ASSET-LIABILITY MANAGEMENT FOR

A GOING CONCERN

Melanie Maier

99 Pages August 2004

Assets and liabilities of insurance companies, risks, asset-liability management, duration, convexity, immunization, techniques and strategies, asset-liability management for a going concern.

APPROVED:

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This thesis describes asset-liability management, basic concepts, applied techniques and strategies. Moreover, it examines asset-liability management on a going concern basis. The first chapter is supposed to give an overview of the assets and liabilities of both life insurers and property/casualty insurers, their valuation principles and the kinds of risks they face. It should explain why asset-liability management gained importance. The second chapter presents the classical and multivariate immunization theory, and its underlying concepts of duration and convexity. Chapter three gives an overview of techniques and strategies of asset-liability management, classifies them into static and dynamic methods, and describes their benefits and weaknesses. The forth chapter examines asset-liability for a going concern. Strategies for selecting the duration of the invested assets in order to protect the shareholder value

of a company are developed, especially taking the impact of competition and future business into consideration.

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MELANIE MAIER

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Melanie Maier

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CHAPTER I

INTRODUCTION

1.1 What is Asset-Liability Management?

The basic ideas of asset-liability management can be traced back to Redington. He linked the concept of duration, developed by Macaulay (1938) and Hicks (1939) and independently reintroduced by Samuelson (1945) and Redington (1952), to insurers' assets and liabilities: Redington suggested an equal and parallel treatment in the valuation of assets and liabilities. His concepts of duration and immunization form the main tools of asset-liability management. Asset-liability management primarily intended to eliminate interest rate risk, which was a major concern in the 1970s, when rates increased sharply and became more volatile. Since several insurers had failed to manage this risk, insurance regulators introduced an obligatory annual analysis to certify their interest rate risk management. An Amendment to the Standard Valuation Law was adopted in 1990 by the National Association of Insurance Commissioners (Laster and Thorlacius, 2000) and required the analysis of the effects of various interest rate scenarios on the asset and liability cash flows. However, much more strategies and techniques where developed that go far beyond meeting this regulatory requirement and included other risks as well.

Therefore, current asset-liability management can be defined as the management of a company so that assets and liabilities are coordinated. It can be seen as an ongoing process of formulating, checking and revising strategies associated with assets and liabilities in order to attain a company's financial objectives, given the company's risk tolerances and other constraints (Baznik, Beach, Greenberg, Isakina and Young, 2003). Asset-liability management has to manage the interest rate risk without neglecting the asset default risk, the product pricing risk and other uncertainties.

1.2 Assets and Liabilities

Before the assets and liabilities of United States (U.S.) life insurance companies and U.S. property/casualty insurance companies are considered, a short overview of their basic lines of business is presented.

The main business of life insurance companies consists of life insurance and annuities, and can be described as follows, according to Black and Skipper (2000).

- *Term insurance*: is a policy that provides coverage for a set time period, usually greater than one year. If the insured dies within the policy term, a specified benefit is paid. If the insured survives the date the policy expires, no benefits are paid. The most popular form is level term insurance, where a fixed premium is offered.
- *Whole life insurance*: offers protection for the whole of the insured's life. The payment of the face amount is made no matter when death occurs. Annual premiums for traditional whole life policies remain constant over time. This means that in early years premiums exceed the actual cost of the insurance, and in later years they are lower. These excess amounts of the early years, together with investment earnings, build up the cash value of the policy. If the policy owner surrenders the policy, he receives this cash value (less any outstanding policy loans).
- *Universal life insurance*: was first introduced in 1979 and designed to offer greater flexibility and shift the investment reward and risk to the policyholder. They offer flexible premium payments and adjustable death benefits. After an initial minimum premium payment, the policy owner can decide what amounts at what times he wants to pay, as long as the cash value covers policy charges.

- *Variable life insurance*: is a type of whole life insurance whose value directly depends on the development of a set of assigned investments. It was first offered in 1976 in the U.S. and aimed to offset the adverse effects of inflation on life insurance policy values.

Annuities are contracts that guarantee a series of payments for a fixed period or over a person's lifetime in order to provide the annuitant with income in the future. An annuity has two phases: First, there is the accumulation period, in which the annuitant pays premiums and the savings grow. Then, the payout period follows, where the annuity provides a steady stream of income for a specified period of time (Alexander, 2003).

- *Fixed annuity*: has a fixed interest rate guarantee for the accumulation period. At the end of the accumulation phase the annuitant can decide between a lump sum, annuitization or reinvestment. Usually, the earnings of this type of annuity are tax-deferred.
- *Equity-indexed annuity*: offers a minimum guaranteed interest rate. Since this annuity is tied directly to some external index, e.g. the

Standard & Poor's 500 Index, it also includes the possibility of stock-market-like gains.

To cover the property and liability losses of businesses and individuals is the primary function of property/casualty insurance. The two largest single lines of business are private auto insurance and homeowners insurance. The Insurance Information Institute (2004) provides the percentage of premiums written by those two different lines of business: 41 percent and 11.6 percent, respectively.

Private auto insurance is designed mainly for non-business automobiles and pays for specific car-related financial losses during the term of policy. The main components of coverage are bodily injury, property damage, collision, comprehensive, medical payments, personal injury and uninsured or underinsured-motorist. The premium depends on the car wished to be insured, the driving record of the client, his/her gender, age and marital status. A policy is generally written for six months, but mostly renewed.

Homeowners policies offer protection for dwelling, personal possessions and personal liability. The premium for such a contract depends on the claim history of the insurer, the value of the home, deductibles and safety measures. The typical term of a homeowner's policy is three years, but an annual or continuous term is also common (Huebner, Black and Cline, 1982).

Life insurers generally offer long-term policies, whereas property/casualty insurers underwrite short-term policies. This is one reason for the difference in their asset portfolio structure.

1.2.1 Assets and Liabilities of Life Insurers

An overview of the main categories of assets life insurance companies hold in 2002 is given in figure 1. The chart is based on numbers provided by the Insurance Information Institute (2004).

The largest investment category of life insurance companies are *corporate bonds*. Publicly traded corporate bonds are characterized by periodical coupons over their lives and the return of their face value to the bondholder at maturity. Since private firms issue corporate bonds, it is of real importance to consider the default risk. Most of the corporate bonds contain options: the call option or the option to convert the bond. Convertible bonds give bondholders the option to convert a bond into a specified number of shares of common stock of the issuing firm. Thus, the bondholder can profit from a positive development of the stock.

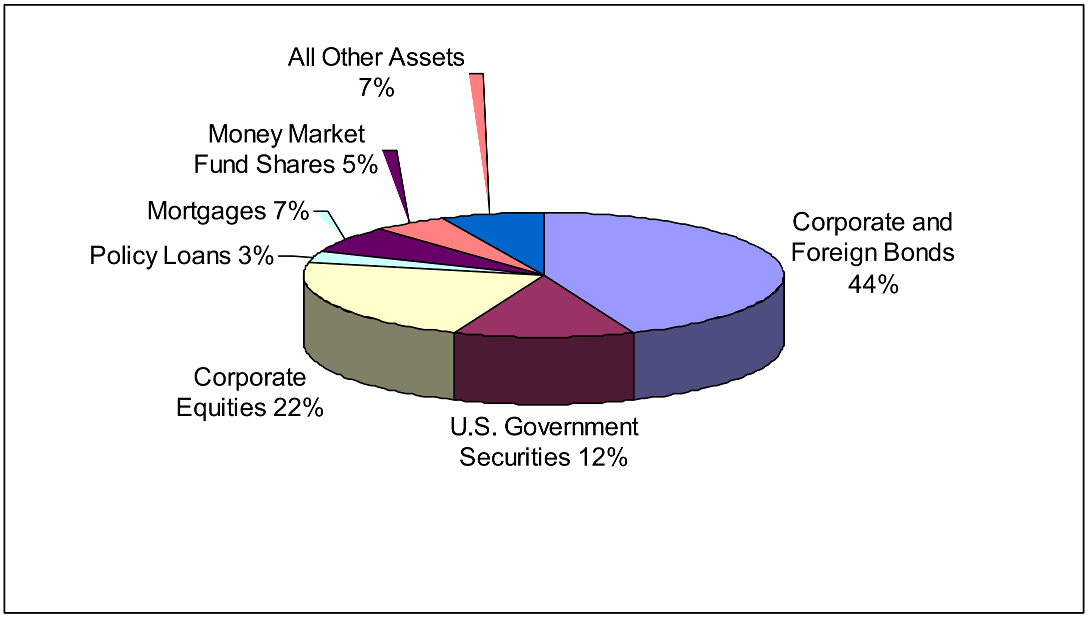

Figure 1: Life insurer assets (Insurance Information Institute, 2004)

Callable bonds give the issuer the option to repurchase the bond at a stipulated call price before the maturity date. If a bond is issued with high coupon rates, and interest rates later fall, the issuer may like to retire the high coupon debt and issue new bonds with lower coupon rates in order to reduce the interest payment. This is called refunding. Changing market rates influence both the probability that the bond may be called and the realization of the market value if a bond is sold before maturity. There are two ways to issue new bonds on the primary market (Bodie, Kane and Marcus, 1996): public offering and private placement. Public offerings are bonds offered to the general investing public that then can later be sold and purchased on the secondary market. A private placement is issued to at most a few institutional investors. Generally, the investor and the issuer directly negotiate the terms of the offering. This and the fact that commissions are avoided are advantageous for both sides, but it also contains risks. Many insurers obtain bonds through private placements. But since a secondary market for private placements does not exist, private placements are therefore generally held to maturity. However, to offset the fact that they are less liquid and marketable than public offerings, insurers expect an increased yield.

