A Probabilistic Analysis of Chapter 7 and Chapter 11 of the U.S. Bankruptcy Code[1]

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[1] This talk is based on recent joint works with Bin Li, Lihe Wang and Xiaowen Zhou.

Contents

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Traditional firm value models

Stemming from Merton (1974, *Journal of Finance*) and Black and Cox (1976, *Journal of Finance*), numerous structural models have been proposed:

- diffusion process
- Lévy (driven) process
- Markov regime-switching models
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Traditionally, bankruptcy and liquidation are treated as the same event that the firm value reaches an absorbing low barrier:

- 0: Gerber and Shiu (ruin theory)
- constant: Longstaff and Schwartz (1995, *Journal of Finance*)
- exponential function: Black and Cox (1976, *Journal of Finance*)
- stationary mean-reverting process: Collin-Dufresne and Goldstein (2000, *Journal of Finance*)

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- Chapter 11 allows the firm to remain in control of its business with a bankruptcy court providing oversight. The court grants the firm a certain observation period during which the firm can restructure its debt.
- The debtor usually proposes a plan of reorganization to keep its business alive and pay creditors over time.
- In case the reorganization plan fails, Chapter 11 will be converted to Chapter 7 of liquidation governed by §1019 of the U.S. bankruptcy code.

The role of Chapter 11

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For related empirical studies of the role of Chapter 11, see also:

- Hotchkiss (1995, *Journal of Finance*)
- Bris, Welch and Zhu (2006, *Journal of Finance*)
- Denis and Rodgers (2007, *Journal of Financial and Quantitative Analysis*)

Literature review - bankruptcy barrier

Under these practical considerations, many recent works in the literature of corporate finance have included Chapter 11 reorganization proceedings and made a distinction between bankruptcy and liquidation.

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See the following works:

- Moraux (2004, *Working paper*, Université de Rennes I.)
- François and Morellec (2004, *Journal of Business*)
- Galai, Raviv and Wiener (2007, *Journal of Banking & Finance*)
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In these works, a firm will be liquidated when the time its value constantly or cumulatively spending under the bankruptcy barrier exceeds a grace period granted by the bankruptcy court.

However, only the bankruptcy barrier was considered and, hence, the firm is not necessarily liquidated even when its value is extremely low, which violates the principle of limited liability.

Literature review - Parisian ruin

It is interesting to notice that a similar research trend has emerged in risk theory independently. Originating from the study of Parisian options, Parisian ruin was first introduced. Parisian ruin is essentially the same as liquidation explained above.

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See the following works:

- Dassios and Wu (2008, *Working Paper*, London School of Economics)
- Czarna and Palmowski (2011, *Journal of Applied Probability*)

Contents

Our considerations

We follow Broadie, Chernov and Sundaresan (2007, *Journal of Finance*) to describe the procedures of bankruptcy and liquidation by incorporating the following:

- the Chapter 11 reorganization
- the Chapter 7 liquidation
- the conversion from Chapter 11 to Chapter 7
- the grace period in Chapter 11

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We follow Broadie, Chernov and Sundaresan (2007, *Journal of Finance*) to describe the procedures of bankruptcy and liquidation by incorporating the following:

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- the Chapter 7 liquidation
- the conversion from Chapter 11 to Chapter 7
- the grace period in Chapter 11

Formally, suppose that the value process is modeled by $X = \{X_t, t \geq 0\}$ with $X_0 = x_0$.

Two stopping times

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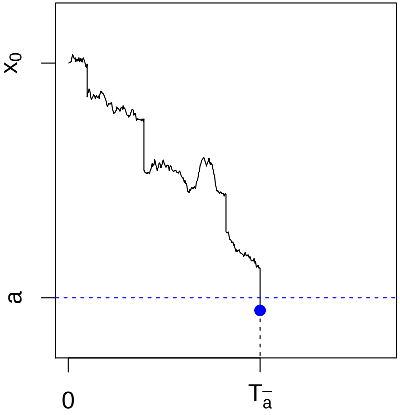

