High Yield Bond index for another 2 equity-indexed annuities, and the Lehman Brothers U.S. Treasury index for 2 more products.

4.1 Stock Indices

The above data shows that the dominating index for equity-indexed annuities is the S & P 500 index. According to David M. Blitzer, managing director and chief investment strategist at Standard & Poor's [Blitzer 2001], the S & P 500 is the index that is used most often by professional money managers and investors. An estimated trillion dollars is indexed to the Standard & Poor's indices. The S & P Index began in 1923, and became the Standard & Poor's Composite with 90 stocks in 1926. In 1957, it was changed to include 500 stocks, 400 of which were industrial values, 40 were stocks from the financial industry, 40 were utility suppliers, and 20 of them were transportation companies. This composition changed in the mid-1980s when this composition was not adequate any more. The fixed numbers were dropped and Standard & Poor's later developed industry classification standards for all their indices. The number of 500 stocks is held constant. That means that if

one company vanishes because of a merger, Standard & Poor's replaces it with a new company's stock. In addition, Standard & Poor's also drops companies from the index for other reasons and replaces them with new ones. These changes amount to about 30 to 40 in an average year, according to Blitzer [Blitzer 2001]. Although drops and additions from or to the index are not investment recommendations, one can identify a correlation between the fact of a company being added or dropped and its financial well being in terms of prices of its stock.

A committee of seven Standard & Poor's staff runs the index by meeting monthly and deciding about companies that are in the index and might merge or about companies that might be dropped. This happens in accordance with several principles that govern these decisions. According to Blitzer [Blitzer 2001], the criteria a company has to meet to be added and kept in the index are:

- The company has to be a U.S. company. This is the reason why, for example, DaimlerChrysler is not in the S & P 500 any more. When Daimler merged with Chrysler, the Standard & Poor's committee judged that the company is not a U.S. company.
- The company should have a market capitalization of at least $3 to $5 billion.

- Half the company's stock should be in public hands.
- It should be liquid.
- It should be a "going concern".

Other criteria whether a company is added to the S & P 500 index are the industry and the sector to which this company belongs. Standard & Poor's wants to keep the mix of industries in the index as close to the overall market as possible.

Liquidity of the company's stock is very important since anytime a stock is added to the index, the index weights of all the other stocks will change. Index funds, which get their name from tracking the index, could not be able to do so by selling and buying stock if a large stock is not liquid. There are several large companies whose stock is not in the index. Probably the best known is Berkshire Hathaway. There is also permanent small under-weighting of technology and internet companies in the index, since Standard & Poor's demands some financial stability and profitability.

Despite the fact that today's requirements make it impossible to add foreign companies to the S & P 500, there are some foreign companies like Royal Dutch Shell, Unilever, and Alcan Aluminum. These companies were added long ago, before the criteria were specified or when they actually were U.S. companies, and today they are kept in the

index for historical reasons. The S & P 500 was always a *market-value-weighted-index*. This means that the portion of shares of a company in this index is weighted according to the market value of its outstanding equity. For example, assume a company, say General Electric, would have issued 1 million shares worth $10 and another company, General Motors, would have issued 25 million shares worth $2. Then General Motors would have five times the weight in the S & P 500, since the value of General Electric's outstanding equity is $10 million and the value of General Motors' outstanding equity is $50 million. To calculate the S & P 500 one needs to calculate the total market value of the 500 companies in the index on one day and the total market value of the 500 companies that were in the index the previous day. The percentage change in the total market value from one day to another equals the change in the index. The change in the index reflects the change in a portfolio of the 500 stocks held in proportion to outstanding market values.

A value-weighted index gives more weight to a company that has more outstanding market value and a lesser price compared to a company, which has a higher-priced stock, but less outstanding market value. For example, if company A's stock costs $10 and the company has issued 1 million stock and company B's stock costs $100, but the company has only issued 10,000 shares, then in a market-value-weighted index, changes in company A's stock will affect the index more

than changes in company B's stock. In a *price-weighted* index, company B's stock would have more effect on the index. Price-weighted indices will be explained later in the chapter since the Dow Jones is such an index.

From 1995 to 2000, only 30% of large cap equity mutual funds outperformed the S & P 500, from 1990 to 2000 this number was even less than 18%, according to Blitzer [Blitzer 2001]. This raises the question why it is hard to beat the index. One explanation might be transaction costs. To be able to explain the costs for financial instruments one has to know the concept of *basis points*. A basis point is one percent of a percent. This means that one basis point is equal to 0.01%. The transaction costs amount to 100 to 150 basis points for a managed mutual fund, including operating expenses and fees. Pension funds incur about the same cost, whereas retail index funds are far cheaper with a cost of approximately15 to 50 basis points. Institutional index funds have even lower transaction costs. Another explanation why index funds are cheaper to maintain might be that if one tracks the index constantly the transaction volume overall, called *turnover*, is lower than with a managed mutual fund. This incurs less transaction cost. A third reason might be that stocks in the S & P 500 tend to get more attention. Investment firms use the index to identify suitable companies for adding stocks to the company's analytical coverage. This means that if one owns the index, one owns popular companies.

Bodie notes in his *Investments* textbook [Bodie, Kane, Marcus 1996] that market value-weighted indices mirror the returns of buy-and-hold portfolio strategies. If an investor bought each share in the index in proportion to its outstanding market value, the index would perfectly track capital gains on the underlying portfolio. On the other hand, the S & P 500 has *survivorship bias.* Survivorship bias is a result of the tendency for poor performers to drop out while strong performers stay in the index. Therefore, if one is analyzing the performance of the index, the sample of current stocks will include those that have been successful in the past, while those that performed poorly and therefore were merged or dropped are not included. The result of survivorship bias is an overestimation of past returns and leads investors to be overly optimistic in predictions of future returns. This fact also makes it impossible to replicate the index with a buy-and-hold strategy, as the holdings must be periodically adjusted for the changes in the index. In addition, as dividends are paid and stock splits happen, appropriate adjustments in the buy-and-hold position must be made, and they involve substantial transaction costs.

The oldest and probably therefore the most popular stock market index is the Dow Jones Industrial Average (DJIA), which dates back to 1896, when it began as a 12 stock arithmetic average. In 1928, its present form was created with 30 stocks. The fact that it is an arithmetic

average makes it unsuitable for an analytical approach for investment analysis. In the beginning, the DJIA was computed as a simple average of the stocks included in the index. Assuming there were 30 stocks in the index, one would add up the value of the 30 stocks and divide the sum by 30. The percentage change in the DJIA would then equal the percentage change of the average price of the 30 stocks. An interpretation of this methodology is that the DJIA measures the return on a portfolio that consists of one share of each stock in the index. Since the percentage change in the average price of the 30 stocks equals the percentage change in the sum, the change in the index equals the change in the portfolio. This methodology is called a *price-weighted average*. The company' share price is the measure for the amount of money invested in the stock.

One problem connected particularly with the DJIA is that it is not any more equal to the average price of the 30 stocks in the index, which it used to be. This change was caused by the way mergers, stock splits, payouts of stock dividends of more than 10%, or replacements are handled. When one of these events happens, the divisor used for the computation is adjusted in order to leave the index unaffected by the event. While this always gives a smooth transition at such an event, which means that the index does not jump or fall because of such an event, it significantly changes the divisor, particularly if such events

happen very often. Usually such a change will decrease the divisor. Assume, for example, that two stocks, which are selling for $5 and $15, respectively, form an index similar to the DJIA. Their average price and therefore the index is equal to $10. Now assume that the second stock is split three-to-one. This means that the number of shares of the second has tripled and the price of the new stock is one third of the old price. Now the second company's stock would be priced at $5. Dividing the sum of the two new prices by two would result in a $5 average price. However, since the index should be kept at the same level, the sum of those two values and the previous index value are used to determine the new divisor. In the example, the new divisor would be one. Therefore, in this example the index divisor would have changed from 2 to 1. The divisor as of June 2002 is 0.014445222 [Dow Jones Indexes 2002]. The problem for index funds is that they cannot simply change their divisor. If they want to replicate the index, they have to sell stocks when a stock is split since they are supposed to have just one share of each stock in their portfolio. This is the replicating strategy for the DJIA. Therefore, the replicating portfolio ends up with a substantially different value than the index. While adjustments can be made every time, this would produce substantial transaction costs, and be quite inconvenient. In practice, as a result of these difficulties, and not being market-weighted, the DJIA is not investable. This issue is also the reason why there exist no futures

on the DJIA. In the previous example, the DJIA would still have an index value of $10, since the divisor has changed. The replicating portfolio would have to sell two of the three shares from the split and the new average value would be $5, which is substantially different from the DJIA value. Since usually successful stocks stay in the DJIA, splits are more likely to keep the stock price at a level that is interesting for investors. Besides the issue of stock splits, dividends are another problem. Theoretically, dividends would have to be invested to buy new stock of the same company since dividends decrease the stock's value. However, a replicating portfolio of the DJIA always keeps just one share of each stock.

Since the DJIA is based only on the relatively small number of 30 companies, the index managers have to pay particular attention to the requirement that the index should represent the broad market. Therefore, the composition of the index has to be changed sometimes to represent sector changes.

The editors of The Wall Street Journal select the companies included in the DJIA. This originates from the fact that The Wall Street Journal is issued by Dow Jones & Co. There are no special criteria for the companies except that they have to be U.S. companies and they should not be transportation or utility companies since for these types of companies there exist separate indices also calculated by Dow Jones.

4.2 Bond Indices

While stock indices have been in existence for more than 100 years, the first bond indices date back only to the 1970s. This is somehow astonishing if one takes into consideration the fact that the value of outstanding U.S. non-municipal bonds exceeds the combined value of equity in the U.S. One might ask oneself what bond indices are used for. First, since sales of fixed income funds have grown dramatically in the last two decades, investors and portfolio managers needed a benchmark to measure their portfolio's performance. Second, similar to equity indices, bond portfolio managers most often have not been able to outperform the aggregate bond market. In addition, the behavior of a particular index is vital to a bond portfolio manager who tries to replicate the performance of this index in his or her portfolio. Another purpose of bond market indices might be the documentation of changes in the market, such as maturity and duration, which affect its risk and return characteristics. In addition, there is a lot of research on fixed income markets because of their size and importance. Indices can provide accurate and appropriate measurement of the risk and return of fixed income securities and the characteristics of the market.

Constructing such a bond market index is far more involved than constructing a stock market index. Several problems have to be addressed when creating and maintaining such an index. The first

problem is that, according to Fabozzi [Fabozzi 1997], the spectrum of bonds is wider and more varied than that of stock. It includes U.S. Treasury issues, agency series, municipal bonds, and corporate bonds in several market segments, rated from high quality bonds to defaulted bonds. Moreover, within each of these groups, issues differ by maturity, coupon, sinking funds, and call features. Therefore, aggregate bond market series can be subdivided into many sub-indices. For example, according to Fabozzi [Fabozzi 1997], the Merrill Lynch index series includes more than 150 sub-indices.

In addition to the first problem, the spectrum of bonds changes constantly. A company may have one stock outstanding, but it usually has several bonds outstanding with different maturities, coupons or other features. This complicates the determination of the market value of bonds outstanding, which is needed for the calculation of market value-weighted rates of return.

Another issue that has to be considered is the variation of the volatility of bond prices across issues and over time. According to Fabozzi [Fabozzi 1997], bond price volatility is influenced by the bond's duration and convexity. The constant change of these factors with maturity, and coupon changes the parameters for the index change in a rather unpredictable fashion, which is an undesirable development for most uses of an index. Many fixed income portfolios are managed with a target

duration and target convexity, and such a target generally cannot be maintained in an indexed portfolio.

One of the most important problems for pricing individual bond issues is the liquidity of the bond. Individual bond issues are generally not very liquid, as opposed to stocks. Stocks are usually traded and listed on exchanges or in an active over-the-counter market. Bonds, however, are traded on a fragmented over-the-counter market without a common quotation system and, more importantly, many large issues, especially private placements, are not traded at all, as many bond buyers hold their bond to maturity. This is a big problem when pricing the bonds since often other sources have to be used instead of prices of real transactions.

For equity-indexed annuities, two types of bond indices are commonly used. The first type is U.S. investment-grade bond indexes. Three companies publish comprehensive investment-grade bond market indices that cover the spectrum of U.S. bonds. These companies are Lehman Brothers, Merrill Lynch, and Salomon Brothers. The firms include more than 5000 bonds in those aggregate indices and the diversity is secured by including Treasuries, corporate bonds, and mortgage securities. This is one more key problem for bond indices. There have to be kept so many issues in an index, because every bond issue by the same company is a completely new bond. The bond

maturities have to be at least one year and the minimum size of an issue ranges from $25 million for Lehman Brothers and Merrill Lynch to $50 million for Salomon Brothers. All the bonds have to be investment-grade, which means that they have to be *rated* BBB or better. A bond rating is simply a grade of creditworthiness. The bonds are graded by big rating agencies like Moody's and Standard & Poor's. The best ratings, which are AAA (by Standard & Poor's) or Aaa (by Moody's), signify extremely high degree of confidence that the investor's principal will be repaid, and that interest is paid in a timely manner. All the bond indices are market value-weighted. A common problem for all three indices is, as mentioned above, that transaction prices are not available for most of the bonds. Here, Salomon Brothers uses the strategy that it gets all the prices from its traders, which means that they will probably be biased. Lehman Brothers and Merrill Lynch use combination of traders and matrix pricing based on a computer model. The indexing companies also treat interim cash flows from the bond differently. Merrill Lynch assumes that cash flows are immediately invested in the instrument that generated them. Salomon Brothers assumes that cash flows are reinvested at the one-month Treasury Bill rate, and Lehman Brothers does not assume any reinvestment of cash flows.