Investments in *foreign securities* have always been very small and amount to approximately 5 percent of total assets. The major part of both long-term and short-term non-U.S. corporate debt securities is invested in Canadian securities by U.S. life insurers.

Life insurers hold large amounts of *government securities*, i.e. Treasury securities and federal agency debts. *Treasury securities* are obligations of the U.S. government issued by the Treasury to meet government expenditures. Marketable Treasury securities can be Treasury Bills, Treasury Notes and Treasury Bonds. Treasury Bills are sold by the U.S. government in order to raise money. Any public investor can buy them at the discounted face value of the Bill, and receives the face value at the maturity date. They do not pay any coupons. The maturities of Treasury Bills are up to one year. Treasury Bills are considered the most marketable of all money market instruments. The U.S. government also borrows money in large parts by selling Treasury Notes and Treasury Bonds. Both provide semiannual coupon payments at a level that enables the government to sell them at or near par value. The maturities of Treasury Notes range from one year to 5 years, the maturities of Treasury Bonds vary from 10 to 30 years. Treasury securities are considered not to contain any credit risk and therefore their expected yield is lower than that of corporate bonds or other fixed-income securities.

Federal agency debts are securities issued by some government agencies to finance their activities. If Congress believes that a sector in the economy does not get sufficient credit through normal private sources, then such agencies are created. The biggest part of this kind of debt is issued in order to support farm credits and home mortgages. Government securities are very liquid, which is advantageous if the sale of assets is required due to cash flow changes.

Corporate equity, or common stock, holdings amount to 22 percent of the total assets of life insurance companies. The total number refers to both the general account and the separate account combined, but 90 percent of the stock held is in separate accounts. Black and Skipper

(2000) define the general account and the separate account as follows. Life insurers divide their assets between two accounts: the general account and the separate account. The general account is linked to obligations with guaranteed, fixed benefit payments, like life insurance policies. The separate account is directly associated with products that pass the investment performance and risk to the policyholder, such as variable life insurance and variable annuities. Therefore, the investment in common stock in connection with separate account is not restricted. Common stocks are ownership shares in a corporation and their cash flows are therefore much more variable and riskier than those of fixed-income securities. The market value of its shares and the non-guaranteed, periodic payment of dividends account for the cash flows of common stock. Common stock is also characterized by its residual claim and limited liability. Residual claim means in case of liquidation the stockholders are the last ones, after all other claimants, who are paid. Limited liability means that the most they can lose is the money they originally invested.

Life insurers invest less than 1 percent of their assets in preferred stock. Preferred stock has characteristics of both equity and debt. It typically promises to pay a fixed dividend each year, which must be satisfied before common stock dividends can be paid. In this sense preferred stock is similar to a perpetuity.

Policy loans comprise 3 percent of life insurer assets. Certain life insurance policies allow policy owners to borrow money against the cash value of their policy under conditions specified when the policy is written. The accumulated cash value of a policy is defined by the total amount of premiums paid, minus the cost of providing insurance protection for the period of time since inception of the policy, plus interest or other benefits accruing on previously paid premiums (Gardner and Mills, 1991).

Mortgages account for 7 percent of total assets of U.S. life insurance companies. Mortgages are loans that require periodic payments of principal and interest with real estate as collateral. A residential mortgage refers to a one- to four family dwelling, whereas commercial mortgages have commercial property, like an apartment building (for more than four families) or a store, as collateral. Commercial real estate mortgages generally are considered fixed-income securities and illiquid investments. Since commercial mortgages are directly negotiated between the insurer and borrower, and the liabilities of life insurers typically are of long duration, mortgages were considered to be an opportunity to match the cash flows. But because of augmenting liquidity requirements, the mortgage loans tend to become closer to 10-year maturities.

Policy reserves are by far the largest category of life insurance liabilities. The prospective definition of the reserve is the amount that, together with future premiums and interest earned, is needed to provide future benefits based on current assumptions of mortality, morbidity and interest (Black and Skipper, 2000). They are the difference between the present value of future benefits and the present value of expected future net premiums. The assumptions and the method of calculation vary with different accounting standards. The valuation of annuity reserves is based on the commissioners' annuity reserve valuation method (CARVM) defined in 1976. The present value of future guaranteed benefits at each duration has to be compared to the present value of future required premiums at that duration. The minimum reserve for the contract is the present value of the greatest excess observed in these comparisons at the valuation date.

A simple example of the commissioners' annuity reserve valuation method applied to a single-premium deferred annuity follows. It is analogous to the example presented by Tullis and Polkinghorn (1996), but all the values obtained here are based on my calculation. Consider the annuity as described below:

Single premium: 10,000

No front end load

Guaranteed Interest: 9% in years 1 through 5

4% thereafter

Surrender charge:

Policy year	Percent of fund
1	7%
2	6%
3	5%
4	4%
5	3%
6	2%
7	1%
8 and later	0%

Valuation interest rate: 8%

Death benefit equal to cash surrender value

First, the value of the fund accumulated at the guaranteed interest rate and the cash value are calculated at the end of each of the first 10 policy years, as shown in table 1.

The present values at the date of issue and at the first four policy anniversaries of each future cash value have to be calculated next, using the valuation interest rate. The results are shown in table 2.

Table 1: Fund and Cash Values

Policy year	Fund	Cash value
0	10,000	9,300
1	10,900	10,137
2	11,881	11,168
3	12,950	12,303
4	14,116	13,551
5	15,386	14,925
6	16,001	15,681
7	16,642	16,475
8	17,307	17,308
9	17,999	17,999
10	18,719	18,719

(Source: Author's Calculation.)

Table 2: CARVM Valuation

Future policy year	Cash value	Policy anniversary of valuation				
		0	1	2	3	4
0	9,300	9,300				
1	10,137	9,386	10,137			
2	11,168	9,575	10,341	11,168		
3	12,303	9,767	10,548	11,392	12,303	
4	13,551	9,960	10,757	11,618	12,547	13,551
5	14,925	*10,157*	*10,970*	*11,848*	*12,795*	*13,819*
6	15,681	9,882	10,672	11,526	12,448	13,444
7	16,475	9,613	10,382	11,212	12,109	13,078
8	17,308	9,351	10,099	10,907	11,780	12,722
9	17,999	9,004	9,724	10,502	11,343	12,250
10	18,719	8,670	9,364	10,113	10,922	11,796

(Source: Author's Calculation.)

For the first 4 years, the cash value, which creates the greatest present value for valuations on each of the first four policy years, is the cash value at the end of the 5th year. Hence, the CARVM reserve, for example at the third policy year, would be 12,795, the present value of the fifth policy year value.

Due to the nature of annuity reserving and the structure of annuity policies, as opposed to life insurance, the largest single category of reserves is for annuities. Pension fund reserves amount to 47 percent, life insurance reserves comprise 29 percent of the total liabilities of life insurance companies (Insurance Information Institute, 2004).

1.2.2 Assets and Liabilities of Property/Casualty Insurers

The investment policy of property/casualty insurance companies can be described as follows (Feldblum, 1989). Long tailed liability lines, such as general liability, products liability and medical malpractice, have slow loss payout patterns and have gained importance in the last decades. Since the investment risk on the assets that corresponds to these loss reserves can not be passed to the policyholder, the insurers try to match their investment and insurance portfolios. The ultimate liability of a property/casualty insurer is generally not expressed in nominal terms. It is determined at the settlement date, and hence depends on the inflation between the accident and the settlement date. This means the liabilities are inflation sensitive. Liabilities increase in case of rising inflation rates, and decrease in case of falling inflation rates. In practice, not all reserves are fully inflation sensitive. Often payments are determined shortly after or at the accident date, and therefore have short durations. But many reserves are inflation sensitive and therefore also similar to short duration assets with regard to the consequences of interest rate changes. Asset-liability matching or immunization, which will be explained in depth in chapter II, requires an asset portfolio of the same duration as the liabilities. Treasury Bills and commercial papers are assets of short duration with returns varying directly with inflation. But generally the yield curve is upward sloping, and therefore assets of longer duration, like corporate bonds, provide higher yields. Thus, the advantages of both immunization and the overall portfolio yield have to be weighted. Common stocks are also inflation sensitive, as are insurance liabilities, and change in the same direction, if the cash flows resulting from current and expected dividends and from price changes or expected dividend changes because of interest rate changes are considered. If inflation and interest rates increase unexpectedly, common stock prices decrease first, but increase later. But if common stocks are reported at their market values their book values fluctuate more than bonds. Therefore, long-term bonds are the primary choice of investment for property/casualty insurers. Most insurers buy long-term bonds, because higher yields can be expected. If long-term bonds are reported at amortized values, they show high and steady returns. However, if they are reported at their market values, long term bonds are risky assets with regard to interest changes. In case of rising interest rates, the market value of long-term bonds decreases. When interest rates fall, the market value of bonds increases.