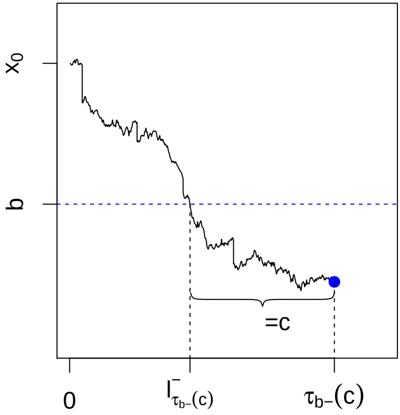

Liquidation time

Let $a < b$ and $c > 0$ be three exogenous constants, with

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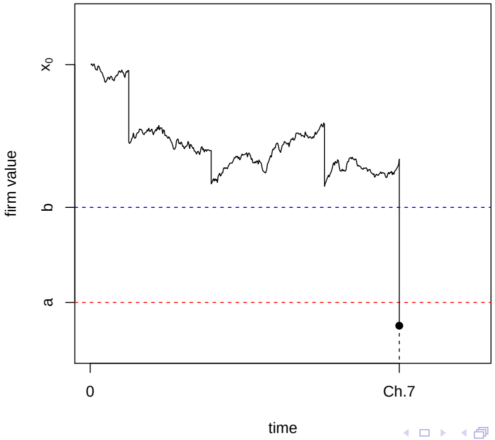
This covers three scenarios of liquidation.

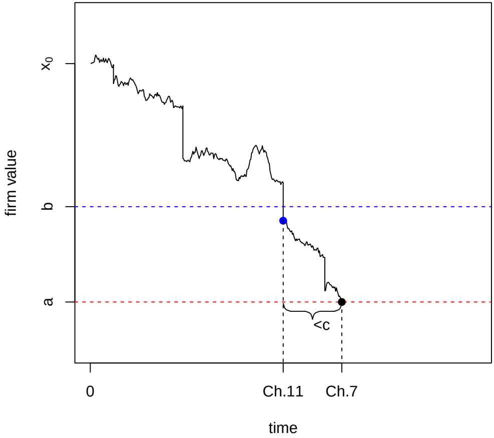
First scenario of liquidation

Declaring Chapter 7 directly: the firm suffers a catastrophic loss causing its firm value jumps from a level above the bankruptcy barrier b to a level below the liquidation barrier a.



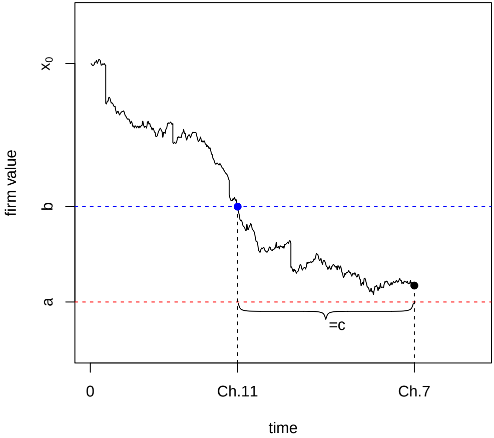
Second scenario of liquidation

Conversion from Chapter 11 to Chapter 7: the firm value drops below the liquidation barrier *a* prior to the end of the grace period *c*.



Third scenario of liquidation

Conversion from Chapter 11 to Chapter 7: the time the firm spends in bankruptcy exceeds the grace period c granted by the bankruptcy court.



The probability of liquidation

The probability of liquidation is defined by

$$q(x_0) = q(x_0; a, b, c) = \mathrm{P}^{x_0}\{T_a \wedge \tau_b(c) < \infty\}. \tag{2.1}$$

This probability of liquidation in the infinite-time horizon provides us with a quantitative understanding of the firm's liquidation risk in the long run.

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This probability of liquidation in the infinite-time horizon provides us with a quantitative understanding of the firm's liquidation risk in the long run.

It is sometimes more convenient to start with the probability of non-liquidation

$$p(x_0) = 1 - q(x_0) = \mathrm{P}^{x_0} \left\{ T_a \wedge \tau_b(c) = \infty \right\}.$$

Some immediate remarks

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Hence, the duration c serves as a bridge connecting the two traditional probabilities of bankruptcy:

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Letting $a \downarrow -\infty$ yields

$$q(x_0; -\infty, b, c) = \mathrm{P}^{x_0}\left\{\tau_b(c) < \infty\right\}.$$

This is essentially the probability of liquidation introduced by François and Morellec (2004, *Journal of Business*) and Broadie and Kaya (2007, *Journal of Financial and Quantitative Analysis*).