The second large type of bond indices is U.S. High-Yield Bond Indices. This type has a shorter history than U.S. investment-grade bond

indices, since this market developed only in 1977, and the indices began in 1984. The problem of nonexistent prices for the bonds is magnified when dealing with high yield bonds since the sample changes in the index usually are larger due to default or redemption. The grade requirement for high yield bonds ranges from BB to CCC. In addition, the illiquidity and bond pricing problems are far more important in the high yield market.

The companies that manage investment-grade bond indices also issue high-yield bond indices. Merrill Lynch includes 735 bonds in its index series, while Lehman Brothers incorporates 624 bonds and Salomon Brothers only 299 bonds. The minimum issue size for a high yield bond is set to $25 million by Merrill Lynch, $50 million by Salomon Brothers, and $100 million by Lehman Brothers. The combination of the highest issue size requirement and a relatively high number of bonds is surprising for the Lehman Brothers index, since one would not expect to necessarily find so many qualified bonds for this index. In addition to the usual characteristics, high yield bonds also differ in the way they handle defaults. Merrill Lynch drops the bonds on the day they default, while Lehman Brothers keeps them for an unlimited period conditioned to size and other constraints. All the indices are market value-weighted. Concerning pricing, Lehman Brothers and Salomon Brothers rely on their traders, whereas Merrill Lynch includes some computer generated

prices. Except for Lehman Brothers all companies assume reinvestment of interim cash flows. Last, the minimum maturity requirement for Lehman Brothers and Merrill Lynch is one year, while for Salomon Brothers it is seven years.

4.3 Conclusions

The indices described above are all possible underlying investments for equity-indexed annuities and most of them are actually used in the equity-indexed annuity market. In fact, index funds are utilized to replicate the indices since one cannot directly invest in an index. The index is just a benchmark number and not an actually traded financial instrument. Theoretically, one could tie equity-indexed annuities to any index. However, there are certain constraints. The most important constraint is that the markets should be liquid, so that an insurance company can trade without any problems. Options and futures should exist for index funds, so that guarantees can be hedged. In addition, there are also several marketing and legal issues. For example, the index should be well-known for the potential customers. All these are reasons why the S &P 500 still dominates the equity-indexed annuity market. While probably far more people know the DJIA compared to the S & P 500, the DJIA is not suitable since DJIA cannot be invested in, and there exist no futures or options on the DJIA. On the

other hand, the S & P 500 market is far more liquid than the bond index market. That is why the S & P 500 is preferred over the bond indices.

CHAPTER V

MATHEMATICAL MODELS OF GUARANTEES IN EQUITY-INDEXED ANNUITIES

The different policy designs of equity-indexed annuities can also be mathematically modeled. This is especially important for the development and the pricing of equity-indexed annuities since the actuary should know what influence the different parameters have on the value of an equity-indexed annuity. A significant paper on this issue was published by Serena Tiong [Tiong 2000] in the *North American Actuarial Journal*. This thesis will present two models out of her paper for two types of equity-indexed annuities that currently dominate the marketplace. To understand Tiong's reasoning [Tiong 2000], one needs to know the concept of Esscher transforms.

5.1 Esscher Transforms

Assume Y is a normal random variable with mean μ and variance σ^2 with probability space (Ω, F, P). For any real number z, the moment

generating function of Y under Esscher transform with respect to parameter h is

$$M_Y(z;h) \equiv E\left(e^{zY};h\right) = \frac{E\left(e^{zY}e^{hY}\right)}{E\left(e^{hY}\right)} =$$

$$= \frac{\exp\left((h+z)\mu + \frac{1}{2}(h+z)^2\sigma^2\right)}{\exp\left(h\mu + \frac{1}{2}h^2\sigma^2\right)} = \exp\left((\mu + h\sigma^2)z + \frac{1}{2}\sigma^2 z^2\right)$$

This is the moment generating function for a normal random variable with mean $\mu + h\sigma^2$.

For A being an event and α an arbitrary real number, the Esscher transform can be applied to parameter h

$$E\left(e^{zY}I(A);h\right) = \frac{E\left(e^{zY}I(A)e^{hY}\right)}{E\left(e^{hY}\right)} = \frac{E\left(e^{zY}I(A)e^{(h+\alpha)Y}\right)}{E\left(e^{(h+\alpha)Y}\right)}\frac{E\left(e^{(h+\alpha)Y}\right)}{E\left(e^{hY}\right)} =$$

$$= P\left(A;h+\alpha\right)M_Y\left(\alpha;h\right)$$

Tiong also shows in a lemma that two independent random variables remain independent under the Esscher transform [Tiong 2000].

5.2 Point-to-Point Designs

Point-to-point designs are also called *European* or *end of term* designs since they compare the index value at issue of the policy to the index value at the end of the policy term, similar to a European option (see chapter IX). The point-to-point design can be slightly modified by

taking the average of a series of weekly or monthly index values at the end of the term instead of the last index value. This variation is called *Asian end* or *average end*. This way the weight of an extreme jump or drop of the last index value is balanced out. The starting point, however, is always the index value at issue of the policy.

Let $S(t) = S(0)e^{Y(t)}, t \geq 0$ be the value of an asset at time t. The asset pays out dividends $\delta S(t)dt, \delta \geq 0$ between time t and $t + dt$. Y(t) is a random variable representing the compounding rate of return on the asset over the time interval [0, t]. α denotes the participation rate. The participation rate can be greater than one, but most often it is less than one. Looking at a policy at time $T, T > 0$ with an initial premium of \$1, the policy pays either $e^{\alpha Y(T)}, \alpha > 0$, or a fixed exercise price $K, K > 0$, whichever is higher. The policy earns either a minimum guaranteed rate of return, which is ln K or a percentage of the realized return on the asset over the term of the policy if it is higher than the minimum guarantee. Therefore, as Tiong shows in her paper [Tiong 2000], the value of this policy can be expressed using Esscher transforms, as $e^{-rT} E\left(\max\left(e^{\alpha Y(T)}, K\right); h^*\right)$, where h^* is the risk-neutral Esscher parameter. A description of Escher transforms can be found in Bingham and Kiesel [Bingham, Kiesel 1998]. Tiong [Tiong 2000] then transforms the expected value using indicator functions to

$$E\left(e^{\alpha Y(T)} I\left(\alpha Y(T) > \ln K\right) + K \cdot I\left(\alpha Y(T) \le \ln K\right); h^*\right) =$$

$$E\left(e^{\alpha Y(T)} I\left(\alpha Y(T) > \ln K\right); h^*\right) + K \cdot P\left(Y(T) \le \frac{\ln K}{\alpha}; h^*\right)$$

and rewrites the expectation on the right-hand side using Esscher transforms as $P\left(Y(T) > \frac{\ln K}{\alpha}; h^* + \alpha\right) e^{\alpha(r-\delta)T + \frac{1}{2}\alpha(\alpha-1)\sigma^2 T}$ with σ being the volatility of the asset. Using the Black-Scholes assumptions that the price process $\{S(t)\}$ is a geometric Brownian motion and $Y(T)$ is normally distributed, Tiong [Tiong 2000] then develops the value of the policy, as

$$P_{pp} = e^{\left((\alpha-1)r - \alpha\delta + \frac{1}{2}\alpha(\alpha-1)\sigma^2\right)T} \times \Phi\left(\frac{\left(r - \delta - \frac{1}{2}\sigma^2 + \alpha\sigma^2\right)T - \frac{\ln K}{\alpha}}{\sigma\sqrt{T}}\right)$$

$$+ e^{-rT} K \times \Phi\left(\frac{\frac{\ln K}{\alpha} - \left(r - \delta - \frac{1}{2}\sigma^2 + \alpha\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

with Φ being the cumulative distribution function of a standard normal variable. If one now assumes the Standard Valuation Law minimum maturity guarantee of 90% of the premium compounded at 3% interest rate, K can be substituted by $K = 0.9e^{0.03T}$. Tiong then studied the resulting function P_{pp} and observed that the value of an equity-indexed annuity with a point-to-point design is an increasing function with respect to the guaranteed minimum K, volatility σ, and

participation rate α [Tiong 2000]. Depending on the participation rate, the value of the equity-indexed annuity can be increasing and/or decreasing with respect to the policy term. For a participation rate of 0.8 for example, the value function is almost perfectly linearly decreasing in T. This simplifies approximations of the value function since one can use linear regression based on the policy term and/or the participation rate.

5.3 The Cliquet or Ratchet Design

For this design, Tiong first develops a more general model [Tiong 2000], where she considers the maximum of two assets. Let $S_i(t) = S_i(0)e^{Y_i(t)}$, $i = 1,2$ be the value of one of two assets at time $t, t \geq 0$. Both assets pay out dividends $\delta_i S_i(t)dt$ between time t and $t + dt$ with $\delta_i \geq 0$. Tiong [Tiong 2000] considers an equity-indexed annuity policy that credits the higher return in each period of these two assets for n periods at a participation rate $\alpha, \alpha > 0$. The final payoff occurs at time T. For simplicity reasons, the periods are assumed to be of equal length $m = T/n$, but they do not necessarily have to be of equal length. The rate of return of asset i in period j is denoted as $Y_{ij} = Y_i(jm) - Y_i((j-1)m)$, $i = 1,2$ *and* $j = 1,2,..,n$. For each asset the periodic returns are assumed to be independent and identically

distributed. However, returns of two assets in the same period may be correlated. For each period, $V = (\sigma_{ik})$ is a 2 by 2 matrix and denotes the common covariance matrix of $Y_j = (Y_{1j}, Y_{2j})^T$ and V is assumed to be nonsingular. Under the risk-neutral measure, the value of this equity-indexed annuity policy at time 0 is $E\left(e^{-rT} \prod_{j=1}^{n} e^{\alpha \max(Y_{1j}, Y_{2j})}; h^* \right)$, $h^* = (h_1^*, h_2^*)^T$.

Tiong [Tiong 2000] rewrites this

$$e^{-rT} E\left(\prod_{j=1}^{n} \left(e^{\alpha Y_{1j}} I(Y_{1j} > Y_{2j}) + e^{\alpha Y_{2j}} I(Y_{1j} \le Y_{2j}) \right), h^* \right)$$

$$= e^{-rT} \prod_{j=1}^{n} \left(\frac{E\left(\left(e^{\alpha Y_{1j}} I(Y_{1j} > Y_{2j}) + e^{\alpha Y_{2j}} I(Y_{1j} \le Y_{2j}) \right) e^{h^{*T} Y_j} \right)}{E\left(e^{h^{*T} Y_j} \right)} \right)$$

Then, she looks at each of the product terms separately

$$\frac{E\left(e^{\alpha Y_{1j}} I\left(Y_{1j} > Y_{2j} \right) e^{h^{*T} Y_j} \right)}{E\left(e^{h^{*T} Y_j} \right)} = \frac{E\left(e^{\alpha Y_{1j}} I\left(Y_{1j} > Y_{2j} \right) e^{(h^* + \alpha 1_1)^T Y_j} \right)}{E\left(e^{(h^* + \alpha 1_1)^T Y_j} \right)} \frac{E\left(e^{(h^* + \alpha 1_1)^T Y_j} \right)}{E\left(e^{h^{*T} Y_j} \right)} =$$

$$= E\left(I(Y_{1j} > Y_{2j}); h^* + \alpha 1_1 \right) M_j\left(\alpha 1_1; h^* \right) = P\left(Y_{1j} > Y_{2j}; h^* + \alpha 1_1 \right) M_j\left(\alpha 1_1; h^* \right),$$

with $1_1 = (1\ 0)^T$ & $M_j(z; h^*) = E\left(e^{z Y_j}; h^* \right)$ and

$$\frac{E\left(e^{\alpha Y_{2j}} I\left(Y_{1j} \le Y_{2j} \right) e^{h^{*T} Y_j} \right)}{E\left(e^{h^{*T} Y_j} \right)} = P\left(Y_{1j} \le Y_{2j}; h^* + \alpha 1_2 \right) M_j\left(\alpha 1_2; h^* \right).$$

For the cliquet design, Tiong [Tiong 2000] uses these general formulas for the maximum of two assets and assumes the second asset to be nonrandom and to earn a fixed rate of return.

Therefore, $Y_{1j} = Y_j$, $Y_{2j} = g/\alpha$, $\delta_1 = \delta$, $\delta_2 = 0$, $h^* = \begin{pmatrix} h^* & 0 \end{pmatrix}^T$ where α is the participation rate and g is the minimum guaranteed rate of return. Applying those values gives

$P\left(Y_j > \frac{g}{\alpha}; h^* + \alpha\right) e^{\alpha(r-\delta)+\frac{1}{2}\alpha(\alpha-1)\sigma^2}$ & $P\left(Y_j \le \frac{g}{\alpha}; h^*\right) e^{g}$. Using those two values in the product, the value of the cliquet policy can be written as:

$$e^{-rn}\prod_{j=1}^{n}\left(P\left(Y_j > \frac{g}{\alpha}; h^* + \alpha\right) e^{\alpha(r-\delta)+\frac{1}{2}\alpha(\alpha-1)\sigma^2} + P\left(Y_j \le \frac{g}{\alpha}; h^*\right) e^{g}\right) =$$

$$\prod_{j=1}^{n}\left(P\left(Y_j > \frac{g}{\alpha}; h^* + \alpha\right) e^{(1-\alpha)r-\alpha\delta+\frac{1}{2}\alpha(\alpha-1)\sigma^2} + P\left(Y_j \le \frac{g}{\alpha}; h^*\right) e^{-(r-g)}\right)$$

CHAPTER VI

CLASSIFICATION OF EQUITY-INDEXED ANNUITIES AND VARIABLE ANNUITIES BY GUARANTEES

In this chapter, the different types of guarantees that are offered with equity-indexed annuities and variable annuities will be examined closer.