Figure 2, based on numbers provided by the Insurance Information Institute (2004), shows the assets of property/casualty insurance companies in 2002.

It is noticeable that their assets, compared to those of life insurance companies, are dominated by municipal securities. Property/casualty insurers invest 21 percent in municipal securities, 21 percent in corporate and foreign bonds and 18 percent of their assets in U.S. government securities.

Municipal bonds are fixed-income securities issued by state or local governments. There are two types of municipal bonds: General obligation bonds are backed by the taxing power of the issuer. Revenue bonds are issued, e.g. by airports, hospitals or port authorities, for financing special projects. They are backed by the revenues of that project, and are therefore riskier. The main reason why property/casualty insurers choose this form of investment is that

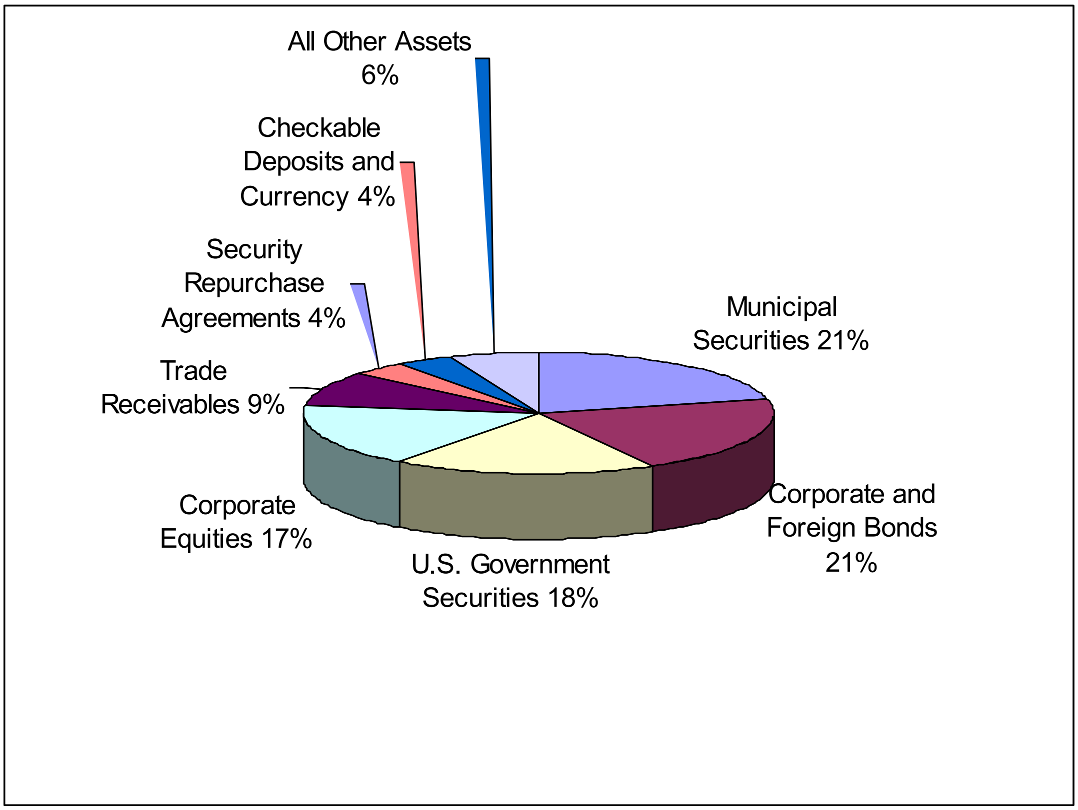

Figure 2: Property/casualty insurer assets (Insurance Information Institute, 2004)

interest income of municipal bonds is exempt from federal income taxes (Bodie, Kane and Marcus, 1996). The interest income also is exempt from state and local taxation in the state where the bond is issued. Capital gains taxes, however, have to be paid at maturity or in case they are sold at a value above the investor's purchase price. Because of this tax-exempt status, investors accept lower yields on these securities. Maturities range from short-term tax anticipation notes to long term municipal debt up to 30 years. But property/casualty insurers have to consider both sides of investing in municipal bonds. In case of good underwriting profits, the tax shelter that these bonds provide is advantageous. In case of underwriting losses, the lower yield on municipals hurts profitability.

17 percent of the assets are held in corporate equities. This is a suitable investment for property/casualty insurance companies because common stock is inflation sensitive, as are their liabilities. Theoretically, the real value of the firm's main assets should not change with inflation. If inflation and interest rates rise, the nominal value of the firm should consequently also rise, so that its inflation-adjusted value should not vary. In practice, the value of a company is determined by its revenue and costs. When inflation and interest rates increase, supply costs rise, but demand may or may not. If inflation is "demand-pull", i.e. a price increase caused by an excess of demand over supply (Webfinance Inc., 2004), demand increases. If inflation is "cost-push", i.e. persistently rising general price levels bought about by rising input costs (Webfinance Inc., 2004), demand may decrease. Furthermore, households tend to

save and not consume more if interest rates rise, what further reduces demand. Thus, the value of the firm and its common stock will decrease. But when interest rates rise, investors often prefer to invest in long-term bonds instead of common stocks to profit for a longer period of time from the high rates. Hence, when inflation and interest rates increase unexpectedly, common stock prices first decrease, but increase later, and are therefore inflation sensitive.

Note that property/casualty companies do not segment funds, as life insurers do. The investment returns of property/casualty insurers have to be enough for the company as a whole, not for a certain block of business (Feldblum, 1989).

Repurchase agreements are a form of short-term, generally overnight, borrowing: a government security dealer sells securities to an investor, in this case the insurance company, with an agreement to buy back those securities by a specified date at a set price. The increase in price is the interest gained (Bodie, Kane and Marcus, 1996).

A *trade receivable* is money owed to the insurance company, whether or not it is currently due, as a result of a trade.

Property/casualty insurers generally make more use of short-term investments than life insurers due to their liquidity needs. Trade receivables amount to 9 percent, checkable deposits and currency to 4 percent, and security repurchase agreements to 4 percent.

Mortgages are not very attractive to property/casualty insurers because of the completely taxable income from mortgage loans and because of its illiquidity (Gardner and Mills, 1991).

If surrender, withdrawals or policy loans entail cash outflows, a company risks losses if assets have to be sold at depressed prices at a time when interest rates have increased. This risk is called intermediation risk and is faced by life insurers (Atkinson and Dallas, 2000). But since property/casualty insurers do not loan to policyholders and offer mostly short-term policies, they do not face disintermediation problems.

The liabilities of a property/casualty insurance company primarily consist of *reserves*. Reserves can mainly be separated into three parts: the loss reserves, the unearned premium reserves and the loss adjustment expense reserves.

The loss reserves are the largest portion of the liabilities. They have been incurred because of claims that have been made but not yet paid. Because estimated losses of property/casualty insurers are not based on mortality and morbidity, they often rely on past experience, with adjustments to reflect increased costs due to inflation or other factors. If an insurer's loss reserves are overestimated, the insurer's profits will decrease, the income taxes may be reduced, and premium rates may be unnecessarily increased. If reserves are too low, underwriting profits will be overstated, income taxes will increase, and premium rates may be cut unwisely. In both cases, the insurance company will have lower than optimal profitabililty.

Unearned premium reserves are obligations for the unexpired terms of new and renewed policies to policyholders who have paid premiums in advance.

The loss adjustment expense reserves contain the fees or salaries paid to claims adjusters, fees paid to investigators, their expenses, e.g. for traveling, legal fees, and other costs associated with settling claims (Cohen and Mooney, 1991).

1.3 Valuation

Valuation informs about the financial condition and current operating results of an insurance company. It measures and compares the insurer's assets and liabilities due to valuation standards, such as interest rates, mortality, morbidity, persistency and expense assumptions. These standards were created by the National Association of the Insurance Commissioners (NAIC) and adopted by each state's legislature (Atkinson and Dallas, 2000).

Two different valuation principles are required for every insurance company: statutory accounting and tax-basis accounting. If the insurance company is publicly traded, GAAP accounting is also obligatory. But since these principles are not sufficient for management decisions, most insurance companies also use managerial accounting.

The state insurance law requires the use of *statutory accounting principles* (SAP) in order to analyze a company's ability to meet its obligations to policy owners. The insurer must annually present financial statements that both proof the economic solvency and the statutory definition of solvency with regard to investments, reserves, and minimum capital and surplus defined by law. The insurer must prove that its assets, future premiums and conservatively estimated interest income will be enough to meet all promises to policy owners. Profits on existing or new lines of business are not considered.