Contents

The firm value process

Suppose that the firm value is modeled by a time-homogeneous diffusion process $X = \{X_t, t \geq 0\}$, with dynamics

$$\mathrm{d}X_t = \mu(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t,$$

where:

- $X_0 = x_0$ is the initial wealth
- $\{W_t, t \geq 0\}$ is a standard Brownian motion (Wiener process)
- $\mu(\cdot)$ and $\sigma(\cdot) > 0$ are two measurable functions satisfying usual conditions of the existence and uniqueness theorem

Denote by $\{\mathcal{F}_t, t \geq 0\}$ the natural filtration generated by $\{W_t, t \geq 0\}$.

The two-sided exit problem

Define

$$G(x) = \exp\left\{-\int^x \frac{2\mu(y)}{\sigma^2(y)}\mathrm{d}y\right\}, \qquad S(x) = \int^x G(y)\mathrm{d}y.$$

The function $S(\cdot)$ is referred to as the scale function of X. To avoid triviality, we assume that $S(\infty) < \infty$.

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It is well known that, for $u < x < v$,

$$\mathrm{P}^x\left\{T_u < T_v\right\} = \frac{\int_x^v G(y)\mathrm{d}y}{\int_u^v G(y)\mathrm{d}y}, \qquad \mathrm{P}^x\left\{T_u > T_v\right\} = \frac{\int_u^x G(y)\mathrm{d}y}{\int_u^v G(y)\mathrm{d}y}.$$

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Letting $v = \infty$ in second relation above yields

$$\mathrm{P}^x\{T_u = \infty\} = \frac{\int_u^x G(y)\mathrm{d}y}{\int_u^\infty G(y)\mathrm{d}y}. \tag{3.1}$$

The main result

Introduce an auxiliary quantity

$$A(a,b,c) = \lim_{\varepsilon \downarrow 0} \frac{\mathrm{P}^{b-\varepsilon}\left\{T_b > T_a \wedge c\right\}}{\varepsilon}. \tag{3.2}$$

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It will be proved later that $A(a,b,c)$ exists, is finite and equals the boundary derivative of the solution of a PDE. Hence, its value can be easily determined numerically.

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It will be proved later that $A(a, b, c)$ exists, is finite and equals the boundary derivative of the solution of a PDE. Hence, its value can be easily determined numerically.

Theorem 3.1 For $a < b \leq x_0$ and $c > 0$,

$$q(x_0) = \frac{A(a, b, c)}{A(a, b, c) \int_b^\infty G(y)\mathrm{d}y + G(b)} \int_{x_0}^\infty G(y)\mathrm{d}y. \tag{3.3}$$

Proof of Theorem 3.1

For $x \geq b$, by the strong Markov property,

$$\begin{aligned} p(x) &= \mathrm{P}^x \left\{ T_a = \infty, \tau_b(c) = \infty \right\} \\ &= \mathrm{P}^x \left\{ T_b = \infty \right\} + \mathrm{P}^x \left\{ T_a = \infty, \tau_b(c) = \infty, \ T_b < \infty \right\} \\ &= \mathrm{P}^x \left\{ T_b = \infty \right\} + \mathrm{E}^x \left[\mathrm{P}^x \left\{ T_a = \infty, \tau_b(c) = \infty, \ T_b < \infty \right| \mathcal{F}_{T_b} \right\} \right] \\ &= \mathrm{P}^x \left\{ T_b = \infty \right\} + \mathrm{P}^x \left\{ T_b < \infty \right\} p(b). \end{aligned} \tag{3.4}$$

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&= \mathrm{P}^x \left\{ T_b = \infty \right\} + \mathrm{E}^x \left[\mathrm{P}^x \left\{ T_a = \infty, \tau_b(c) = \infty, \ T_b < \infty \middle| \mathcal{F}_{T_b} \right\} \right] \\
&= \mathrm{P}^x \left\{ T_b = \infty \right\} + \mathrm{P}^x \left\{ T_b < \infty \right\} p(b). \qquad (3.4)
\end{aligned}$$

It follows that

$$\begin{aligned}
p'_+(b) &= \lim_{\varepsilon \downarrow 0} \frac{p(b+\varepsilon) - p(b)}{\varepsilon} \\
&= q(b) \lim_{\varepsilon \downarrow 0} \frac{\mathrm{P}^{b+\varepsilon} \left\{ T_b = \infty \right\}}{\varepsilon} \\
&= q(b) \frac{G(b)}{\int_b^\infty G(y) \mathrm{d}y}, \qquad (3.5)
\end{aligned}$$

where in the last step we used (3.1).