6.1 Guarantees in Equity-Indexed Annuities

Equity-indexed annuities characteristically have built-in guarantees in the contract. This is the reason why they are considered fixed annuities. Since they only credit upward movements in the underlying index, they have a built-in downward protection. Typically, an equity-indexed annuity will offer a *guaranteed minimum death benefit* (GMDB), which is a rising floor protection in the case of death of the annuitant, and a *guaranteed minimum accumulation benefit* (GMAB), which is a rising floor protection of the annuity's value until the end of the accumulation phase. The minimum guarantee for the death and the accumulation benefit is usually the minimum prescribed by the

Standard Valuation Law, which is 90% of the premiums paid minus any withdrawals accumulated at 3% interest. In addition, the account value is usually calculated according to the different methodologies presented in chapter II. Since this is an essential enhancement of the guarantees, this has to be considered as an additional guarantee. For example, an equity-indexed annuity could have a point-to-point design or an annual ratchet design with monthly or weekly averaging or no averaging. The different design choices offer the actuary a wide variety of choices when he or she develops the equity-indexed annuity. These different design features are exactly the characteristics, which determine an equity-indexed annuity's guarantee. The different index-based interest crediting methods used are point-to-point, high watermark, low watermark, and ratchet. On the market, there are almost only point-to-point designs and ratchet designs, which are usually annual ratchets. Most often, insurance companies use averaging over several index values, which are determined prior to the end of the term. For example, the average of the last 52 weekly values is common. Another design feature that determines the guarantee is the participation rate, which was defined in chapter II. The participation rate is usually locked in at the beginning of the contracts and is often guaranteed for the whole term of the annuity, except for annual ratchet designs. The participation rate for annual ratchets is normally determined on a yearly basis. In this chapter

different products will be presented to show the different types of design and therefore also the different types of guarantees.

The first product is called Powerhouse and it is issued by Allianz Life Insurance Company of North America. The following information can be found in the Powerhouse annuity brochure [Allianz Life Insurance Company of North America 2002]. The only underlying index that can be chosen for this single premium equity-indexed deferred annuity is the S & P 500. The minimum interest that is credited to the annuity is 2.5% and it is calculated on 100% of premium. At the beginning of the policy, a participation rate is fixed and guaranteed for the first ten years. After that, the participation rate is set yearly. Index-based interest is credited on each policy anniversary, so essentially this policy has an annual ratchet design with averaging. Adjustment factors are the participation rate and the cap. As of June 2002, the participation rate is 125% of the average of the last 12 monthly values with a 12% cap, which is guaranteed for the first 5 years. The cap is guaranteed to be never less than 3%. As an example for averaging, assume the policy was issued on January 1, and the S & P 500 closing prices on the last day of each month of the following year were 510, 530, 550, 570, 590, 610, 610, 590, 570, 550, 530, and 510. The average of these 12 values is 560. Now assume, the S & P 500 was at 500 when the policy was issued. The increase from 500 to 560 is a 12% increase. The participation rate for the

Powerhouse annuity is currently set at 125%. This means that 125% of 12% increase would be credited. However, since 15% is greater than the cap of 12%, the index-based credited interest would be 12%. The annuity's value calculated according to the above example is paid out if the policyholder annuitizes and is therefore called *annuitization value*. The *cash surrender value* is the value the policy owner receives if the annuity is surrendered and a lump sum payment is taken. The Powerhouse annuity calculates the cash surrender value as the value that is in the annuity at the time of surrender minus a surrender charge, which is 10% at policy issue, decreases monthly by 0.07% for 12 years and is 0% thereafter. In order to avoid surrender charges, the minimum requirement is that the policy be held for five years and then payouts are annuitized over at least the next ten years. The death benefit is the greater of annuitization value and 110% of the cash surrender value if it is taken over at least 5 years. The Powerhouse annuity also offers free withdrawals of up to 15% of the premium paid. No surrender charge applies to withdrawals if they are made at least twelve months after issue and twelve months before surrender or annuitization. The policy also offers policy loans at 2% net interest for up to 50% of the cash surrender value capped at $50000. In addition, if annuitization is chosen, annuity payments have been received for at least two years by the policyholder and the policyholder becomes disabled, the annuity increases its

payments by 60%. In case the policyholder enters a nursing home after the first policy year, the annuity can be annuitized over five years without surrender charges.

Allianz offers several other indexed annuities. One of them is the FlexDex annuity, which is a flexible premium equity-indexed deferred annuity. Information about the FlexDex annuity can be found on the Advantage Group's website [The Advantage Group 2002]. There are several differences compared to the Powerhouse annuity. First, this annuity allows the policyholder to make several payments and the payments in the first five years are granted a 5% premium bonus. This means that for the premiums paid in the first five years the insurance company adds 5% of premium. The FlexDex annuity is also an annual reset design with monthly averaging, but the maximum interest rate is capped at 10% and the participation rate is 100% of the index movement. The minimum guaranteed interest rate is 3% on 75% of the first year's premium. Thereafter 87.5% of the following year's premiums are the interest-crediting basis. Both the Powerhouse annuity and the FlexDex annuity can be tax-qualified as explained in chapter III or non-qualified.

Great American Life Insurance Company offers a product, which is called EquiLink. This product is a point-to-point design with averaging. The minimum guaranteed value in case of death or surrender is the

nonforfeiture minimum of 90% of the premium with 3% interest. Index-based interest is credited according to the following method. The average value of the S & P 500 is calculated over the last six months of the policy term. Then the index increase is determined as the ratio of the average value at the end of the term over the value of the S & P 500 at the beginning of the term. This ratio is multiplied with the participation rate, which is 80% as of July 2002. This result is the basis for a vesting schedule and it is multiplied with a factor for vesting. For the first three years, none of the interest is vested, from the fourth year on the vested index participation is gradually increased, starting with 10% and ending with 100% vested at the end of the term. If the vested index-based interest is less than the minimum guarantee, then the minimum guarantee applies. The vesting part in the contract is a security measure that is advisable for point-to-point designs offered by an insurance company since early surrenders might be a big risk otherwise.

6.2 Guarantees in Variable Annuities

Variable annuities usually do not have the guarantees built in the contract. Instead, the customer can purchase them optionally as add-on features, so-called *riders*. Variable annuities offer guarantees, which can be categorized in the three main categories *guaranteed minimum death benefits* (GMDB*), guaranteed minimum accumulation benefits* (GMAB) and

guaranteed minimum income benefits (GMIB). The last type is a guarantee that the policyholder will receive a minimum payment upon annuitization even if his account value is used up.

The typical death benefit of a variable annuity is the maximum of the account value and the premiums paid minus the proportional impact of withdrawals. Some companies offer enhanced death benefits in addition. In an article published in 2001, Moshe Milevski and Steven Posner [Milevski, Posner 2001] claim that a simple return-of-premium death benefit is worth between one and ten basis points while the median Mortality and Expense risk charge for return-of-premium variable annuities is 115 basis points. They use risk-neutral option pricing theory to value this guaranteed minimum death benefit.

The following information was found on the website AnnuityFYI [Raymond James Financial Services 2002]. Allmerica Life, American Skandia, ING – Golden American, Kemper Life, and Sun Life of Canada offer similar enhanced death benefit programs. The standard guarantee for all variable annuities of these companies is the typical guarantee as described above. The first enhanced option offers the policyholder the highest anniversary value of his or her account. Only American Skandia combines this option with a 5% minimum interest guarantee and offers the maximum of those two. The second rider that is offered guarantees a minimum interest rate for the total investments. This interest rate is 5%

for Allmerica and Sun Life of Canada, 7% for ING – Golden American and Kemper Life. American Skandia basically enhances its previous rider and offers 7.2% interest instead of 5%. The next level of riders then offers the maximum of the previous optional riders. The enhanced death benefit riders typically come at an additional fee of 0.15% to 0.45%.

Several companies also offer living benefits in form of guaranteed minimum accumulation benefits. It is very interesting that due to the recent downturn in the stock market several companies now offer living benefits implicitly in their contracts. These contracts usually come at a higher fee of 0.25% to 0.50% more premium. For example, Transamerica insurance company offers its variable annuities Landmark, Extra, and Freedom with a living benefit minimum guarantee of at least 6% compounded interest on the account value. MetLife insurance company offers the maximum of the account value compounded at 6% compounded interest and the account's highest previous anniversary value. American Skandia offers the highest anniversary value, whereas Manulife Financial also offers the maximum of the highest anniversary value and 6% compounded interest on the account value. ING's variable annuities are supplied with a 7% compounded interest guarantee with a cap at double the premium.

The LIMRA organization [Weston 2002] examined 16 products that offer a guaranteed living benefit in a survey. Nine products were offering

a guaranteed minimum accumulation benefit and seven were offering a guaranteed minimum income benefit. The LIMRA report also mentions that in 2000 the state of California stopped the sales of variable annuities referring to the fact that the insurance industry did not find an agreement on the amount of cash reserves that insurance companies should set aside to support guarantees in variable annuities. A working group set up by the American Academy of Actuaries reported to the National Association of Insurance Commissioners with recommendations on cash reserve requirements for insurance companies offering guaranteed benefits. The California Insurance Commissioner lifted the ban and said the department would follow the AAA's reserve recommendations. Insurance companies still face the challenge of pricing guaranteed living benefits properly.

According to the LIMRA report, 11 companies are offering an earnings-related death benefit (ERDB) in 27 products. An earnings-related death benefit is a contract feature in which a predetermined percentage of the investment gains is added to the sum the beneficiary receives upon the annuitant's death. The purpose of this benefit is to provide money needed for any tax payments that become due at the annuitant's death. In 2000, no company offered earnings-related death benefits. However, in 2002 in addition to the 11 companies that offer them seven companies are going to introduce them within the next six

months. The earnings-related death benefit is popular amongst customers because it can be illustrated easily and customers perceive a real value in it.

LIMRA examined 27 different products that providing earnings-related death benefits and the most common earnings-related death benefit percentage offered is 40 percent. Every earnings-related death benefit surveyed in the report reduces the percentage at a certain age and will not allow the earnings-related death benefit to be purchased beyond a higher age. Most often, the reduction in percentage occurs at age 70, and no contract allows the earnings-related death benefit to be purchased after age 80. The basic charges for an earnings-related death benefit range from 0.15% to 0.25%.

Allianz Life Insurance Company of North America, for example, offers a living benefit in form of a guaranteed minimum accumulation benefit in its variable annuity called Alterity. The living benefit comes at a cost of an increase of the mortality and expense charge of 0.30% and guarantees a 5% annual increase of premium paid minus withdrawals or the highest anniversary value reduced by the percentage withdrawn. This living benefit is offered only for the fixed options within the variable annuity and it is offered only up to age 81. The policy has to persist for at least 7 years and the payout option can only be exercised within 30 days

following a contract anniversary. If a certain payout period is chosen, the annuity has to persist for at least ten years.

Pacific Life insurance company offers a living benefit in form of a guaranteed minimum income benefit in its products Pacific Portfolios and Pacific Value. The age restriction here is 80 years. Each annuitant has to be 80 or younger. The guaranteed minimum income benefit is sold as a rider. This rider compares the premium paid adjusted for withdrawals with a compounded interest of 5% annually up to age 80 and the net value of the annuity plus 15% of the net contract value of the annuity minus premiums paid in the preceding 12 months. The cost for this guarantee is 0.30%.

Allianz Life Insurance Company of North America offers an earnings-related death benefit in its variable annuity called Dimensions. The policyholder has two choices. He or she can choose an earnings guaranteed minimum death benefit, which adds 40% of the minimum of earnings or premium to the death benefit. This percentage is decreased to 25% if the policyholder is older than 70 years at issue. The double principal guaranteed minimum death benefit equals the maximum of the contract value and the highest contract anniversary value up to age 81. If the annuity persists for more than five years, this benefit is doubled. The protection comes at a cost of 0.20% for the earnings guarantee and 0.30% for the double principal guarantee.

Hartford Life insurance company offers an earnings-protection death benefit in its variable annuity Director Edge. The age restriction here is 76 years. If the annuitant is younger than 70, 40% of the earnings are added to the contract. If the policyholder is 70 to 75, this percentage is decreased to 25%. This benefit is capped at 200% of the contract value before the benefit was added. The cost for this guarantee is 0.20%.

CHAPTER VII

RESERVING FOR EQUITY-INDEXED ANNUITIES

The valuation and certification of an insurance company's liabilities are two crucial actuarial functions since the liabilities of an insurance company have a very specific character [Tullis, Polkinghorn 1996]. The main portion of a life insurance company's liabilities originates from the contingent benefits that are guaranteed in policies and contracts with a long-term contract period. Almost 90% of a life insurance company's liabilities are reserves. The impact of a small change in the reserves is a significant change of the company's period earnings and equity value.

Reserves are liabilities for amounts an insurance company is obligated to pay as defined in an insurance policy or annuity contract. The time of payout and/or the exact amount are usually uncertain or contingent. Reserves can be classified as claim reserves (or loss reserves) or policy reserves.

Claim reserves are established for insured events that have already happened, but their payout amount is not known yet.

Policy reserves are established for insured events that have not yet happened, but the insurance company has an obligation to pay if they occur.

This thesis deals only with policy reserves, which can also be called actuarial reserves. They are determined performing an actuarial valuation. Loss reserves are insignificant for life insurance in general and they are zero for annuities.

Due to the contingent character of policy reserves one cannot specify with certainty the exact amount necessary to fulfill all future payout obligations. One has to use probabilities of future events to calculate the reserves. The calculation of actuarial reserves relies heavily on the Law of Large Numbers. Consequently, actuarial reserves are only meaningful and valid if calculated for a large number of policies. Although it is possible to calculate the reserve for a single policy and to establish a real liability to the insurance company that way, the theory behind actuarial reserves holds only for large portfolios of policies. Based on the assumptions and methodologies used, results of an actuarial valuation may vary widely, but still may be legitimate.