The usefulness of statutory reports is restricted for two reasons. First, results are reported at a specific point in time under some static assumptions that neglect possible changing economic conditions in the future. However, a NAIC model law requires the proof of an adequate reserve for various economic scenarios. Because traditionally most insurers held bonds to maturity and did not even intend to sell them earlier, bond values are recorded at amortized values rather than market values. During periods of relatively high interest rates, the asset values are overestimated and artificially stabilized by these amortized values. Thus, it is possible that the insurer's statement shows solvency whereas the insurer actually is not able to meet his future obligations, and vice versa.

Second, statutory accounting is not adequate for investors and creditors, because it is balance sheet oriented and treats the insurer as if he were about to be liquidated. Additionally, the use of conservative assumptions used for the valuation of the insurer's liabilities generally neglects the possible profit that can be generated by in-force policies in the long run.

The use of *generally accepted accounting principles* (GAAP) is required by the Securities and Exchange Commission for publicly traded insurance companies and is a condition for listing on major stock exchanges. The main purpose of GAAP accounting is to report the financial results for an insurance company such that it is comparable to those of other companies and of other reporting periods. This comparability is particularly important to investors in order to judge alternative investment possibilities and to predict future financial results.

Although GAAP accounting presents assets, liabilities and cash flows, it does not recognize possible future profits generated by existing and future policies. Therefore it is not appropriate to evaluate the long-term financial impact of current management action, and hence it is often considered not to be adequate as a financial management tool.

More detailed, GAAP includes the lock-in principle, which does not allow restating the assumptions of interest, expense, and mortality for traditional policies in force. Only interest-sensitive products and participating business can periodically be reevaluated. Furthermore, unrealized capital gains and losses are not reflected in the GAAP income statement. Finally, GAAP as well as SAP, do not recognize the future impact of current events, since surrenders may cause increased current-period earnings, and do not reflect the loss of future profits on lapsed policies.

The main difference in SAP and GAAP accounting concerning the valuation of assets is the distinction between admitted and non-admitted assets. Assets approved by state regulatory authorities and accepted by the NAIC Annual Statement are called admitted assets. These are rather liquid assets, such as bonds, stocks, mortgages and real estate. Non-admitted assets are usually either illiquid, e.g. furniture and equipment, or not allowed by statute, e.g. certain kinds of securities above the statutory limit. Only admitted assets may be listed on the statutory balance sheet. However, on the GAAP balance sheet, all, i.e. both admitted and non-admitted assets, may be reported. Roughly, SAP reports only admitted assets and those at amortized values, while GAAP recognizes all assets at market values.

As far as the valuation of liabilities is concerned, the property/casualty situation is much different from the life insurance case: Property/casualty insurers report both statutory and GAAP reserves at undiscounted values. Whereas the valuation of statutory reserves for life insurers is defined by law and the valuation of GAAP reserves is based on the company's and industry's experience (Black and Skipper, 2000).

Generally, GAAP recognizes liabilities later or at a lower value and recognizes assets earlier or at a higher value. GAAP accounting treats the business more as a going concern, whereas SAP accounting rather treats it as if it were about to be liquidated.

The third valuation principle, the *tax-basis accounting*, is required by the Internal Revenue Service. It necessitates the calculation of the reserve liability in order to determine the taxable income in accordance with the Internal Revenue Code and its interpretations (Atkinson and Dallas, 2000).

Since all three regulatory accounting principles do not properly represent the performance of lines of business adequately for management decisions, most insurers have adopted *management-basis accounting* (Dicke, 1996). These three principles do not completely recognize the following issues either:

- Changes in present value of cash flows according to changes in interest rates
- Embedded options in assets and liabilities
- Lost future profits due to surrenders or additional profits due to new sales in the long run
- Expected future profits on future business
- Actual market values of assets held in the investment portfolio

These deficits resulted in the development of economic value analyses, which is now often used in insurance firm management. The most essential ones are the value-added and return-on-equity methods.

1.4 Risks

Most insurance companies identify and manage the risks they face according to the classification developed by the Society of Actuaries (Ostaszewski, 2002):

- *Asset default (C-1) risk*: is the risk of a decrease in the insurer's investment asset value. It can either be caused by the default of borrowers in payment of interest or principal, or by declining market values of assets (if not based on interest rate movements).
- *Insurance pricing (C-2) risk*: is the risk of losses from increasing claims and pricing deficiencies. The latter may occur if the actual mortality, morbidity, lapse or expense experience is higher than the expected, i.e. if future results do not match the assumptions implicit in product pricing.
- *Interest rate (C-3) risk*: is the risk that changes in interest rates affect assets and/or liabilities in a negative way. This will be discussed more detailed in the next chapter.
- *Business (C-4) risk*: represents miscellaneous risks that are not mentioned in C-1 through C-3, e.g. market risk from

expansion into new lines of business, changes in taxation or regulation, insurance fraud, mismanagement and law suits.

The primary concern of early insurance companies was the C-2 risk, since they were not able to predict their benefit disbursements. But with the development of the principles of actuarial sciences the importance of the C-2 risk gradually decreased. In the 1950s and 1960s almost all claim-related cash flows were known. And also other cash flows, such as lapses, surrenders, new business or investment returns, were stable and therefore predictable (Ostaszewski, 2002). Since interest rates stayed in a narrow range from the Great Depression until the mid-1960s – the yield on long-term U.S. government securities e.g. stayed between 2 and 4.5 percent – they were not problematic either. This stable environment ended in the 1970s when inflation accelerated and became unpredictable, and the volatility of financial markets, especially interest rates, increased. In the early 1980s, the short-term interest rates were at unprecedented height. High rates combined with greater volatility encouraged more and more individuals to borrow against their life policies and reinvest the proceeds at higher rates elsewhere. Policyholders changed their behavior with regard to the options embedded in their contracts. They started to exercise the options more frequently and opportunistically.

Some examples of options embedded in insurance policies are (Laster and Thorlacius, 2000):

- Settlement option: allows the beneficiary the choice of the form of benefit payment, e.g. lump sum or annuity.
- Policy loan option: offers the policyholder the right to borrow, at specified terms, against the accumulated asset value of an insurance policy.
- Over-depositing option: enables the policyholder to pay higher premiums than required, which will be credited at a pre-specified interest rate.
- Surrender privilege: permits the policyholder to stop paying premiums and to halt the insurance contract earlier.
- Renewal privilege: allows policyholders to continue an insurance contract or halt the agreement at the end of the policy period.

When interest rates were stable, these options were not very valuable. Hence, many insurers did not adjust their assets and liabilities and were therefore not prepared for the risks these options posed when interest rates became volatile.

In the mid 1980s, the level of nominal interest rates declined dramatically and as a consequence, many insurers' portfolios were refinanced and prepaid. At the end of the 1980s, insurers that followed higher yields often took too much credit risk in their investment portfolio.

Another response to high interest rates and increased competition at the end of the 1970s was the rise of new, interest-sensitive policies. Annuities were historically not an important part of life insurance industry and just used as a source of income after retirement. But annuities gained more importance than traditional insurance, since the main purpose of purchasing life insurance was no longer protection but investing. Single, flexible premium-deferred annuities, variable annuities, universal life and other interest-sensitive products exposed insurers to new sorts of risk that some have not been able to manage. As a consequence, the National Association of Insurance Commissioners adopted an Amendment to the Standard Valuation Law in 1990. It required a basic asset-liability analysis, known as cash flow testing, to verify that the insurer holds enough reserves. Thereby, the effects of various different interest scenarios on the assets and liabilities are tested.

Thus, these changes simultaneously caused the relative decline of the C-2 risk, except for the catastrophe risk of property/casualty insurance companies, and the increase in the significance of the C-1 and C-3 risk.

CHAPTER II

INTEREST RATE RISK

The term C-3 or interest rate risk denotes the risk of losses because of changes in interest rates – changes in either the level of interest rates or the shape of the yield curve.

In order to understand what this means, consider a block of insurance business and its associated assets. The asset cash flow in any future time period consists of the investment income and capital maturities (principal repayments) expected to occur in that time period. The liability cash flow in any future period consists of the policy claims, policy surrenders and expenses minus the premium income expected to occur in that time period. Therefore the net cash flow is the difference between the asset cash flow and the liability cash flow. If the net cash flow is positive, the asset cash flow exceeds the liability cash flow, which generates excess cash for (re)investment. If interest rates are below the initial rates when the net cash flows are positive, the cash flows may have to be reinvested at lower rates and thus losses may occur. This is related to as *reinvestment risk*. On the other side, negative net cash flows denote shortages of cash needed to meet liability obligations. In this case assets have to be liquidated or borrowed (within or without the company). If interest rates are above the initial level when the net cash flows are negative, losses can occur due to the fact that bonds and other fixed-income securities whose values have fallen must be liquidated. This is called *disinvestment risk* or *price risk*. The various interest rate options embedded in the assets and liabilities further aggravate the C-3 problem; this means that both the asset and the liability cash flows are functions of interest rates. When interest rates rise, more policyholders are expected to surrender their policies (to obtain higher returns by reinvesting the cash values elsewhere) or make use of their policy loan options. On the other side, when interest rates decline, bonds are more likely to be called, and bonds can be prepaid earlier than expected (Shiu, 1993).