Proof of the Theorem 3.1 (Cont.)

Similarly, for $x \in (a, b)$ we have

$$p(x) = \mathrm{P}^x \{T_a = \infty, \tau_b(c) = \infty\} = \mathrm{P}^x \{T_b \leq T_a \wedge c\} p(b).$$

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It follows that

$$\begin{aligned} p'_-(b) &= \lim_{\varepsilon \downarrow 0} \frac{p(b) - p(b - \varepsilon)}{\varepsilon} \\ &= p(b) \lim_{\varepsilon \downarrow 0} \frac{\mathrm{P}^{b-\varepsilon} \{ T_b > T_a \wedge c \}}{\varepsilon} \\ &= p(b) A(a, b, c). \end{aligned} \tag{3.6}$$

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Thus, the conjunction of (3.5) and (3.6) gives

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Substituting (3.7) into (3.4) and using (3.1), we obtain

$$p(x) = \frac{\int_b^x G(y)\mathrm{d}y}{\int_b^\infty G(y)\mathrm{d}y} + \frac{\int_x^\infty G(y)\mathrm{d}y}{\int_b^\infty G(y)\mathrm{d}y} \times \frac{G(b)}{A(a,b,c)\int_b^\infty G(y)\mathrm{d}y + G(b)}.$$

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Thus, relation (3.3) follows from $q(x) = 1 - p(x)$.

Contents

A PDE

Consider the modified two-sided exit probability function

$$\phi(x, t; a, b) = \mathrm{P}^x \left\{ T_b \le T_a \wedge t \right\}, \qquad a < x < b, t \ge 0.$$

The following theorem establishes a PDE for this function:

A PDE

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The following theorem establishes a PDE for this function:

Theorem 4.1 Suppose $h(x, t)$ solves

$$h_t(x, t) = \mu(x) h_x(x, t) + \frac{1}{2}\sigma^2(x) h_{xx}(x, t), \qquad a < x < b, t > 0,$$

with the boundary conditions $h(b, t) = 1$ and $h(a, t) = 0$ for $t \ge 0$ while $h(x, 0) = 0$ for $a < x < b$. Then

$$h(x, t) = \phi(x, t; a, b), \qquad a \le x \le b, t \ge 0.$$

Existence and finiteness of A(a,b,c)

By the well-known regularity theory of PDE (see, e.g. Theorem 4.22 of Lieberman (1996)), we immediately have the following:

Corollary 4.1 It holds for every fixed $t > 0$ that $\phi_x(x, t; a, b)|_{x=b}$ is finite and continuous with respect to t. In particular,

$$A(a, b, c) = \lim_{\varepsilon \downarrow 0} \frac{\mathrm{P}^{b-\varepsilon}\{T_b > T_a \wedge c\}}{\varepsilon} = \lim_{\varepsilon \downarrow 0} \frac{1 - \phi(b - \varepsilon, c; a, b)}{\varepsilon} = \phi_x(x, c; a, b)|_{x=b}$$

exists and is finite.

Contents

Numerical PDE

Recall formula (3.3) for the default probability $q(x_0)$:

$$q(x_0) = \frac{A(a, b, c)}{A(a, b, c)} \int_b^\infty G(y)\mathrm{d}y + G(b) \int_{x_0}^\infty G(y)\mathrm{d}y. \tag{3.3}$$

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The only implicit part is the quantity $A(a, b, c)$.

Theorem 4.1 and Corollary 4.1 enable us to compute $A(a, b, c)$ numerically via a PDE. We use the Crank-Nicolson method to solve $A(a, b, c)$:

- It is a second-order implicit finite difference method, which is unconditionally convergent and stable.
- The local error is of order $O(\triangle x^2) + O(\triangle t^2)$, implying that the error for $A(a, b, c)$ is of order $O(\triangle x) + O(\triangle t^2)$.