7.1 Types of Valuations

There are three main types of valuations in the U.S.: statutory valuation, GAAP valuation, and tax valuation.

The main purpose of *statutory valuation* is to ensure the financial health of an insurance company. An insurance company in the U.S. has to be licensed in each state separately to do business in it. Part of the requirements for the license is that the insurance company has to file a financial report annually with the insurance regulator using statutory valuation for this report, which is specified and published by the National Association of Insurance Commissioners (NAIC). The law defining statutory valuation is the Standard Valuation Law. Since determining and ensuring solvency is the main idea, statutory valuation relies on conservative assumptions and methodologies, which produce larger liabilities than the other types of valuation. U.S. valuation law is explicit concerning assumptions and methodology allowed for statutory valuation, sometimes even prescribing specific mortality tables or interest rates. Nevertheless, there is a trend of shifting more responsibility to the valuation actuary. The valuation actuary concept is designed to make sure the insurance company has sufficient provisions for future obligations not only under expected experience but also under a number of different scenarios that might be plausible. The responsibility for this is placed on the valuation actuary. [Tullis, Polkinghorn 1996]

Generally accepted accounting principles or GAAP valuation is required for publicly traded U.S. stock companies. The objective of this

type of valuation is to correctly assign income to the period in which it is earned. Therefore GAAP valuation does not focus as much on conservative assumptions as statutory valuation, although GAAP assumptions for traditional products are required to be reasonable and conservative. Statutory valuation does not give an accurate picture of an insurance company's financial situation, especially concerning trends, since it is sometimes too conservative to be used for management decisions. Therefore most companies that do not have to file GAAP financial statements produce "GAAP-like" financial statements for the internal use of management to accurately assess the performance of the company utilizing GAAP principles with adjustments for their particular needs.

The last main type of valuation is *tax reserve valuation*. This type of valuation serves to calculate the reserve liability in order to determine taxable income. It is carried out by calculating the federally prescribed tax reserves. Minimum permissible statutory reserves and the highest interest rate and most recent mortality table allowed by at least 26 states have to be used. If an interest rate, which is prescribed in the valuation requirements, is higher than the highest interest rate in the 26 states, the prescribed interest rate has to be used. Deficiency reserves are not to be used for this calculation. Deficiency reserves are reserves that may be

required if the gross premium is below a certain level, for example the valuation net premium [Tullis, Polkinghorn 1996].

In addition to the three main types of valuation there is another type, which is called *gross premium valuation*. This type of valuation is probably least conservative and its purpose is to give a realistic best estimate value of the company's liabilities. It is often used for internal purposes or acquisition and mergers.

7.2 Valuation Requirements in the United States

The annual filing of an Actuarial Opinion of Reserves was revised in 1990 by adopting the Standard Valuation Law.

> "Every life insurance company doing business in this state shall annually submit the opinion of a qualified actuary as to whether the reserves and related actuarial items held in support of the policies and contracts…are computed appropriately, are based on assumptions which satisfy contractual provisions, are consistent with prior reported amounts, and comply with applicable laws of this state. The commissioner by regulation shall define the specifics of this opinion and add any other items deemed necessary to its scope."

The actuarial opinion has to be on the adequacy of reserves in aggregate, which means that components of the reserves can offset each other. Since the actuary may be personally liable for this statement, it will explicitly state reliance on others. The reserves calculated in the statement, which is filed in any particular state where the company is doing business, in the aggregate, must satisfy the laws of that state, and presumably also satisfy the regulations of that particular insurance department. This may cause practical problems because different states interpret the law differently.

The 1990 Standard Valuation Law also requires an actuarial analysis of reserves and assets supporting such reserves. This part is based on New York Regulation 126 and means that asset adequacy analysis is required. One possible method for asset adequacy analysis is cash flow testing, which will be discussed in more detail in chapter VIII.

Equity-indexed deferred annuities guarantee a minimum interest accumulation rate on a part of the customer's premium payments and on a part of the growth of an index that is based on equity. The time period during which these guarantees are valid is specified in a policy term within the contract. In addition, equity-indexed annuities also guarantee a minimum death benefit amount and a nonforfeiture value.

Equity-indexed immediate annuities guarantee a minimum annuitization amount and offer the opportunity to participate in the

growth of an index by receiving additional periodic payments if the index goes up. These guarantees have to be valued and reserves have to be set aside for the company to be able to fulfill its promises.

The legal basis for the valuation of annuities is the Standard Valuation Law. Within the Standard Valuation Law, in section 5a, paragraph B, the Commissioner's Annuity Reserve Valuation Method (CARVM) is defined:

> "Reserves according to the Commissioners annuity reserve method for benefits under annuity or pure endowment contracts, excluding any disability and accidental death benefits in such contracts, shall be the greatest of the respective excesses of the present values, at the date of valuation, of the future guaranteed benefits, including guaranteed nonforfeiture benefits, provided for by such contracts at the end of each respective contract year, over the present value, at the date of valuation, of any future valuation considerations derived from future gross considerations, required by the terms of such contract, that become payable prior to the end of such respective contract year. The future guaranteed benefits shall be determined by using the mortality table, if any, and the interest rate, or rates, specified in such contracts for determining guaranteed benefits. The valuation considerations are the portions of the respective gross considerations applied under the terms of such contracts to determine nonforfeiture values."

Following is an example of the Commissioner's Annuity Reserve Valuation Method applied to a single-premium deferred annuity similar to the example in Tullis and Polkinghorn [Tullis, Polkinghorn 1996]. The annuity's policy features are as follows:

- Single premium: 10000

- Guaranteed Interest: 10% in years 1 through 4
  4% thereafter
- Surrender charge:

Policy year	Percentage of fund
1	7%
2	6%
3	5%
4	4%
5	3%
6	2%
7	1%
8 and later	0%

- Valuation interest rate: 8%
- Death benefit equal to cash surrender value

First, the fund balance is projected forward at the guaranteed basis in the policy and then the cash value of the policy is calculated for the end of each policy year. Here, the first 10 policy durations are chosen.

Table 3: Fund Values and Cash Values

Policy year	Fund	Cash Value
0	10000	9300
1	11000	10230
2	12100	11374
3	13310	12645
4	14641	14055
5	15227	14770
6	15836	15519
7	16469	16304
8	17128	17128
9	17813	17813
10	18526	18526

Source: Tullis, Polkinghorn 1996

Then, for each policy anniversary a valuation of future benefits is carried out. The calculation procedure uses the cash value of the future policy year and discounts it back to the appropriate policy anniversary using the valuation rate. For example, if one wants to know the value of the policy benefit in the fifth policy year valued at the policy's third anniversary, one has to take 14770 and discount it back two times with 1.08 to get 12663. This is done here for the first four policy anniversaries.

Table 4: CARVM Valuation

Future policy year	Cash value	Policy anniversary of valuation				
		0	1	2	3	4
0	9300	9300				
1	10230	9472	10230			
2	11374	9751	10531	11374		
3	12645	10038	10841	11708	12645	
4	14055	*10331*	*11157*	*12050*	*13014*	*14055*
5	14770	10052	10856	11725	12663	13676
6	15519	9780	10562	11407	12319	13305
7	16304	9513	10274	11096	11984	12943
8	17128	9254	9994	10794	11657	12590
9	17813	8911	9624	10394	11225	12123
10	18526	8581	9268	10009	10810	11675

Source: Tullis, Polkinghorn 1996

For each of the first 4 policy anniversaries, the cash value that results in the largest present value is shown in italic. These are the CARVM reserves at the respective policy anniversaries since the CARVM reserve has to be the greatest of the net present values of future guaranteed benefits.

Naturally, this is only a simplified version of a real CARVM calculation since there might be dozens of annuity options, different benefits, and policy anniversaries on which those benefits could be used.

CARVM applies to equity-indexed annuities since these products offer implicitly different guarantees to the customer. However, application of CARVM to equity-indexed annuities is more problematic than with

traditional fixed annuities because the guarantees in an equity-indexed annuity are fixed but the future development of the equity index is not known. This combination of guaranteed and therefore deterministic parameters and uncertain, probabilistic index development complicates the application of CARVM.

Therefore, the National Association of Insurance Commissioners (NAIC) published Actuarial Guideline 35, which addresses the application of CARVM to equity-indexed annuities and is effective since December 1998. The guideline interprets the standards set by CARVM for the valuation of reserves for equity-indexed annuities. It defines methodologies for the computation of reserves, which meet the intent of the Standard Valuation Law.

7.3 Hedged as Required

A prerequisite for the discussion of the computational methods is to understand the *hedged as required* operational criteria. The computational methods are divided in Type I and Type II methods. Type I methods are applicable only if the hedged as required criteria are met, otherwise an insurance company has to use Type II methods. To meet the hedged as required criteria, the appointed actuary must certify quarterly that the equity-indexed annuity meets either “Basic” or “Option Replication” criteria.

The Basic criteria consist of five conditions. First, the option contracts held and the contract-immanent options must have equivalent characteristics regarding feature like the underlying index, term, averaging methods, strike price, etc.

Second, the company must purchase an amount of hedge close to the date of contract issue that is at least a specified percentage of the contract's account value at the time of contract issue. The specified percentage depends on the length of the option guarantee in the annuity and allows the insurance company to assume up to 3 percent per year of elective benefit decrements. The Commissioner can agree to a higher limit. For example, for an annual ratchet product, the specified percentage would be: $(1-0.03)^1 = 97\%$. Note that, even though the annual ratchet product might have a term of several years, the participation rate is only guaranteed for one year and this causes the term for this purpose to be 1 year.

The third condition is that the insurance company must define a plan to hedge risks caused by interim death benefits.

Fourth, the insurance company must have a system to monitor the company's hedging strategy's effectiveness, so that it can identify critical divergent developments in its hedge portfolio.

The last condition is that the insurance company must state a maximum tolerance for divergences between the expected performance and the actual performance of the hedge.

The Option replication criteria also define five conditions that have to be met. The first condition requires the characteristic contract features of an option replication strategy to be equivalent to the options embedded in the liabilities. This is somehow similar to the first condition of the Basic criteria, but here instead of options, an option replication strategy is used.

Second, the value of the target option replication strategy should be at least a specified percentage, which is defined exactly the same way as in the Basic criteria. Another example could be, for a seven-year point-to-point product, the specified percentage would be:

$(1-0.03)^7 = 81\%$.

The next two requirements are the same as in the basic criteria. Interim benefits should be considered and a monitoring plan must be put in place.

The last condition that has to be met is that the company must state the criteria for measuring the deviation of the reality from the plan. However, in the option replication case the fifth condition is further specified. A maximum tolerance test and a compliance evaluation test

are performed and must meet some requirements. The compliance evaluation requirements are checked weekly in an retrospective correlation test, in which the insurance company compares the change in the market value of the hedging portfolio to the change in the market value of the options embedded in the liabilities. The testing period is the calendar quarter. The difference dollar amount between these two changes must be less than or equal to 10% of the market value of the embedded options in the liability portfolio at the beginning of the testing period.

Actuarial Guideline 35 specifies an action plan if this limit is exceeded. It distinguishes three scenarios. If the difference exceeds 10% twice in a testing period and in both cases is less than 25% of the embedded options' beginning of period market value then the Commissioner of Insurance in each state in which the insurance company is registered must be notified. The notification must include the amount of reserves that are hedged be the replicating strategy. In the second scenario the difference exceeds 25% once in a testing period. This triggers also a notification to all Commissioners of Insurance in the states in which the insurance company is registered. In addition, the insurance company must include the impact on the surplus if the reserves would be reported based on CARVM with updated market values (CARVM-UMV), which is a Type II method. The third scenario describes

the actions necessary if the difference is bigger then 35% in one testing period. The insurance company then must switch to CARVM with updated market values; notify all Commissioners of Insurance in the states in which the insurance company is registered and state the impact on surplus of reporting the reserves based on CARVM with updated market values.

All these requirements are geared towards the situation in which the actual hedge underperforms relative to the expected hedge performance. If an insurance company over-hedges, the excess hedging portfolio is not used for measurements that are required in the last item of the Hedged as Required Criteria. Over-hedging in this context means that the value of the hedge portfolio exceeds the value of the liabilities that are hedged.

The hedged as required criteria are being used to determine which computational methods are permitted. The computational methods can be classified into two groups, the Type I group and the Type II group. A method from the Type I group can only be applied if the hedged as required criteria are met. If those criteria are not met, only Type II methods can be applied.

7.4 Type I Methods

In the Type I group, the Enhanced Discounted Intrinsic Method (EDIM) is the only explicitly specified method in Actuarial Guideline 35. This method consists of four steps. In the first step, the fixed component of an equity-indexed annuity at issue is the reserve obtained by applying either CARVM-UMV or MVRM and the fixed component at the end of the term equals the minimum benefit, which is actually being hedged. In step two the initial value and the ending values are used to calculate an interest rate which would match up those two, respectively, and then the intermediate values of the fixed component are calculated as in the following example: Options are purchased under the assumption that 15% of the policyholders will annuitize at maturity and 85% of the policyholders will surrender at maturity. Then, the fixed component is the sum of 15% of the fixed component that accumulates to the floor of the annuitization benefit and 85% of the fixed component that accumulates to the floor of the surrender benefit. In step three the equity component is calculated by discounting the intrinsic value of the options at the valuation rate from the valuation date to the end of the term. The intrinsic value used for the discounting is the intrinsic value taken at the valuation date. The valuation interest rate should be consistent with other Actuarial Guidelines, such as Actuarial Guideline 33, which is used for valuation of annuities with elective benefits and, concerning the valuation interest rate, refers to section 4b of the Standard Valuation

Law. The reference index for the valuation interest rate is Moody's investment grade corporate bonds index. The last step eventually defines the reserve as the sum of the fixed component and the equity component.