Important measures of interest rate risk are duration and convexity. These two concepts are essential tools for asset-liability management.

2.1 Duration

Both duration and convexity are based on the assumption that only one interest rate is used, which means we assume a flat yield curve. Hence we examine the sensitivity to small parallel shifts in the yield curve, not bends or twists in rates.

Let P denote the price of a security, portfolio or liability, and let i denote the interest rate. Then, the *duration* of the security is defined as (Baznik, Beach, Greenberg, Isakina and Young, 2003):

$$D \qquad \frac{1}{P}\frac{d}{di}P \qquad \frac{d}{di}\ln(P).$$

Thus, the duration is the negative of the percentage change in price P per unit change in interest rate. In case of assets or liabilities with deterministic cash flows that do not contain embedded options this expression is referred to as *modified duration*. In case of securities with cash flows dependent on interest rates, i.e. with embedded option, it is called *effective duration* and can be approximated by

$$D_E \approx \frac{P(i-\Delta i) - P(i+\Delta i)}{2(\Delta i)P(i)},$$

where P(i) is the price of the security as a function of the interest rate i and Δi is the interest rate change (Gajek, Ostaszewski and Zwiesler, 2004).

A positive duration means that when interest rates increase, the price of the instrument decreases, which is the case for most fixed-coupon, fixed-income instruments and for liabilities with reasonably well-defined cash flows. Negative duration means that the price of the instrument will increase when the interest rate increases.

If the cash flows CF_t at time t of the security are certain, then the present value of the security, its price, is given by .

The derivative of the price with respect to the interest rate i is

.

This expression is called *modified dollar duration* and represents the absolute sensitivity of a position in dollars, instead of a percentage change in price, to a 1 percent change in rates (Baznik, Beach, Greenberg, Isakina and Young, 2003).

If we assume that the cash flows of a security are fixed and certain and that the security has no embedded options, we can introduce one of the first concepts of asset-liability management: the *Macaulay duration* (first appeared in Macaulay's work (1938), independently reintroduced by Samuelson (1945) and Redington (1952), respectively) where CF_t are the certain cash flows, the price P is the present value of the discounted cash flows , and i is the interest rate used to discount them (Gajek, Ostaszewski and Zwiesler, 2004).

The Macaulay duration can be expressed in terms of the interest rate i (as shown above) or in terms of the force of interest δ, since $1+i=e^{\delta}$ and therefore $\frac{d\delta}{di}=1+i$:

$$D_M=-\frac{d}{d\delta}\ln(P)=-\frac{1}{P}\frac{d}{d\delta}P=\frac{\sum_{t\geq 0} t\cdot e^{-\delta\cdot t}CF_t}{\sum_{t\geq 0} e^{-\delta\cdot t}CF_t}.$$

Note that $P=\sum_{t\geq 0}\frac{CF_t}{(1+i)^t}=\sum_{t\geq 0}e^{-\delta\cdot t}CF_t$ is still the present value of the security.

The Macaulay duration can be interpreted as the weighted average time-to-maturity of the security's cash flows, where the weights are the present values of each cash flow. In the special case of a non-callable, default-free, zero-coupon bond, the Macaulay duration is always the time to maturity.

There are some more characteristics of the Macaulay duration that are worth mentioning. First, if the maturity increases, the Macaulay duration also increases. But because of the use of the present value in its calculation the Macaulay duration increases more slowly than does maturity. Second, if the interest rate increases the duration decreases. This makes sense since discounting at higher interest rates has a greater effect on later cash flows and their relative importance declines. Third, higher coupon or interest payments yield to a lower Macaulay duration. This is understandable; if the payments at the beginning are larger, the cash flows are received earlier and the present value weights of those cash flows are higher.

2.2 Convexity

Convexity (Baznik, Beach, Greenberg, Isakina and Young, 2003) is a second order term and measures the change in price from the duration estimate for a small change in interest rates. If an instrument has positive duration and no embedded options, positive convexity means that the duration gets longer when interest rates fall, and the duration shortens when interest rates rise. This is the case for fixed cash flow bonds. Securities with embedded options may have regions with negative or reduced positive convexity. For example, home mortgages may have negative convexity as rates lower and increase the likelihood of prepayments, which results in lower duration as rates fall. Convexity may turn positive from lower likelihood of prepayment or extension, and result in greater duration as rates rise.

The *Macaulay convexity* of a security with certain cash flows CF_t at time t and price $P = \sum_{t \geq 0} e^{-\delta \cdot t} CF_t$ is the convexity measure with respect to the force of interest δ (Gajek, Ostaszewski and Zwiesler, 2004):

$$C_M = \frac{1}{P}\frac{d^2}{d\delta^2}P = \frac{\sum_{t \geq 0} t^2 \cdot e^{-\delta \cdot t} \cdot CF_t}{\sum_{t \geq 0} e^{-\delta \cdot t} \cdot CF_t}.$$

It indicates the sensitivity of the duration measure with respect to changes in the force of interest.

Because $P \cdot D_M = -\frac{d}{d\delta}P$, where D_M is the Macaulay duration, we have

$$C_M = -\frac{1}{P}\frac{d}{d\delta}(P \cdot D_M) = -\frac{1}{P}\left[P\frac{d}{d\delta}D_M + D_M\frac{d}{d\delta}P\right] = -\frac{dD_M}{d\delta} - \frac{1}{P}\left[-\frac{1}{P}\frac{d}{d\delta}P\right]\frac{d}{d\delta}P$$

$$= -\frac{dD_M}{d\delta} + \frac{1}{P}\frac{d}{d\delta}P\frac{1}{P}\frac{d}{d\delta}P = -\frac{dD_M}{d\delta} + D_M^2.$$

The quantity $M_M^2 = -\frac{dD_M}{d\delta} = C_M - D_M^2$ will be termed *Macaulay M-squared.* The Macaulay M-squared can be seen as a measure of dispersion of the cash flows of the security. The Macaulay convexity is the sum of the Macaulay M-squared and the square of the Macaulay duration. This means that the more dispersed the cash flows are, or the longer the duration cash flows are, the greater the convexity (Ostaszewski, 2002). For a zero-coupon bond maturing in T years, the Macaulay convexity is T^2. Since the Macaulay duration is T, the M_M^2 of this zero-coupon bond is $M_M^2 = T^2 - T^2 = 0$.

For assets or liabilities with embedded options the *effective convexity* can be approximated by the following expression:

$$C_E \approx \frac{P(i - \Delta i) - 2P(i) + P(i + \Delta i)}{(\Delta i)^2 P(i)},$$

where P(i) is the price of the security as a function of the interest rate i, and Δi is the change in the interest rate.

2.3 Immunization

The British actuary Frank M. Redington coined the term "immunization" in his 1952 paper "Review of the Principles of Life-Office Valuations" (Redingtion, 1952). His paper intended to be about valuation rather than strategies for matching assets and liabilities. In this paper, he proposed the use of a similar basis for the valuation of both assets and liabilities. He suggests the equation of the mean term of assets to that of the liabilities, while a greater spread for the cash flows of the assets is required in order to immunize the surplus value of a block of business against interest rate changes. Redington's idea of mean term had already occurred in the work of Frederick R. Macaulay (1938), and his term "duration" is the one generally used today (Panjer, 1998). Macaulay's duration is the main concept used in Redington's theory of immunization.

Consider a block of long-term insurance or annuity policies and its related assets at time $t = 0$. Let A_t be the asset cash flow expected to occur at time t: interest income, dividends, rent, capital maturities, repayments and prepayments. Let L_t be the liability cash flow expected to occur at time t, i.e. policy claims, policy surrenders, policy loan payments, policyholder dividends, expenses, and taxes, less premium income, policy loan repayments, and policy loan interest. Let the assets be more dispersed than the liabilities, i.e. choose assets with greater convexity than the liabilities. Redington argues that the same interest rate i should be applied to discount both the asset and liability cash flows to figure out their values, since every insurance liability cash flow can be considered the negative of an asset cash flow. Let i denote the given interest rate. Then the present values of the assets and liabilities are the sums

$$\sum_{t \geq 0} \frac{A_t}{(1+i)^t} \quad \text{and} \quad \sum_{t \geq 0} \frac{L_t}{(1+i)^t}.$$

Let $S(i)$ be the surplus of this block of business evaluated at the interest rate i. The surplus or net worth is the difference between asset and liability values, and therefore is

$$S(i) = \sum_{t \geq 0} \frac{A_t}{(1+i)^t} - \sum_{t \geq 0} \frac{L_t}{(1+i)^t} .$$

Redington's (1952) immunization theory can directly be attributed to the notion of an equal and parallel treatment in the valuation of assets and liabilities. Due to the definition of a derivative,

$$S(i + \Delta i) \approx S(i) + S'(i)\Delta i$$

is an approximation formula for small interest rate changes Δi. As a consequence of structuring assets and liabilities such that

$$S'(i) \quad 0,$$

we would obtain

$$S(i \quad\quad i) \quad S(i).$$

This means the surplus roughly remains unaffected or is immunized with respect to small interest rate changes of size $\quad i$.