Numerical example 1

First, we assume that the firm value follows a linear Brownian motion,

$$\mathrm{d}X_t = \mu \mathrm{d}t + \sigma \mathrm{d}B_t,$$

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Parameters: $\mu = 0.1$, $\sigma = 0.25$, $a = 0.1$, $b = 0.2$, and $c = 1$.

mesh	$A(a,b,c)$	$q(x_0)$	time (s)
$\triangle x = \triangle t = 0.005$	8.5534038	$1.3801420\mathrm{e}^{-3.2x_0}$	0.09655
$\triangle x = \triangle t = 0.001$	8.4987776	$1.3777311\mathrm{e}^{-3.2x_0}$	3.94097
$\triangle x = \triangle t = 0.0005$	8.4919795	$1.3774294\mathrm{e}^{-3.2x_0}$	32.5367
$\triangle x = \triangle t = 0.00025$	8.4885830	$1.3772786\mathrm{e}^{-3.2x_0}$	267.074

Next, we propose a reorganization plan during bankruptcy:

$$
\begin{aligned}
\mathrm{d}X_t &= \mu(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t, \\
\mu(x) &= \mu 1_{\{x>b\}} + \left(1 - \frac{b-x}{2(b-a)}\right)\mu 1_{\{a\le x\le b\}}, \\
\sigma(x) &= \sigma 1_{\{x>b\}} + \left(1 - \frac{b-x}{2(b-a)}\right)\sigma 1_{\{a\le x\le b\}}.
\end{aligned}
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$$\mathrm{d}X_t = \mu(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t,$$
$$\mu(x) = \mu 1_{\{x>b\}} + \left(1 - \frac{b-x}{2(b-a)}\right)\mu 1_{\{a\le x\le b\}},$$
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This reorganization plan concerns the priority of the debt holder over the shareholders during bankruptcy by reducing $\mu(\cdot)$ and $\sigma(\cdot)$. Meanwhile, the ratio $\mu(\cdot)/\sigma(\cdot)$ remains constant.

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mesh	$A(a,b,c)$	$q(x_0)$	time (s)
$\triangle x = \triangle t = 0.005$	8.2173101	$1.3649425e^{-3.2x_0}$	0.55676
$\triangle x = \triangle t = 0.001$	8.1639584	$1.3624470e^{-3.2x_0}$	4.42222
$\triangle x = \triangle t = 0.0005$	8.1574008	$1.3621387e^{-3.2x_0}$	34.5876
$\triangle x = \triangle t = 0.00025$	8.1541313	$1.3619848e^{-3.2x_0}$	267.169

Numerical example 2

Second, we assume that the firm value follows a geometric Brownian motion,

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}B_t,$$

and that the capital structure remains unchanged during bankruptcy.

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$$q(x_0) = \frac{A(a,b,c)}{A(a,b,c) + 2\mu/\sigma^2 - 1} \left(\frac{b}{x_0}\right)^{2\mu/\sigma^2 - 1}.$$

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Parameters: $\mu = 0.1$, $\sigma = 0.25$, $a = 0.1$, $b = 0.2$, and $c = 1$.

mesh	$A(a,b,c)$	$q(x_0)$	time (s)
$\triangle x = \triangle t = 0.005$	11.495846	$0.014815100 x_0^{-2.2}$	0.46036
$\triangle x = \triangle t = 0.001$	11.145638	$0.014590922 x_0^{-2.2}$	4.36286
$\triangle x = \triangle t = 0.0005$	11.101517	$0.014562175 x_0^{-2.2}$	31.4908
$\triangle x = \triangle t = 0.00025$	11.079429	$0.014547740 x_0^{-2.2}$	276.484

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dX_t &= \mu(X_t)dt + \sigma(X_t)dW_t, \\
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mesh	$A(a,b,c)$	$q(x_0)$	time (s)
$\triangle x = \triangle t = 0.005$	12.348142	$0.015332581 x_0^{-2.2}$	0.56785
$\triangle x = \triangle t = 0.001$	11.970123	$0.015107802 x_0^{-2.2}$	4.43208
$\triangle x = \triangle t = 0.0005$	11.922681	$0.015079068 x_0^{-2.2}$	32.4584
$\triangle x = \triangle t = 0.00025$	11.898945	$0.015064648 x_0^{-2.2}$	258.393

Potential future works

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Thank You Very Much!!!