7.5 Type II Methods

Type II methods can be used when "Hedged as required" is not met. The first method that will be described is the Commissioners' Annuity Reserve Valuation Method with Updated Market Values. (CARVM - UMV) Here, the first step determines the market value of the appropriate call option for each duration and each benefit at which an index-based benefit is available. A call option is appropriate if it exactly hedges the floor of the benefit at that specific time, which means that its payoff exactly equals the difference between the specific benefit available at that specific moment and the guaranteed minimum of that benefit. The market value should be established with an appropriate pricing technique, for example Black-Scholes or a stochastic scenario method. In the second step, all the call options' market values are projected forward to the expiration date of the call options using the appropriate valuation interest rate, corresponding to other Actuarial Guidelines. In step three the guaranteed amounts of each option are added to the projected market values. In the last step, a traditional CARVM calculation is done

according to Actuarial Guideline 33 and any other regulations or guidelines that apply.

The second available Type II method is the Market Value Reserve Method (MVRM). In this method the projected index value at the end of the term has to be calculated first in a manner such that the projected index value at the end of the term equals the sum of the current market value of a call option. The option fully hedges the index-based benefit and the contractual benefit guarantee at the end of the term, assuming equal annual percentage increases in the index. The call options used should have the same expiration terms as the options embedded in the liabilities, such as participation rates or spread, for example. In step two the current index level and the projected index level at the end of the term are used to calculate an implied compound constant index growth rate from the valuation date to the end of the term. Then index levels at intermediate anniversaries are calculated using this implied growth rate. Now the index levels define all annuity benefits. Eventually, a traditional CARVM calculation can be performed.

A variation of the Market Value Reserve Method is the Market Value Reserve Method using Black-Scholes Projection. This method is introduced to accommodate products for which the index-based benefit is redetermined within the term. In this case, the first thing to do is to calculate the cost of a fully hedging call option as a percentage of the

account value for the period in which the benefit level is guaranteed, accumulate the percentage to the end of that period at the risk-free interest rate. This accumulated percentage is then used as the account value's projected growth rate during that period. This calculation is done for all the periods within the term, taking into account benefit guarantees, forward interest rates, forward index volatility, and index dividend levels. The projected account level on each anniversary is then used to determine the index level based on the applied benefit determination method. The last step is the same as in the original Market Value Reserve Method, a traditional CARVM calculation.

7.6 General Conditions

There are some general conditions for the use of all computational methods. First, the policy must be structured in a way that there is a single predominant benefit. Predominant refers to the benefit being most likely to be provided under this policy taking into account all contracts features.

The predominant benefit defines the term end point, which is used for the computational method and for complying with the "Hedged as required" criteria.

The above-mentioned monitoring plan or hedging policy should define the risks, the actions that have to be taken and it should consider

the risks involved for the hedges. The possible risks include the liquidity risk if a company needs to sell quickly, the credit risk of the counterparty, the market risk, the pricing risk, the legal risk that the instrument be allowed, and the operations risk.

A description of the investment policy and hedging for equity-indexed annuities can be found in chapter IX.

Options in both the Commissioners' Annuity Reserve Valuation Method with Updated Market Values and the Market Value Reserve Method should be valued at market value and bonds should be valued at book value.

The minimum guarantees in equity-indexed annuities are usually backed up with assets in bonds. Bonds are traditionally held at book value, which equals amortized cost. This means that their book value in the beginning equals expenses for the bond and then it is written up or down depending on what will be received as cash from it as a maturity value.

It is not desirable to use stocks to back up the non-guaranteed part since stocks have a risk based capital requirement of 30% of the market value, which means that an insurance company has to set aside 30 cents for each dollar invested in stocks to maintain its risk based capital ratio. Risk based capital is a regulatory requirement that is meant to ensure solvency. The basic idea of risk based capital is that the

amount of capital required for a company depends on the risk the company is taking. If the risk based capital ratio falls below certain levels there are certain actions specified, that are triggered. Although holding stocks is a legitimate hedging strategy insurance companies tend to not do that because of the high risk based capital requirements.

Options have to be purchased in any case since they either have to provide the equity component if bonds are bought, meaning the positive difference between the index value and the minimum guarantee provided by the bond, or they have to offset the fixed guarantee in the case when stocks are bought.

Most important is the consistency of methodologies in valuing assets and liabilities. If assets are valued at market value, the liabilities should also be valued at market value.

CHAPTER VIII

RISK MANAGEMENT, ASSET LIABILTY MANAGEMENT AND CASH FLOW TESTING

Equity-indexed annuities are exposed to several different types of risk. The American Academy of Actuaries practice note [American Academy of Actuaries 1999] lists risks that are commonly considered by actuaries. Although the risks presented can also be found in other insurance products, each of the risks has some aspects that are unique to equity-indexed annuities.

First, and very important for equity-indexed annuities is the *disintermediation risk*. This is the risk that more policies than assumed lapse before the end of the index term. Since disintermediation risk is such an important problem for equity-indexed annuities, it will be addressed separately in chapter X. The other risks are presented below.

8.1 Hedge Mismatch Risk

Another risk that often has to be taken into account by valuation actuaries is the *hedge mismatch risk*. Hedging plays such an important

role in the context of equity-indexed annuities that it will also be discussed separately in chapter IX. At this point, it is sufficient to know that hedging is a countermeasure against negative equity market developments carried out by using option replication strategies or Put-Call Parity. According to Bodmayr [Bodmayr 1998], hedge mismatch risk occurs if an insurance company's hedging position does not fully hedge the equity-indexed annuity liability or if the index development is not fully in line with the company's expectations. From an insurance company's perspective, equity-indexed annuities add some components, which are unique for this type of product, to the company's general hedge mismatch risk for other fixed products. The reason for that is that equity-indexed annuities typically are associated with an investment in a unique combination of options and fixed income assets. Hedge mismatch risk consists of two parts. First, risk could arise if the insurer does not cover the full amount of possible payout. Second, risk could also arise if there is not a 100% correlation of the hedge to the index. In general, one can never assume a 100% correlation of the hedge to the index, so that this is some kind of permanently present basis risk. Product design is one thing that hedge mismatch risk usually depends on. Annual ratchet products have for example less market price volatility than point-to-point products. The basis risk could be higher for the insurance company if it uses an index, which has less liquid securities than the S & P 500 index.

The insurance company might have to use derivatives that are not very liquid or are not publicly traded, since they are issued on an index whose securities are traded far less than the ones based on the S & P 500. These risks have to be kept in mind by the insurance company when it designs its equity-indexed annuity since they can be partially managed at the designing stage by using modifiers, such as participation rates, caps, and averaging as described in chapter II. It is crucial for the insurance company to be able to adjust these parameters periodically to the market situation. Generally, these are guaranteed for the whole term, which is usually more than ten years. However, point-to-point annual reset designs are usually set up in a way that allows for adjustment of participation rates or caps after each index crediting period. Using an option replication strategy instead of buying a long-term option, which is usually not very liquid since it is most probably an over-the-counter option, could also reduce hedge mismatch risk. This is called dynamic hedging and will be discussed in more detail in chapter IX. As an example, an insurance company could buy a series of short-term options with terms of usually six to twelve months and buy the next series at expiration of the previous options instead of buying one option as a hedge for the whole index term. Nevertheless, this strategy might be more costly than the single option approach, which is called static hedging and will be presented in more detail in chapter IX. There might be a higher

transaction cost for dynamic hedging since it involves more trading. In addition, if the expected volatility of the equity market increases, the cost of replication renewal will also increase. Moreover, the risk associated with the correlation of the hedging portfolio to the index might increase since more transactions take place with different kind of derivatives.

According to the American Academy of Actuaries [American Academy of Actuaries 1999], actuaries assess hedge mismatch risk by considering the interaction of the existing options, the strategies that a company uses for reinvestment and disinvestments, and the projected capital market with two kinds of benefits. The two different equity-indexed benefits used for the interaction testing are the benefit at the end of the index term and the benefits prior to the end of the index term like death benefits, annuitization benefits or withdrawal benefits. Regardless whether a company uses an option replication strategy, this interaction is considered serious.

The actuary has to find different methods to model hedge mismatch risk. Many of the currently used techniques will involve varying combinations of equity index-based and fixed interest rates. This is practically cash flow testing, which will be described in later in this chapter. These techniques usually take into account both assets and liabilities. Liability modeling usually includes lapse assumptions and enhanced guaranteed and non-guaranteed benefit and product features.

The liability assumptions used often depend on economic variable and management strategies, which can be applied to the non-guaranteed elements of the benefit. Asset modeling assumptions typically depend on several parameters, like the company's reinvestment and disinvestments strategies, future market volatility within predictable scenarios, liquidity of the options, option strategy, and availability of management information needed to control the hedging program. Actuaries model the company's reinvestment and disinvestments strategy particularly in combination with dynamic hedging since they believe it is preferable that the model includes the company's tolerance set for holding on to these strategies or to diverge from them. According to the American Academy of Actuaries [American Academy of Actuaries 1999], the model should also include the portion of future market volatility, which can be predicted for different scenarios, because it has an impact on the risk and cost of assets, which might have to be traded in the future. Liquidity of the options has an enormous impact on the future cost of hedging the liabilities and should therefore be included in the model. The hedging strategy should be considered because different strategies have different levels of complexity and flexibility. Finally, the actuary should also consider the availability of management information, which is necessary to monitor the hedging program and to apply corrections if necessary, since it can also have an impact on hedging cost and risk. The particular

information should usually include regular and accurate actual experience rates and market values of assets hedging inforce business.

8.2 Enhanced Benefit Risk

Typically, equity-indexed annuities offer the policyholder the possibility to withdraw vested and sometimes non-vested index values because of some insured event like death or nursing home admittance. In addition, policyholders may access their money for specific purposes like annuitization. All the above benefits are enhanced benefits causing *enhanced benefit risk* according to the American Academy of Actuaries [American Academy of Actuaries 1999] because the occurrence of the events and the decision of the policyholder are a-priori unknown. For instance, the policyholder can often choose within a window period of 60 days following the expiration of a term whether he or she wants to surrender the policy for a single lump sum payment or whether he or she elects annuitization for a certain period of time. The same options are offered to the beneficiary in case the policyholder dies before the end of the term. Usually a portion of the assets is invested in index options or other derivatives. Therefore, the insurance company is exposed to the risk that the fixed portion of the assets will not be sufficient to fund death benefits under the scenario that the index is down at the time of death. If the death benefit of a contract guarantees crediting of part of or

all of the index growth, the actuary will usually need to consider interim index values to quantify the death benefit exposure. Another issue is the possibility of the need to sell a relatively small number of options to provide for death benefits, which may be economically unattractive to sell. The window period mentioned above also contributes to the enhanced benefit risk, since it usually is attached to the term and may extend beyond the date when the insurance company would need to buy new hedges for the annuity. In some cases, the hedge may be insufficient to fund the surrender value. The surrender amount may also be too small to justify selling hedges for it. In addition, some equity-indexed annuities allow the policyholder to transfer between various index and term choices at different times during the contract. Often, insurance companies limit the timing and the amount of these options. Actuaries should address this risk by extending their modeling period and considering policyholder behavior.

8.3 Guaranteed Element Risk

Many insurance companies guarantee different factors that influence credited index increases in equity-indexed annuities. These guarantees usually have duration of one year or the whole term of the policy. Some contracts also include guarantees concerning renewal terms such as participation rates or caps. The guarantee of credited index

affecting factors in equity-indexed annuities is similar to the guarantee of current interest rates in a fixed annuity. If an insurance company guarantees these factors for the whole term of the policy, the risk is equivalent to guaranteeing a fixed interest rate for the whole policy term. The insurance company takes the risk that it can manage the investment portfolio, which should earn the interest and index credits, which originate from the guarantees to the policyholder. Guaranteed element risk is especially important to consider in combination with disintermediation risk and hedge mismatch risk.

Actuaries asses this risk by paying attention to the length of the modeling period, the availability of options and the impact of renewal guarantees, according to the American Academy of Actuaries [American Academy of Actuaries 1999], which also says that many actuaries think it is necessary to extend the modeling period over a phase in which all important guaranteed elements are covered. The length of the modeling period depends on the length of the index credit guarantees and on the period of the hedging assets. If there are assets in the portfolio, that hedge several terms of index credit or if there are index crediting guarantees extending over several terms, the modeling period is selected accordingly. Usually it is advisable for the actuary to have the end of the modeling period coincide with the end of an index term.

8.4 Market Liquidity Risk

If the policy includes exotic crediting methods or uses an illiquid index, the insurance company *faces market liquidity risk* since it will probably require customized options, which might not be available in the future. Actuaries should have a backup investment strategy in case the needed special options cannot be found. If the nominal amount of an option is small, there might occur additional cost, which should be considered by the actuary.

8.5 Counterparty Risk

If an insurance company uses exchange-traded derivatives, there is no *counterparty* risk, since the clearinghouse, which is an agent of the exchange and with whom the derivative issuer has to deposit a security margin payment, guarantees the transactions. Unfortunately, the exchange-traded options at the Chicago Board of Options Exchange are all short term derivatives. Therefore, insurance companies might choose especially customized over-the-counter options to back up their exotic or long-term guarantees in equity-indexed annuities. In this case, the counterparties are typically investment banks. Therefore, there is a risk that the counterparty might default, which has to be considered by the actuary.