If the cash flows do not depend on interest rates, then the condition that the first derivative of the surplus is equal to zero is equivalent to

$$\frac{\quad}{\quad} \qquad \frac{\quad}{\quad} .$$

This tells us that the first moment of the asset and liability cash flow streams are equal, which forms the fundamental of Redington's immunization strategy. If the present value of the assets additionally equals the present value of the liabilities, this means the Macaulay durations of the assets and liabilities are matched; the changes in asset values will be exactly offset by the changes in liability values.

The deficiency of this model is that it allows arbitrage opportunities. To see why, we use the three-term Taylor approximation formula (Panjer, 1998)

$$S(i \quad i) \quad S(i) \quad S'(i)\ i \quad \frac{1}{2}S''(i)(\ i)^2.$$

If both the conditions

$$S'(i) \quad 0 \qquad \text{and} \qquad S''(i) \quad 0$$

are satisfied, then, for small changes in the interest rate, this results in

$$S(i \quad i) \quad S(i).$$

Thus, changing interest rates always imply an automatic increase in surplus. This automatic "free lunch" in the model results from the use of the same interest rate applied to discount all cash flows. The model always assumes a flat yield curve, i.e. it does not distinguish between short- and long-term interest rates (Panjer, 1998).

Immunization also assumes the knowledge of all future asset and liability cash flows. However, in reality the future cash flows, especially of an insurance company, are hard to predict with regard to both their values and timing.

Moreover, Macaulay duration explicitly assumes deterministic cash flows. Thus it can not meaningful be used in cases where cash flows may depend on interest rates. But this is true for assets and liabilities that contain embedded options; policy owners may surrender their policies when interest rates rise, bonds may be called and mortgages repaid when interest rates fall and so on, as discussed earlier. In this case, the concept of *option-adjusted duration* and *option-adjusted convexity* (also called *effective duration* and *effective convexity*) should be applied (Ostaszewski, 2002).

Another difficulty with this approach is that duration varies with interest rate movements and over time. Even if the duration of the assets and liabilities has initially been matched, their durations might differ once the interest rate shifts. As a consequence, immunization requires continuous rebalancing of the assets and liabilities. Since convexity measures how rapidly duration changes in response to a change in interest rates, matching both durations and convexities is also advantageous. This way, an insurer hedges its interest rate risk to a second degree of precision (Laster and Thorlacius, 2000).

A further aspect is the question why a company should pursue a strategy where it minimizes its surplus. If both

$$S'(i) \quad 0 \qquad \text{and} \qquad S''(i) \quad 0$$

hold true, this means that the surplus function has a local minimum at the current level of interest, and every small change in the interest rate adds surplus. As it turns out immunization in practice pursues the point of maximum wealth and rather maximizes interest rate risk (Ostaszewski, 2002).

The strength and weakness of the traditional immunization strategy resulted in the development of all the modern techniques of asset-liability management.

2.4 Yield Curve and Multivariate Immunization

In the preceding chapters, we used only one interest rate to discount the cash flows of all maturities. However, in reality the interest rates for discounting cash flows for different maturities are not the same. The yield curve, or term structure of interest rates is the pattern of interest rates for discounting cash flows of different maturities. The *yield curve* generally represents the functional relationship between the time to maturity and the corresponding interest rate, whereas *term structure of interest rates* usually describes the fact of interest rates varying for different maturities. If longer-term bonds offer higher yields, which is generally the case, the yield curve is said to be *upward sloping*. If interest rates for various maturities do not differ, we call this a *flat* yield curve. And if shorter-term maturity interest rates are higher than longer-term rates, which occurs rarely, the pattern is called *inverted* yield curve. In practice, estimates of the yield curve are based on Treasury Bills, Treasury Notes and Treasury Bonds, since they are considered to be risk-free. Actually, there are three ways to define the yield curve. The first one is called *bond yield curve* and assigns to each maturity the coupon rate of a (generally newly issued) bond of that maturity trading at par. The second one is called *spot curve*. It assigns to each maturity the interest rate on a zero-coupon bond of this maturity. This interest rate is called *spot rate*. The third definition of the yield curve applies short-term interest rates in future periods of time implied by current bond spot rates. A *short-term interest rate* or *short rate* represents an interest rate that is used for a short period of time, i.e. for an instantaneous rate over the next infinitesimal period of time or a period of time up to one year. Applying the one-year rate as the short rate, we can derive forward rates and the relationship between these two:

$$(1+s_n)^n = (1+f_1)(1+f_2)...(1+f_n),$$

where s_n is the spot rate for maturity n and f_i is the forward rate from time $i-1$ until time i, $i = 1,...,n$.

We also have

$$1+f_n = \frac{(1+s_n)^n}{(1+s_{n-1})^{n-1}}.$$

The yield curve can also be defined for continuously compounded interest rates, i.e. for the force of interest. Here, the force of interest $\delta = \delta(t) = \delta_t$ is a function of time. To explain the difference between the spot force of interest δ_t for time t and the forward force of interest φ_t at time t, their mathematical relationship is presented. The accumulated value at time t of a monetary unit invested at time 0 is

$$(e^{\delta_t})^t = e^{\int_0^t \varphi_s ds}.$$

Thus we have $\delta_t = \frac{1}{t}\int_0^t \varphi_s ds$, which means the spot rate for time t is the mean value of the forward rates between 0 and t (Gajek, Ostaszewski, Zwiesler, 2004).

To remove one weakness of the traditional immunization theory, Ho (1990) and Reitano (1991a, 1991b) generalize duration and convexity to the multivariate case. Instead of using one single interest rate parameter i, a yield curve vector $\vec{i} = (i_1,...,i_n)$ is used, where the coordinates of this vector refer to certain “key” rates, e.g. based on observed market yields at various maturities. Then, the price $P(i_1,...,i_n)$ of a security is a function of these key rates. We implicitly assume that they are independent, i.e. each of them has its derivative with respect to the other equal to zero. But this is certainly not the case for various maturity interest rates.

The negative partial logarithmic derivatives of $P(i_1,...,i_n)$ are called *partial durations* (Reitano, 1991a, 1991b), or *key-rate durations* (Ho, 1990). The *total duration vector* then is

$$-\frac{P'(i_1,...,i_n)}{P(i_1,...,i_n)} = -\frac{1}{P(i_1,...,i_n)}\left(\frac{\partial P}{\partial i_1},...,\frac{\partial P}{\partial i_n}\right).$$

The second derivative matrix defines the *total convexity*:

$$-\frac{P''(i_1,...,i_n)}{P(i_1,...,i_n)} = -\frac{1}{P(i_1,...,i_n)}\left[\frac{\partial^2 P}{\partial i_k \partial i_l}\right]_{1\le k,l\le n}.$$

The surplus S of a company can be expressed in terms of the key interest rates chosen: $S = S(i_1,..,i_n) = S(\vec{i})$. Analogous to the one-dimensional case, the multivariate immunization can be applied using multivariate calculus. To protect the surplus level from changes in interest rates, the first derivative has to be set equal to zero

$$S'(\vec{i}) = \left(\frac{\partial S}{\partial i_1}, \ldots, \frac{\partial S}{\partial i_n} \right) = \vec{0},$$

and the second derivate matrix has to be made positive definite (Gajek, Ostaszewski, Zwiesler, 2004).

CHAPTER III

STRATEGIES AND TECHNIQUES FOR ASSET-LIABILITY MANAGEMENT

An overview of the strategies and techniques for asset-liability management is provided by Van der Meer and Smink (1993). They present both simple methods and methods applying more difficult theories from finance or actuarial science. Further, they explain the basic benefits and weaknesses.

The analyzed strategies and techniques can be classified in two groups: static and dynamic. Since strategies necessitate rules that specify how to make decisions they are considered dynamic, whereas techniques are considered static.

3.1 Static Techniques

The static techniques are ordered at increasing level of sophistication. Most of the methods are commonly applied in banking and insurance. The reason therefore might be the fact that they are relatively easy to understand and implement. These methods concentrate on a complete match between assets and liabilities, especially in the case of cash flow matching. The usefulness of these approaches is limited as they do not consider the possibility of a consistent trade-off between risk and return. Risk and return are not explicitly measured by these techniques. Moreover, those methods are based on the assumption of complete predictability of cash flows, which makes them not very useful for insurance companies, especially not for property/casualty insurers whose liability reserves are inflation sensitive (Ostaszewski, 2002).

- *Cash flow payment calendars*:

  Cash flow payment calendars provide an overview of all cash inflows and cash outflows of a company. It is a tool for identifying main imbalances between cash flows that result from assets and liabilities.