8.6 Asset Liability Management

Ostaszewski [Ostaszewski 2002] defines asset-liability management as a tool for the insurance industry that should not only eliminate or control interest rate risk, but increasingly incorporates asset default risk, product pricing risk, and other uncertainties of the business. The ideas of asset-liability management can be traced back to Redington's [Redington 1952] theory, which used the concept of *duration* and first introduced the technique of *immunization*. Bodie [Bodie, Kane, Marcus 1996] states that if one wants to deal with the ambiguity of the "maturity" of a bond making many payments, one needs a measure of the average maturity of the bond's promised cash flow, which should serve as a useful summary statistic of the effective maturity and the sensitivity to interest rate changes of the bond. Macaulay [Macaulay 1938] coined the expression *duration* of a bond for the effective maturity concept, and he also suggested that duration is calculated as a weighted average of the times to each coupon or principal payment made by the bond. He suggested that each payment time should be weighted with the proportion of the total value of the bond accounted for by that payment. This proportion equals the present value of the payment divided by the bond price. This is known as *Macaulay duration*: $\sum_{t\geq 0} \frac{CF_t (1+i)^{-t}}{\sum_{t\geq 0} CF_t (1+i)^{-t}} \cdot t$,

where t are times of cash flows, CF_t is the cash flow at time t, respectively, and $(1+i)^{-t}$ is the discount factor from the future point in time to time 0. The denominator equals the value of this bond at time 0, since it is the sum of all discounted future payouts. Note that this definition applies only when the cash flows of the bond are deterministic, when they do not depend on interest rates themselves.

Redington [Redington 1952] addressed interest rate risk by the ingenious idea of applying basic ideas from elementary calculus. If $f(x)$ is a function of a variable x, and if the derivative $f'(x)$ exists, then the following approximation holds true: $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$. If the derivative in this relationship equals zero, a small change in x, denoted by Δx, will not change the value of the function. Redington applied this principle to an insurance company's surplus. If $A(i)$ denotes the market value of the company's assets with i being the effective annual interest rate, and $L(i)$ denotes the market value of the company's liabilities, one can express the surplus of the company as market value of assets minus market value of liabilities: $S(i) = A(i) - L(i)$. This is clearly a function in i, the effective annual interest rate. If the above reasoning is applied to the surplus function, one can automatically see that assets and liabilities should be managed such that $A'(i) = L'(i)$. This would immunize the

insurance company from small changes in interest rates, which means that the value of the surplus would not change if small changes in the interest rate occurred.

Even though asset-liability management is a comprehensive tool for the management of a company by using projected assets and liabilities, it is typically associated with the management of interest rate risk. The National Association of Insurance Commissioners developed a risk-based capital formula, which came into effect in 1993 as a possible answer to several life insurance company bankruptcies in the early 1990s. Risk-based capital should establish a minimum capital level for each insurance company based on the risk the company is taking [Morgan Stanley & Co., Inc. 1993]. The risk-based capital formula establishes target surplus amounts that are required above reserve requirements. These amounts are calculated using four major factors related to four major categories of risk facing an insurance enterprise:

- C-1: Asset quality and payment default risk
- C-2: Insurance pricing risk
- C-3: Interest rate risk, often generalized as asset-liability management risk
- C-4: Miscellaneous business risks

Morgan Stanley [Morgan Stanley & Co., Inc. 1993] provides the numerical formulae for the particular components. The actual *risk based capital* is then calculated as: $RBC = (C-4) + \sqrt{(C-2)^2 + ((C-1)+(C-3))^2}$. The insurance company's adjusted capital, which equals statutory capital and surplus, asset valuation reserve, plus voluntary reserves and half of the policyholder dividend liability, is divided by the risk-based capital to determine the risk-based capital ratio. Insurance regulators use this ratio to determine a company's capital adequacy. As this model implies, asset-liability management has been traditionally related to interest rate risk or C-3 risk.

There are several different strategies to manage interest rate risk. Van der Meer and Smink [Van der Meer, Smink 1993] have categorized and described some of them. The strategies and techniques are classified in three distinct groups: static, value driven, and return driven. Van der Meer and Smink distinguish between techniques, which they consider being essentially static, and strategies, which are dynamic since they require some set of decision-making rules. Dynamic strategies then are divided into value driven and return driven strategies.

8.6.1 Static Techniques

The first class considers static techniques and they are ranked at increasing level of sophistication required. Most of the methods in this class are applied by banks and insurance companies since they are very simple and relatively easy to implement. They all concentrate on a complete match between assets and liabilities. This is particularly true for cash flow matching. All those methods lack the possibility of a consistent trade-off between risk and return. These techniques do not explicitly measure risk or return.

- *Cash flow payment calendars*

Cash flow payment calendars give a maturity overview of all cash inflows and outflows of an insurance company. They help detect major disparities between cash flows resulting from assets and liabilities. This means that an insurance company can spot timing differences in cash inflows and outflows.

- *Gap analysis*

Gap analysis is a tool from bank asset-liability management. The Gap can be defined as the balance sheet value difference between fixed and variable rate assets and liabilities. A non-zero Gap implies interest rate exposure. As an example, if a company owns more variable rate assets than liabilities, then a decline in rates will result in a loss in net

operating income. Gap analysis can also account for maturity differences between assets and liabilities.

- *Segmentation*

Liabilities can be segmented according to their characteristics. In addition, each segment of liabilities gets its own portfolio of assets, designed to meet the particular characteristics of the particular liability segment.

- *Cash flow matching*

Cash flow matching usually applies linear programming to minimize the inequalities between all asset and liability cash flows. From a selection of assets, a portfolio is compiled to meet all liability payments with certainty, within a minimal acceptable time span, and with minimal cost. In practice, there are several problems with this technique. For instance, a complete match may not always be available in the market, particularly since insurance companies typically are dealing with long-term liabilities. This problem will show up once more in chapter IX because the same problem exists for derivative instruments. Because of this issue, the programming problem might not have a solution. Second, this technique does not allow adjusting the risk that a company is willing to take in expectation of higher returns. Another problem is the stochastic character of most of the liabilities of an insurance company.

Cash flow matching, as well as all other presented techniques assumes full knowledge of timing and amount of cash flows, which might not be true for example for claims.

8.6.2 Value Driven Dynamic Strategies

The basic idea underlying all immunization type strategies was coined by Macaulay [Macaulay 1938] and is described above: the Macaulay duration, which measures the interest rate sensitivity of the value of fixed cash flow assets or liabilities. Redington [Redington 1952] then defined a strategy to maintain the surplus of a portfolio consisting of assets and liabilities with fixed cash flows, the so-called immunization. There are several types of immunization strategies:

- *Standard immunization*

Standard immunization matches the interest sensitivities of assets and liabilities. As described above, this means equating the first order partial derivatives of asset and liabilities with respect to the yielding interest rate. In addition, the second derivative of the assets, called convexity, has to be at least as large as that of the liabilities. Since the approximation formula used for immunization is only true for infinitely small changes in the flat term structure interest rate and for a small instant of time, immunization requires continuous rebalancing of the portfolios. This is what makes it a dynamic strategy. A major weakness of

this model is that it assumes a flat term structure. If one could be immunized with greater asset convexity than liability convexity, then any change in interest rates would produce value from nothing and violate the no-arbitrage assumption. At least, this is the impression one could get at a first glance. However, in practice, this so-called arbitrage gain is mostly the result of a risk-return trade-off. Convexity increases with more dispersed cash flows. The assumption of only one relevant interest rate implies that only parallel shifts in the term structure are measured, which means that all yield points move in the same direction and by the same amount. In reality, however, non-parallel shifts are important as well, since in general, interest rates earned on assets and liabilities will differ for different maturities and depend on the so-called term structure of interest rates. The impact of non-parallel yield curve shifts on assets and liabilities will increase with convexity. In addition, Macaulay durations explicitly assume deterministic cash flows. Therefore, if cash flows are interest rate dependent, Macaulay duration cannot be meaningfully applied.

- *Model conditioned immunization*

This model is a modification of the standard immunization to iron out the term structure "flaw" of the standard model. The modifications depend on assumptions regarding the stochastic process behind the development of the term structure. The resulting strategies used for

immunization differ only in the duration and convexity measures used. Advantages of this type of strategies are their potential accuracy and the possibility to include derivative instruments. The major disadvantage is the non-stationarity of the factors driving the model. This causes potential risk related to the validity of the model and the need to monitor the driving factors.

- *Key rate immunization*

Key rate immunization is almost completely similar to standard immunization except for the fact that it considers non-parallel term structure shifts. This is achieved by segmentation of the cash flows, which is achieved by the key rate immunization strategy. The shape of the term structure is characterized by a limited number of key interest rates, from which the other values are obtained by interpolation.

Active immunization strategies want to guarantee a minimum floor value for the asset portfolio. In the case of asset liability management, this floor is determined by the value of the liabilities at the end of the term. There are several active immunization strategies, which were most often originally designed for equity portfolios.

- *Contingent immunization*

Contingent immunization combines active portfolio management with portfolio matching. The underlying idea is that a portfolio of assets

can be immunized at any point in time, but as long as the portfolio's value meets the liabilities, it can be managed actively to increase performance. If the portfolio's value drops to a previously specified value, then the portfolio is immunized with an immunization strategy.

- *Portfolio insurance*

Based on option pricing theory and the Black-Scholes option pricing formula, a strategy using stocks and bonds can be used, allowing for the upside potential of stock investments with the downside protection of the portfolio's value against a previously specified level. The idea behind this strategy is the synthetical creation of a put option on a stock portfolio. The idea of option replication is described more detailed in chapter IX. This strategy, however, is probably not very feasible for the insurance company because of the high risk-based capital requirements of stock.

- *Constant Proportion Portfolio Insurance*

This is a variation of portfolio insurance, which holds the proportion of risky assets in the portfolio constant. This means that exposure to the risky asset is reduced compared to regular portfolio insurance, in the case that the value of the risky asset increases. The risk free asset in this case also grows to provide the floor at maturity.

8.6.3 Return Driven Dynamic Strategies

Different from the immunization type strategies, which concentrate on the value of assets, the strategies in this section are determined by returns or spreads. This often causes neglect of the value monitoring, which is inbuilt in immunization strategies. Therefore, these methods may not represent all risk correctly.

- *Spread management*

This method tries to maintain a yield spread between assets and liabilities. It uses the idea of segmentation and buy-and-hold-investment-strategies. Advanced spread management relates differences in spreads to spread determining factors like duration differences. The spread management used for asset liability management is based on market value and should be included into a comprehensive risk-return framework.

- *Required rate of return analysis*

This method considers the future cash flows of the liabilities and based on these it determines the return required on the current cash balance of the liability to meet these cash flows. These returns are then used to select an appropriate asset portfolio.

8.6.4 Other methods

Besides the strategies and techniques above, there are methods that can either not be classified clearly or that deserve special attention.

- *Multiscenario analysis*

Multiscenario analysis is a static technique, but scenario dependent actions can be prepared. Multiscenario analysis produces projected cash flows of assets and liabilities under different assumptions for the development of some key variables like interest rates or inflation. This method reveals scenarios under which cash flows are not matched.

- *Risk-return analysis*

The underlying principle of risk-return analysis is that if two portfolios with assets and liabilities have different returns, they either have different risks or one of the portfolios is not efficient. An investor will only consider efficient portfolios in the universe of possible portfolios

and the investor chooses the portfolio that best matches the risk-return preference of the investor.

8.7 Cash Flow Testing

As already mentioned in chapter VII, the 1990 Standard Valuation Law also requires an actuarial analysis of reserves and assets supporting reserves as part of statutory valuation, which was defined in chapter VII. The part of the law requiring this analysis is based on New York Regulation 126 and means that asset adequacy analysis is required. Actuarial Standard of Practice Number 22 [Actuarial Standards Board 2002] states that "both the type and depth of asset adequacy analysis will vary with the nature and significance of the asset, obligation, and/or investment-rate-of-return risks".

Cash flow testing is a method of asset adequacy analysis. In Actuarial Standard of Practice Number 7, the American Academy of Actuaries defines cash flow testing as the "process of projecting and comparing, as of a given date called the *valuation date*, the timing and amount of asset and obligation cash flows after the valuation". An introduction to valuation can be found in chapter VII or in Tullis & Polkinghorn [Tullis, Polkinghorn 1996].

Cash flow testing is usually more appropriate for products where future cash flows might vary under different economic or interest rate

scenarios. Since the Standard Valuation Law cannot consider all the possible events that can happen in the market, the concept of the valuation actuary has evolved. The valuation actuary is given the responsibility to make sure that reserves not only meet legal requirements but that the assets supporting the reserves are sufficient to cover outstanding liabilities by testing them and valuing them. The valuation actuary should consider several different factors that can affect adequacy of reserves, one of them being cash flow testing. Because equity-indexed annuities offer a unique combination of equity and fixed interest rate returns, according to the American Academy of Actuaries [American Academy of Actuaries 1999] many actuaries perform cash flow testing to assess asset adequacy for equity-indexed annuities. For regulatory testing purposes, the Standard Valuation Law specifies seven interest rates scenarios:

1. The interest rates remain level, exactly where they are now, for the period of testing.
2. Interest rates are uniformly increasing 5% over 10 years and then level.
3. Interest rates are uniformly increasing 5% over 5 years, uniformly decreasing 5% over 5 years and then level.
4. Interest rates jump up 3% and then level.

5. Interest rates are uniformly decreasing 5% over 10 years and then level.
6. Interest rates are uniformly decreasing 5% over 5 years, uniformly increasing 5% over 5 years and then level.
7. Interest rates jump up 3% and then level.

Those seven testing scenarios have to be performed at least to meet regulatory requirements. In addition, the actuary may want to pick other scenarios to test asset adequacy. In all the prescribed scenarios interest rates are floored by 4% and capped by 25%. As starting interest rates the valuation actuary can use actual interest rates at the time of valuation or he may base them on an index. In addition to deterministic scenarios, it is also possible and often advantageous to use stochastic interest rate scenarios. Within each of these interest rate scenarios, equity market movements can be considered by randomly generating them or using a formula to model index movements in relationship to the fixed interest rate. Therefore, many actuaries choose *Monte Carlo simulation* to randomly generate equity market movements and carry out cash flow testing. A Monte Carlo method can be characterized by the use of random numbers and probability statistics to investigate problems. One of the earliest applications of random numbers was the calculation of integrals. The idea underlying this method is that if one generates a large number of random numbers and applies them to the probability statistic

one can approximate the integral. A good source for Monte Carlo simulation is Ross [Ross 2002].

The model used for cash flow testing should satisfy several conditions. It should consider relevant product design, features and risks, and interaction of different factors. For example, interaction of early surrenders with volatility in interest rate and equity markets may result in a wider distribution of possible outcomes. The tested scenarios should cover the whole distribution of possible outcomes and reflect the expected return and volatility.

It is important for the actuary to choose specific equity market and interest rate scenarios based on experience, product features and inherent risks. The particularly chosen scenario can help determine influential quantities for the results of the model and they can address risks that occur too infrequently to be uncovered by a reasonable number of stochastically generated scenarios. However, those deterministic scenarios should only be used in addition to stochastically generated scenarios, since they can only accomplish the picture that is formed by running random scenarios, but they should not be the sole basis of cash flow testing for equity-indexed annuities, according to the American Academy of Actuaries [American Academy of Actuaries 1999].

If extreme scenarios are chosen deterministically, and represent extreme, most dangerous developments, this is called *stress testing* or *resilience*

testing. Stress testing is not prescribed in the U.S., but Canada requires its actuaries to do stress testing. Usually, some of the scenarios used for stress testing are chosen based on historical experience. Threatening scenarios that have actually occurred can be used or slightly modified to test the behaviour of the cash flows under these scenarios.

CHAPTER IX

INVESTMENT STRATEGIES FOR EQUITY-INDEXED ANNUITIES

Since the liabilities evolving in an equity-indexed annuity are a priori uncertain, the insurance company is exposed to a risk in its liabilities, which cannot be deterministically quantified. A company has several possibilities to deal with this risk. It can take the risk, it can transfer it fully or partially through reinsurance, or it can manage the risk. One possible risk management strategy is called hedging. The Webster dictionary [Merriam-Webster Inc. 2002] defines “to hedge” as “to protect oneself from losing or failing by a counterbalancing action”. The one specific risk of equity-indexed annuities that is added to the portfolio of an insurance company’s risks is the risk that the assets underlying an equity-indexed annuity perform worse than the index. The insurance company can take counterbalancing actions to protect itself against losses from this risk. For instance, it can reinsure all the risk of possible liabilities arising from a strong index performance and the high payouts associated with this performance. This is a choice for insurance companies, which is actually elected by several companies in the market.

It is dependent on the willingness of a reinsurance company to take such a risk.

Another way for insurance companies to make sure they can pay their liabilities arising from index performance is using synthetic options. It is possible to purchase a portfolio of the stocks of the index, and then buy and sell these stocks using option replication theory. This is called dynamic hedging and will be discussed in section 7.4. A source for option replication theory is Panjer [Panjer 1998].

The most practicable alternative for insurance companies to hedge equity-indexed products is using derivatives. This method will be presented in section 7.3. To be able to discuss hedging by using derivatives one has to be familiar with financial terminology.

9.1 Financial Terminology

Hull [Hull 2000] defines a *call option* as the right to buy a certain security, called underlying security or just underlying, at a predetermined price at some point in time in the future. A *put option* is the right to sell an underlying security at a predetermined price at some point in time in the future.

If someone owns a call option with a *strike price* of 100 on a company, which matures in a year, this person has the right to buy a share of that company for $100 either during the next year or at the end

of the next year. If the option can only be exercised at the end of its life, it is called a *European option*, according to Hull [Hull 2000]. If it can be exercised throughout its life, it is called an *American option*.

Such an option can also be issued on an index, for example the S & P 500. This is the right to receive or pay the difference between an index level and a predetermined price. The main difference between the option on a security and the option on an index is that the index is not a security and therefore cannot be bought in the market. It is possible to replicate the index by buying all the stocks in the portfolio, but one still does not own an index then.

The *exercise date* is the date on which the option can be exercised. This means that on this date the owner has the right to go to the writer of the option and buy or sell the underlying security at the predetermined price.

The *maturity date* is the final exercise date. For example, *European options* have one exercise date, and that is the same as the maturity date. *American options*, which can be exercised at any point in time, have several exercise dates but only one maturity date. That is the last date one can use this option.

If an investor is *long* in options that means he or she owns an option. That means he or she bought the right to receive the difference between the strike price and the index value at that time. If an investor is

short in them, that means he or she sold them, which means that somebody else has the right to expect him or her to pay the difference between the underlying index and the strike price.

The price paid for an option is called the *option premium.* According to Hull [Hull 2000], an option is *in the money* if immediate exercise of the option would result in a positive cash flow. In the case of a call option, this means that the price of the underlying is greater than the strike.

If the value of the index is above strike price and the call option is in the money, and if the option bearer has the right to exercise on that date, he or she can demand the payout from the option issuer. An option is *out of the money* when immediate exercise of the option would lead to a negative cash flow. In the case of a call option, the option is in the money when the price of the underlying asset is less than the strike. If the S&P 500 index were at 930 points and an investor had a call at 1000 then he or she has the right to buy the S&P at 1000, which one would not exercise since the option is out of the money. If an investor had a put on the S & P 500 with a strike of 1000 and the S &P were at 930, the option would be in the money and it would be exercised.

Call options are the most practical way to hedge equity-indexed annuities. The portion of the interest rate that is credited according to the index performance can be supplied by holding a call option, since both have the same characteristics. If the index is below a certain level,

for example below the level at the beginning of the term, the portion of the interest credited due to index performance is equal to zero, if it is above the starting level, interest is credited as a linear function of index performance. Assume now, one is holding a call with strike price equal to the level of the index at the beginning of the term and maturity date equal to the end of the term. If the index is below the strike price at maturity, the call is out of the money, will not be exercised and has a payout of zero, if it is above, the value of the call is a linear function of the index price. This shows that call options can be used to hedge the portion of the credited interest, which depends on index performance.

9.2 Hedging in the Context of Asset Liability Management

According to the equity indexed products task force of the American academy of actuaries [American Academy of Actuaries 1998a], insurance companies usually use asset liability management for their general account on a company wide-basis. Asset liability management for equity-indexed annuities in general is discussed in chapter VIII. That means that they first allow the different blocks of business to offset each other in terms of an economic variable and then fine-tune their exposure to this variable. The reason for choosing this approach is that often one general account liability reacts exactly the opposite way from another liability when exposed to a specific economic variable. For example,

deferred annuities and immediate annuities are exposed exactly the opposite way to changes in index movements. While rising indices typically require decreasing amounts of assets needed to provide for immediate annuity liabilities, increasing indices might cause an increase in assets needed to provide for deferred annuity liabilities, if the increase is credited to the contract holder. An insurance company will usually measure its exposure to risk from a change in index after combining those two blocks of business, which partly offset each other in terms of asset requirements due to index changes.

Liabilities caused by equity-indexed annuities increase with increasing equity markets. Usually, one cannot find other liabilities in an insurance company's general account, which would have an opposite response. Therefore, insurance companies usually hedge the equity exposure of these liabilities separately. The hedging of those liabilities is often realized in a way that several equity-indexed annuities are grouped together. That is the reason why equity-indexed annuities usually are issued every two weeks. Grouping together those contracts facilitates hedging. This comes close to individual policy hedging. For example, the initial equity index value used to calculate the interest credited to the contract is usually the index value on a certain day of the week following the issue date.

When an insurance company mismatches its assets and liabilities, the volatility and possible trend movements of equity markets can transform any mismatch into a big risk. Hull [Hull 2000] defines the *hedge ratio* as the ratio of the size of the position taken in hedging instruments to the size of the exposure. This hedging ratio need not necessarily be 1.0. In fact, if an investor wants to minimize risk, often a hedge ratio different from 1.0 will be optimal. Under-hedging means that the hedge ratio of the real hedge is less than the hedge ratio which would minimize risk. Under-hedging is risky and harmful if share prices are rapidly rising, since not the whole possible payout is hedged. Over-hedging means that the hedge ratio is greater than the ratio, which would minimize risk. Over-hedging can be a problem if share prices are falling. Over-hedging means that an insurance company hedges more than 100 % of the payout and this can also cause problems with rising share prices since the overhead portion of the options loses worth in addition to the losses caused by the options needed for the regular hedge.

9.3 Static Hedging

According to the equity indexed annuity task force [American Academy of Actuaries 1997], an insurance company will try to match the liabilities arising from an equity-indexed annuity by investing in assets

that will provide an equivalent income cash flow. Typically, liabilities arising from a general account equity-indexed annuity at maturity are the maximum of the floor guarantee and the value due to the performance of the underlying index and computed by a formula, which is specified in the contract. Usually, the floor guarantee is the nonforfeiture law minimum for single premium fixed deferred annuities, which means that 90% of the premium are accumulated at an interest rate of 3%. Since this guarantee is a certain liability, the company can purchase fixed income securities, such as zero coupon bonds to provide for it. Purchasing zero coupon bonds of the same term and having a final payout equal to the maturity floor guarantee can be a hedging strategy for the floor guarantee of each annuity expected to persist to the end of the term. Assume, for example, an equity-indexed annuity with a term of seven years and a single premium of $1000. The maturity guarantee can then be hedged by purchasing a zero coupon bond which pays $1106.89 after seven years. To fulfill the nonforfeiture law requirements, $900 are accumulated at 3% for seven years, which equals $1106.89.

In reality, insurance companies may mismatch on purpose in their portfolio of fixed income securities. Instead of zero coupon bonds, the companies might buy coupon bonds, in order to get a higher yield or to create an additional income stream. In addition, mortgage-backed securities are also used sometimes as an alternative asset, because they

often provide higher yield in exchange for a high risk of default, which can cause problems for the insurance company. If a company mismatches its assets in the way that is described above, it deliberately takes reinvestment risk in order to achieve higher yield or liquidity.

In addition, equity-indexed annuities guarantee supplementary interest based on the performance of an underlying equity index. The granted supplementary interest is zero if the index performs poorly and ends up at a level that is below the guarantee level in the annuity, and the interest grows in proportion to the excess of index performance over minimum guaranteed performance. This is equivalent to a call option. If the underlying index performs poorly and ends up below some benchmark, it will pay nothing. On the other hand, if the index grows above the predetermined benchmark, it pays proportionately larger amounts. Consequently, the portion of the liability that is due to the performance of an equity index is hedged by purchasing call options. Assume, for example, an equity-indexed annuity with a seven-year term, point-to-point design, an 80% participation rate on the S & P 500 and a single premium payment of $1000. The floor guarantee according to the nonforfeiture law can be hedged by buying a seven-year zero-coupon bond with payout of $1106.89. The equity-indexed part of the payout can be hedged by purchasing a seven-year S & P 500 European call option with a strike price of 113.361% of the initial index value and an assumed

amount of $800. The strike price is determined by dividing the notional floor guarantee increase by the index participation rate (10.68865% / 0.80 = 13.36081%). As a result, the floor guarantee is hedged by the zero-coupon bond, and the supplemental guarantee is hedged by the call option. The assumed amount for the call option is only $800 because the index participation rate of 80% effectively allows only $800 of the $1000 to participate in the index performance. The participation rate is one parameter the insurance companies can use to adjust their option prices. The smaller the index participation rate, the higher the strike price and the lower the notional amount and consequently the cheaper the option. On the other hand, one does not want the participation rate to be too low since this is a problem from a marketing perspective. Ideally, an index participation rate should be somewhere in the range of 70 to 100%.

The equity indexed products task force [American Academy of Actuaries 1997] mentions also other points an insurance company might want to incorporate in its equity hedging strategy, for example equity participation in the case of death and the effect of vesting of equity participation in the case of surrender or lapse. Most equity-indexed annuities guarantee the maximum of the account value based on the full index performance and the nonforfeiture minimum upon death. The policy is treated as if it was the end of the term and the ending index is the index value on the date of death. The company has a liability that

has to be matched with a shorter duration than if it would run to maturity. A possible solution for this problem is to purchase a series of calls with shorter durations than maturity and in amounts according to the expected deaths in each policy year. Since the guaranteed value in this case does not have as much time to grow as it would have until maturity, the guaranteed floor value will be smaller than at maturity and therefore the strike prices for the call options would be lower. This would imply a higher option premium.

There are several problems connected to hedging by using options. First, there is only a limited range of standardized options, which can be traded on an exchange. Only standardized European vanilla options are traded at the Chicago Board Options Exchange, which is the predominant market for options in the U.S. In addition, the traded options are available only for short durations when compared to the term of an equity-indexed annuity. The exchange-traded options on the S & P 500 with the longest durations are S & P 500 Long-Term Equity Anticipation Securities (LEAP), based on one tenth of the S & P 500 index value. They usually expire in 2 to 3 years and the expiry month range is typically limited. Because of this problem, one cannot directly hedge most equity-indexed annuities with exchange-traded options. For example, if a company sells a seven-year point-to-point equity-indexed annuity, it will not find a seven-year exchange-traded European option to

hedge its products guarantees. However, these options can supplement an *over-the-counter* option plan and thus hedge risks like disintermediation or early terminations. They can also be used to replicate other options, which will be described in the section about dynamic hedging.

Over-the-counter options can be purchased from a number of investment banks. They are manufactured and customized to the buyer's specifications and can be designed for long durations. Since they are especially customized, they can match unusual features and options, which cannot be found at an exchange. However, this special manufacturing usually comes at a significantly higher cost than an exchange traded option. Over-the-counter options inherently come with a greater counter party risk than exchange-traded options, which are backed by the options or futures clearinghouse, which protects the option holder against default by requiring the deposit of substantial margins.