- *Gap analysis*:

  Gap analysis is routed in bank asset-liability management. The gap is defined as the difference between the values of the fixed and variable rate assets and liabilities on the balance sheet. Maintaining a gap as close to zero as possible is a risk-minimizing stratgey, a non-zero gap indicates interest rate risk exposure. For example, when the variable rate assets exceed the liabilities, then decreasing rates will result in a loss in net operating income. More helpful than pure gap analysis is a refined version that accounts for maturity differences between assets and liabilities, i.e. the company's asset and liability categories are classified according to when they will be reprised and when they will be placed in groupings called time buckets.

- *Segmentation*:

  Segmentation is a method used by insurance companies. For this method, liabilities are partitioned due to differences that result from product characteristics. Then, a separate asset portfolio is assigned to each segment, which has to reflect the particular structure of the liabilities, e.g. concerning the cash flow pattern.

- *Cash flow matching*:

  Cash flow matching is a technique that intends to minimize the imbalances between all asset and liability cash flows, generally using the method of linear programming. The scheduled negative cash flows generated by the liabilities are projected and an asset portfolio that generates the same cash flows is selected. The asset portfolio has to match the liabilities with certainty, within a very small acceptable time span, and with minimal cost (Ostaszewski, 2002).

Cash flow matching may cause several practical problems. First, it is not always possible to match the liability cash flows completely, e.g. in case the liability cash flows have very long maturities and corresponding assets are not available in the market. Second, a complete cash flow match may be too restrictive for managing the portfolio since it does not consider the higher return, which results from undertaking a higher degree of risk. But this is especially important for highly priced liabilities whose associated costs have to be regained by the asset portfolio. Finally, this technique supposes a complete and exact knowledge of the timing and value of all cash flows, whereas this is often not possible. For example if the cash flows depend on interest rates or other stochastic factors, cash flow matching is not achievable for all possible future scenarios.

Multiscenario analysis is not a purely static technique (Van der Meer and Smink, 1993). Since it forms a link between the static techniques presented and the dynamic strategies, it is mentioned here. The projected development of the cash flows of the liability and the asset portfolios is the focus of multiscenario analysis. These projections are made under varying future scenarios with regard to interest rates, inflation and other variables that are in part responsible for dependencies between assets and liabilities. The analysis reveals under which scenarios cash flows are not matched and what the consequences are for the company. Multiscenario analysis gives a deeper insight into the different sorts of risks the company faces, but the user tends to be biased towards scenarios which seem to be more likely, while other scenarios may be more distressing. This may also be the case if scenarios are randomly generated.

3.2 Dynamic Strategies

The second group of asset-liability management strategies, the dynamic strategies, can be classified in value driven and return driven methods. The methods of both categories explain the relationship between assets and liabilities, and other factors that have an influence on their balance.

3.2.1 Value Driven Dynamic Strategies

The value driven dynamic strategies are all based on Redington's (1952) classical notion to protect the company's surplus from interest rate risk. With regard to the identified pitfalls of his immunization strategy solutions have been provided for some cases and several modified immunization strategies have been developed. But they still have the goal to dynamically replicate a risk free asset, i.e. to preserve the surplus value.

Further, they can be divided into two groups: passive and active value driven dynamic strategies. Passive value driven dynamic strategies are:

- *Immunization*:

  The immunization (also called standard immunization) principle involves duration matching and is discussed in the preceding chapter.
- *Model conditioned immunization*:

  The model conditioned immunization drops the assumption of a flat yield curve and modifies the standard immunization theory by using a stochastic process that determines the development of the yield curve over time. The process that determines the development can depend on one or more factors. In case of the single factor immunization, only the short term interest rate is defined by a stochastic process, and the long term structure of interest rate is defined as by Cox, Ingersoll and Ross (1985). In case of the multi-factor immunization, a limited number of independent factors determines the shape of the term structure. Such factors can for example be observed forward rates or the long term average of the short term rate. The sensitivities to changes in the yield curve can be defined by model specific durations. The immunization concept used is the same as in the standard immunization theory. The model conditioned immunization strategies only differ with respect to the duration and convexity measures applied.

The advantage of these strategies is their possible precision and the chance to integrate derivative instruments in the same term structure environment. The main disadvantage is the non-stationarity of factors. This may result in a risk related to the validity of the model and may necessitate monitoring these factors.

- *Key rate immunization*:

  Key rate immunization is very similar to standard immunization except that it allows non-parallel term structure shifts. Instead of using one interest rate i, a limited number of key interest rates $i_1, i_2, \ldots, i_n$ are used, which shape the term structure and which can be interpolated to determine the other values. Thus, the price P of a security will be a function of several variables:

$$P \quad P(i_1, i_2, \ldots, i_n).$$

Since we may have different changes in interest rates ${}_1, {}_2, \ldots, {}_n$, key rate immunization requires to equate the corresponding sets of partial derivatives of $A(i_1, i_2, \ldots, i_n)$ to those of $L(i_1, i_2, \ldots, i_n)$ (Ostaszewski, 2002).

The active immunization strategies intend to guarantee a minimally acceptable level of surplus while allowing active asset portfolio management in order to achieve higher asset returns. They also implicitly assume that a positive surplus is available.

- *Contingent immunization*:

  Contingent immunization was developed by Leibowitz and Weinberger (1982, 1983). It allows active portfolio management while meeting the requirements of portfolio matching. The main idea is the immunization of the asset portfolio at any point in time. While the asset portfolio exceeds the amount needed to meet the liabilities, the asset portfolio can be managed actively in order to attain better performance. But if the asset portfolio drops to a specified minimum value, the portfolio has to be managed through an immunization strategy.

- *Portfolio insurance*:

  Using the option pricing theory and the derivation of the Black-Scholes option pricing formula, Leland and Rubinstein (1981) developed a strategy of replicating an option on a capital asset. It permits possible gains from asset investments, but preserves the portfolio value above or at a specified level.

  Ostaszewski (2002) describes this in depth. A European call is the right to buy a security at a predetermined price at some future point in time. A European put is the analogous right to sell a security. Let C denote the price of the European call, and P denote the price of the European put, both with exercise price X in time T in the future. Let the capital asset be S and assume it does not provide any dividend or other sort of income. A portfolio consisting of one unit of capital asset S and one put P on that capital asset will have a terminal value of S if $S \geq X$ or of X if $X \geq S$ at time T. But if we have a portfolio of a call C on one unit of capital asset and cash that will, together with the interest earned at the risk-free rate, equal the present value of the exercise price PV(X), then the portfolio will have the same value at time T as the portfolio described before:

  $$C + PV(X) = S + P.$$

This is called put-call parity (Panjer, 1998) and describes the relationship of a portfolio of cash and capital asset to a portfolio of call and put, since the equation can be rearranged to

$$S - PV(X) = C - P.$$

The payoff of the portfolio consisting of cash in the amount of PV(X) and the call will be the same as the one of the portfolio presented first, if we use the following strategy: If between now and time T the price of the capital asset is above PV(X), keep the capital asset. If the price drops to PV(X), sell the asset and keep the cash. If the price increases again, purchase the capital asset at price PV(X). This way, we will also have S if $S \geq X$ and X if $X \geq S$ at time T. The inconvenience of this strategy may be its dependence on market liquidity, which may be missing when needed.

- *Constant proportion portfolio insurance*:

  Constant proportion portfolio insurance combines contingent immunization and portfolio insurance. It also guarantees a specified minimum acceptable floor. The guarantee of this floor at the end of the investment period is achieved by one part of the portfolio, called reserve account, which is invested in a risk-free strategy. The remaining part, the active account, is partly invested in a risky asset or portfolio to provide upside potential. But the proportion of the active account invested risky is fixed over time, which is the basic difference to portfolio insurance. Portfolio insurance increases the exposure to the risky asset when the asset value increases, whereas constant proportion portfolio insurance keeps the same proportion and therefore constantly increases or decreases the exposure depending on the proportion value.

3.2.2 Return Driven Dynamic Strategies

The return driven dynamic strategies mainly focus on returns earned by a company.

- *Spread management*:

  Spread management concentrates on preserving a yield spread between the asset and liability portfolio. Generally, both asset and liability portfolio yields are related to term structure derived Treasury Bond yields. Advanced spread management additionally involves differences in spreads to spread determining factors, e.g. duration differences. This lead to the development of option adjusted spread analysis and spread duration. Nevertheless, spread management should be integrated into a broader risk-return framework.

- *Required rate of return analysis*:

  Required rate of return analysis has been derived by Miller, Rajan and Shimpi (1990). A required rate of return is defined according to the future cash flows of the existing liabilities, and this is the basis for the choice of a particular asset portfolio. The choice may depend on a number of scenarios in combination with a risk criterion. The advantage of this approach is its simplicity and the possibility to discover trading opportunities. The disadvantage is that it focuses on return and may thereby not recognize other risks, e.g. higher returns are generally related to additional mismatch risk.

Since the *risk-return analysis* generalizes the required rate of return analysis, it will be mentioned here. It is related to the ideas of Markowitz (1952), the Capital Asset Pricing Model (Lintner, 1965) and Arbitrage Pricing Theory (Ross, 1976). The main idea is that when two portfolios of assets and liabilities differ with respect to their expected returns, then they either contain different risk, i.e. higher return corresponds to higher risk, or one of the portfolios is not efficient. If a rational investor can choose from all possible portfolios, he will only choose from the efficient portfolios, and he will take the one that offers him the most attractive risk-return trade-off relating to his underlying risk-return preference.