Hull [Hull 2000] shows in his book, that a portfolio of one European call plus an amount of cash, which equal to the present value of the strike price of the option, has the same value today as a portfolio consisting of one European put option, with the same strike price and the same maturity date as the call, plus one share of a non-dividend paying stock, since both portfolios have the same value at expiration of

the options. The value of these portfolios at expiration of the options is the maximum of the stock price at expiration and the strike price. This means that those two portfolios are equivalent. If this fact would not hold true, there would exist arbitrage opportunities. This relationship is called *Put-Call Parity*.

The Put-Call Parity says that in an arbitrage-free market, buying a put option and an underlying stock is equivalent to buying a call option and a zero-coupon bond. However, the insurance companies usually buy calls to hedge the equity-linked part of an annuity's guarantee. The reason for this approach is that it is disadvantageous for insurance companies to hold stock. If they used Put-Call Parity, they would have to hold a substantial amount of stock, and the risk-based capital requirements for stock are very high. An insurance company has to hold capital equal to or greater than 30% of the stocks initial worth. According to the equity-indexed annuity task force [American Academy of Actuaries 1997], the same risk-based capital requirements are imposed without regard to whether the stock is held for investment return or as part of a hedge. Risk-based capital is explained in chapter VIII. Therefore, the interesting situation arises that equity-indexed annuities have liabilities that are based on stock, but their assets are not invested in stock. Instead, insurance companies try to replicate their liabilities.

9.4 Dynamic Hedging

Dynamic hedging, also known as option replication, is an alternative approach to hedging guarantees in equity-indexed annuities. Through dynamic hedging the insurance company itself creates the call option needed to hedge its liabilities. In order to produce this call option it follows a trading strategy, which is designed to provide the insurance company with the amount of index equity needed at maturity to cover the liability. The cost of this strategy is presumed to be less than or equal to the cost of purchasing an option at issue of the equity-indexed annuity, provided market volatility and interest rates remain stable.

Effectively, the insurance company is managing a portfolio, the replicating portfolio of the call option. The market value of this portfolio has to track the market value of the option that is replicated. This is a field for investment banks and in which insurance companies do not have a lot of experience. The replicating portfolio is always equal in value to the replicated option, during the whole term and especially at maturity.

If one wants to use dynamic hedging as a strategy one needs some measurements for the risk in the option position. For each dimension of risk there is a separate measurement represented by a Greek letter. That is why those risk measurements are commonly referred to as the *Greeks*.

Hull [Hull 2000] defines five risk measurements, which are all partial derivatives of the option price with respect to different variables.

The first, and most fundamental Greek is the *delta*. Delta is defined as the option price rate of change with respect to the price of the underlying security. This can be interpreted as the slope of a graph, which depicts the option price as a function of the underlying security's price or it can equivalently be perceived as the first derivative of the option price with respect to the underlying security's price. If a portfolio has a delta of 0, it is said to be *delta neutral*. Since delta changes constantly, a portfolio can only be delta neutral for a short period. The portfolio needs to be adjusted from time to time. This process is called *rebalancing*. For example, assume that the price of a certain stock equals $50, the price of the option is $5 and the delta of the option is 0.7. Assume further, an insurance company has sold 10 option contracts, which entitle the holder to buy 1000 stocks since an option contract consists of 100 options. The insurance company could hedge this position with 0.7*1000 = 700 shares, which it would have to buy. Then, the loss in one position would exactly offset the gain in the other position, no matter if the markets go up or down. The portfolio would be delta neutral, provided delta stays constant. However, this exactly is not the case. Assume now, the stock price increases to $55. Since delta is a partial derivative at a specific point, it only approximates the slope in a

small environment around that point. That means, that if the stock price changes stronger, the previous delta is not valid any more and if it is still used, it will lead to hedging errors. Therefore, the portfolio needs to be rebalanced with a new delta. Assume now, the new delta is 0.65, which means that it increased by 0.05. The insurance company would now need to purchase 0.05 * 1000 = 50 additional shares to remain delta hedged. According to Hull [Hull 2000], this delta-hedging scheme is also referred to as *dynamic hedging* scheme. It requires the hedging portfolio to be adjusted regularly as opposed to static hedging where once the hedging portfolio is set up, it is never changed. That is why the latter is sometimes also called *hedge-and-forget* scheme. In reality, often times futures contracts are used for delta hedging rather than the underlying security, since future prices are a function of current security prices. This implies that a multiple of futures has the same sensitivity to stock price movements as one share of the stock.

Second, *gamma* is a logical continuation of delta. Gamma is the second derivative of the option price with respect to the price of the underlying security. It is somehow similar to convexity for interest rates, which is the second derivative with respect to the interest rate of a fixed-income security. Gamma measures the sensitivity of the rate of change of the option price to the price of the underlying security. Adjustments to

keep a portfolio delta neutral are necessary only seldom if gamma is small, since this means that delta changes slowly.

Vega is the third measurement and actually not a Greek letter, even though it is also in this group. Vega is the sensitivity measure of an option with respect to market-implied volatility.

The fourth Greek is *theta*, which is the rate of change of the option with respect to time decay.

The last risk measurement is *rho*. This is the rate of change of the option with respect to the interest rate. Rho is the analogue of duration for interest rates, which is the first derivative of a fixed-income security with respect to the interest rate.

If a company hedges perfectly, all those measurements should be zero for the combined asset-liability portfolio of a company. In reality, however, it is almost impossible to achieve this since one cannot find options to be used as hedges that can be traded in the required volume at a reasonable price in order to make all of the Greeks equal to zero. Usually, delta is set to zero daily by trading the underlying security and gamma and vega are monitored and if they move out of certain bounds, either up or down, some countermeasures are taken.

The cost of dynamic hedging is uncertain and is known only at the end of the term period. The trading strategy makes the insurer buy stock when the price is rising and sell stock when the price is falling, which is

called a *buy high, sell low* strategy according to the equity-indexed annuity task force [American Academy of Actuaries 1997]. Option replication will be more expensive than expected if the volatility of stock prices is higher than expected, since the insurance company will have to buy high more stocks than expected and sell low more stocks than expected. The underlying principle of delta hedging is that the portfolio is structured so that the change in its market value resulting from a change, for example, in an index matches the change in the market value of another portfolio, the option portfolio that is being replicated, for example. As mentioned above, delta hedging a call requires the insurance company to be long a portfolio of index futures and short-term interest-bearing securities. The initial value of the interest-bearing securities is equal to the theoretical value of the replicated option. This value can be determined by employing an option pricing model, which is also used to find the values of delta throughout the replication scheme. With changing futures prices there are positive or negative daily cash flows into or out of the futures account. If delta were exactly matched and the futures price increased, the positive cash flow into the futures account should roughly be equivalent to the increase in the replicated option's value. The futures price increase would cause an increase in delta, and theoretically, one would need to buy more futures. The additional futures do not require any other investments.

Although delta changes constantly, the number of futures contracts is only adjusted periodically since futures contracts are very large, e.g. $1,000,000. The daily change in delta may not require a change of even a single futures contract. Frequent trading would imply higher transaction costs. If a replication portfolio deviates from the delta of the option because the trading is done less frequently, tracking error cost will be created. This cost can only be reduced by matching other option sensitivity measures on top of delta. For example, insurance companies may try to hedge delta and gamma, or delta, gamma and vega. If an insurance company opts to match more Greeks than delta, it might have to use additional types of derivatives. The problem with more complicated matching strategies is that they will involve higher transaction costs.

CHAPTER X

DISINTERMEDIATION RISK FOR EQUITY-INDEXED ANNUITIES

The Equity Indexed Products Task Force of the American Academy of Actuaries [American Academy of Actuaries 1997] defines *disintermediation risk* as the risk that a contract is surrendered before the end of the contract term. Disintermediation risk deserves some special attention in the context of equity-indexed annuities. It is more complex than with fixed products since it not only depends on interest rates, but also on equity market movements. Disintermediation risk is a substantial risk since the insurance company might have hedging portfolios associated with the equity-indexed annuity. These portfolios are set up under the expectation that the equity-indexed annuity will persist until the end of the term. If the policyholder lapses before the end of the term, there may be a big risk since the portfolios may not have developed to support the liability before the end of the term. Falling equity markets might cause some equity-indexed annuity holders to lapse their policy. Bear markets usually go along with widespread

pessimism. Some contract holders may conclude that small returns in the recent past mean small return in the future. Therefore, they might judge that it is better for them to surrender and reinvest the income in some better form of investment from their point of view. If the bear market is accompanied by increasing interest rates, which is very possible, fixed income investments might be attractive. If the equity gains of the contract since issue are very small or the equity gains are largely vested, the insurance company is exposed to this type of lapse since the policyholder does not loose a lot by surrendering.

The product design of an equity-indexed annuity has a big influence on the degree of disintermediation risk. A point-to-point design might experience problems if the equity markets since issue are level and then suddenly drop. A policyholder might think that the market will not recover and end up below the level at issue of the policy and therefore he or she might as well surrender, get the minimum guarantee, and reinvest the proceeds in another form of investment. The inducement to surrender grows with the level of interest rates at this point. Some policyholders might even reinvest in the same type of equity-indexed annuity since it would then have a much lower starting point. Point-to-point designs are also subject to another bullying scenario. If equity markets rise from issue and then suddenly fall back close to the starting level, some policyholders may become discouraged and surrender. The

inducement to do so is smaller than in the first scenario, but it can still be considerable, especially if fixed interest investments are then offering attractive returns.

A high watermark design is exposed to the same scenarios as the point-to-point design. If the equity markets rise from issue and then suddenly fall back close to the starting level, some policyholders might believe that the high watermark for their equity-indexed annuity has already been set for the term and that they do not profit from persisting. This additional risk is only present if the policy vests part of the equity-based interest.

Annual ratchet designs are assumed to be the design that is least vulnerable to disintermediation. Only the recent year's index performance influences the credited equity-based interest. As opposed to the other two designs, one bad year's performance will not transfer into the next, each year starts new. Actually, a lot of policyholders might assume greater future performance after a downturn or market crash because the next year's starting point is the low endpoint of the bad year.

Early terminations can cause considerable loss to an insurance company. If, for example, the equity market falls shortly after the insurance company has issued equity-indexed annuities and at the same time interest rates rise, the bonds and the call options, which should hedge the fixed and the index-based portion of the credited interest rate,

respectively, will have low market values. The company has a substantial risk, if many policyholders terminate at that time. The insurance company might decide to hedge this disintermediation risk by buying bond and equity index puts. In this scenario, both puts would rise. However, since equity-indexed annuities are such a new product and the insurance companies do not have experience on the possible level of disintermediation for equity-indexed annuities, the quantity to buy is very uncertain.

The insurance company might have to pay out a portion of the equity-based interest to early surrendering contracts, if the insurance company vests a portion of the equity performance before the end of the term. Theoretically, the number of surrenders can be estimated by policy year, and the insurance company could buy shorter duration calls with lower notional amounts and lower strikes since only part of the annuity's account value might be vested and therefore lower minimum guarantees would apply.

In practice, there are some problems with this approach. One should remember that equity-indexed annuities are issued every two weeks and hedged in those biweekly groups. Expected lapses for such a group might be too small to justify buying an exact array of shorter-dated options. In addition, although death might be predictable fairly well by using appropriate mortality assumptions if the number of policyholders

is large enough, voluntary terminations are far more uncertain and the lapse experience is not yet on hand. More importantly, lapse experience varies depending on market and other economic conditions, and the level of disintermediation related to certain market situations is unknown, according to the Equity Indexed Products Task Force of the American Academy of Actuaries [American Academy of Actuaries 1997].

The same Equity Indexed Products Task Force [American Academy of Actuaries 1997] suggests a practical approach to this problem. In practice, an insurance company might assess the risk of early terminations by buying full term call options at issue to hedge 100% of issued policies, even though some deaths will occur almost certainly, and probably some surrenders will occur. If terminations then occur, the company sells the overhead of the options. This approach has some disadvantages. At the beginning, it is more expensive since more call options are bought than necessary. One might expect terminations to occur during falling markets. If this happens, the options sold are worth less than when they were bought. Advantages of this approach are that the company is hedged against the risk of lower-than-expected surrenders and in increasing markets the company can profit from having bought more call options than necessary. Given that historically markets had gone up more often than down, in the long run, this is a profitable strategy. However, in the short run strong market downturns

happen quite regularly, and a prolonged downturn will cause the company to have to unwind large positions, most likely under conditions of severe lapses.

One question that arises is what special considerations must be taken into account by actuaries when they model disintermediation risk. Many actuaries give special attention to the modeling of disintermediation risk for equity-indexed annuities, particularly to policyholder behavior and valuation of underlying assets. Because of the additional factors, which affect both policyholder behavior and option values, sensitivity testing might become more and more important for the evaluation of options in periods of changing volatility. The model used for policyholder lapse often includes features such as the sensitivity to the movement of the underlying index, the contract term, and the vesting pattern of the interest credits, the underlying interest rate guarantees and the economic impact of surrender on the policyholder. This model often indicates the relative advantage to the policyholder of surrendering now versus persisting to the end of the term. Typically, the assumptions also take into consideration differences in contract provisions. For example, penalties for early surrender differ significantly from contract to contract and may depend on index performance from the beginning of the term. In this case, past index performance and current value are typically both considered. In addition, the model used by the actuary to

evaluate disintermediation risk often reflects the value of the assets supporting equity-indexed annuities. Typically, the guaranteed part of the credited interest is supported by fixed investments and the excess part due to index increases is backed by options. The value of options is typically modeled by using factors including interest rates, index levels, implied volatility, and dividend rates. All these factors usually are incorporated in the model to evaluate disintermediation risk.

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