Risk-return analysis is a general framework and may be applied for evaluation of several of the techniques and strategies presented. For example Wise, Wilkie and later Leibowitz and Langeteig used the portfolio selection principals to match the assets and liabilities, and considered the ultimate surplus as key variable.

Since all these methods have their potential advantages and disadvantages, it is most appropriate to use several of those simultaneously to meet a particular need for asset-liability management purposes.

CHAPTER IV

ASSET-LIABILITY MANAGEMENT ON A GOING CONCERN BASIS

Asset-liability management aims to coordinate the assets and liabilities of a company in order to reduce its overall risk exposure. Risk should not be avoided entirely, but an optimal trade-off between risk and return has to be determined. The rate at which the net worth or surplus of a company, the excess of the value of the assets over the value of the liabilities, increases over time is called capital growth rate. Since the values of the growth rate measured under GAAP, statutory and economic accounting will converge in the long run, it is not important under which accounting principle they are measured. Maximizing the capital growth rate is a major objective in most companies. Applying strategies and techniques of asset-liability management is advisable, since measuring and controlling exposure to risk has a positive effect on a company's capital growth rate. If a company with a high average return on equity does not manage its risks, its surplus may increase at a slower rate than a company with a lower average return but less risk. Thus, asset-liability management helps a company to increase its surplus (Panning, 1987).

A conventional balance sheet for a company may not completely reflect its real assets and liabilities. Consider a property/casualty insurance company selling an auto policy. Generally, a policy sold to a first-time client is almost never profitable because of the costs for initially selling and underwriting it. However, renewal business is virtually always profitable, because the costs for updating and claim costs are lower. On the one hand, the policyholder gets older and therefore tends to have fewer accidents. On the other hand, a higher risk policyholder has to pay higher premiums or will not be able to renew its policy. Hence, the economic gain to the insurer from selling a new policy is the present value of the negative net cash flow of the initial year plus the present value of the positive net cash flows of the following years. Therefore, policyholder retention (or persistency for life insurers) is important for auto insurers. Retention rates for the most profitable auto insurers generally exceed 90 percent annually (Panning, 1993). Consequently, when conducting asset-liability management, it is important to see the company as an ongoing concern. In this case, the assets and liabilities, which refer to future business not yet written and which normally do not occur on the balance sheet of the company, have to be taken into account. Although these hidden assets and liabilities are not recognized by current accounting conventions, they are real and they are reflected in the market value of the company. But they only occur on the balance sheet if the company is acquired. Taking future business into account, management responses to interest rate changes have to be considered as well. If future cash flows can change in response to interest rate changes, the response of competitors and customers to these changes also has to be integrated in asset-liability management (Panning, 1987).

In the following, we want to consider future business and apply asset-liability management to a going concern. To simplify, we mainly focus on interest rate risk, although asset liability management goes beyond interest rate risk. It has to deal with other kinds of risk as well, such as inflation risk, which is especially important to consider for property/casualty insurers.

Asset-liability management is based on two fundamental principles: First, the real economic value of a company is the present value of the cash inflows from its assets less the present value of the cash outflows from its liabilities. Second, the main objective of asset-liability management is to manage the sensitivity of the real economic value of the company to changes in interest rates (Panning, 1993).

4.1 The Policy

Consider a simple property/casualty insurance policy: a single premium payment occurs now at time $t = 0$, the time the policy is written, net of expenses are paid simultaneously, and a single loss payment occurs T periods later at time $t = T$. The results that will be obtained can be generalized to multiple premium, expense and loss payments, but in order to draw clear conclusions the focus will be on that simple policy. Furthermore, assume that the term structure of interest rates is flat, that taxes are not considered, except for the pre-tax return on surplus, and that the analysis takes place at time $t = 0$.

The following notation will be used in this chapter:

N – premium net of expenses paid at time $t = 0$, in dollars

L – expected loss payment at time $t = T$, in dollars

T – time at which the loss payment will occur

s – T-period spot rate

r – required pre-tax return on surplus

k – surplus required per dollar of ultimate loss payment

Since the actual value of the loss payment is uncertain, surplus is needed to write this policy. The surplus needed at time T will be kL dollars due to the definition of k. The surplus needed now at time $t \quad 0$ is the discounted value $kLe^{\ sT}$.

At time $t = T$, the surplus and the accumulated premium must be sufficient to pay the loss L and generate the required return on surplus:

$$kL + Ne^{sT} = L + (kLe^{-sT})e^{rT}.$$

Therefore, the *premium* N that satisfies this condition is

$$N = Le^{-sT}[1 - k + ke^{(r-s)T}].$$

4.2 The Company's Nominal Balance Sheet

Assume that the company has written the same above defined policy each year in the past, and has now at time $t \quad 0$ just sold another policy. Then there are T such policies in force, each with an expected loss payment L, and the *nominal reserves* of the company for its past and newly written policies are $R = LT$. The nominal reserves used here contain both the loss reserve and the loss component of the unearned premium reserve for the newly written policy.

We assume that the *nominal surplus* required for the nominal reserve R is kR.

Moreover, we assume that the company's *assets* A are equal to the sum of its nominal reserves and its required nominal surplus, i.e.

$A = R(1+k)$. All the company's assets are invested, because the premiums are paid immediately.

4.3 The Company's Economic Balance Sheet

In order to describe the company's economic balance sheet we have to determine the *market value of its assets* V(A), and the present value of its liabilities V(R). For simplification purposes we assume that the market and book value of the assets are the same, i.e. $V(A) = A$. Given that the term structure is flat and we wrote the same policy each year in the past, we have

$$V(R) = \sum_{t=1}^{T} Le^{-st}.$$

Using $d = e^{-s}$ and the geometric sum, we can simplify this expression to

$$V(R) = L(d + d^2 + ... + d^T) = Ld(1 + d + d^2 + ... + d^{T-1})$$

$$= Ld\frac{1-d^T}{1-d} = Le^{-s}\frac{1-e^{-sT}}{1-e^{-s}} = Le^{-sT}\frac{e^{sT}(1-e^{-sT})}{e^s(1-e^{-s})}.$$

Thus, we obtain the *present value of the liabilities*:

$$V(R) = Le^{-sT}\frac{e^{sT}-1}{e^s-1}.$$

The company's *total economic surplus* is $V(A) - V(R)$. But since the *committed surplus*, which equals $kV(R)$, has to produce the required return on surplus, the company's *uncommitted surplus* is $V(A)-(1+k)V(R)$. Since R is not discounted, we have $R > V(R)$, and since $V(A)=A=(1+k)R$, we obtain $V(A)-(1+k)V(R)=(1+k)R-(1+k)V(R)$ $=(1+k)(R-V(R))>0$. This shows that the uncommitted surplus will be some positive amount.

To protect the uncommitted surplus from interest rate changes, we use the immunization theory described in chapter 2.3. If the first derivative of the uncommitted surplus is equal to zero, the uncommitted surplus will not change with a small parallel shift in the yield curve. The (Macaulay) *duration of* the *reserves*, denoted by D(R), is

$$D(R)=-\frac{1}{V(R)}\frac{d}{ds}V(R)=-\frac{1}{Le^{-sT}}\frac{e^{s}-1}{e^{sT}-1}L\frac{d}{ds}\left[\frac{1-e^{-sT}}{e^{s}-1}\right]$$

$$=-\frac{e^{s}-1}{e^{-sT}(e^{sT}-1)}\left[\frac{(e^{s}-1)(Te^{-sT})-(1-e^{-sT})e^{s}}{(e^{s}-1)^{2}}\right]=-\frac{T(e^{s}-1)-(e^{sT}-1)e^{s}}{(e^{sT}-1)(e^{s}-1)}$$

$$=-\frac{T}{e^{sT}-1}+\frac{e^{s}}{e^{s}-1}$$

Thus, the value of the *uncommitted surplus* $V(A)-(1+k)V(R)$ will not change with a small parallel shift in the yield curve, if

$$\frac{d}{ds}[V(A)-(1+k)V(R)]=\frac{d}{ds}V(A)-\frac{d}{ds}(1+k)V(R)=\frac{d}{ds}V(A)-(1+k)\frac{d}{ds}V(R)=0.$$

Since the *duration of the assets* is defined as

$$D(A) = -\frac{1}{V(A)}\frac{d}{ds}V(A),$$

and therefore

$$\frac{d}{ds}V(A) = -V(A)D(A),$$

the following equation must hold true:

$$V(A) \cdot D(A) = (1+k)V(R) \cdot D(R).$$

This tells us how we have to select the duration of the assets:

$$D(A) = \frac{(1+k)V(R)D(R)}{V(A)}.$$

As $V(A) = (1+k)R > (1+k)V(R)$ and therefore $\frac{(1+k)V(R)}{V(A)} < 1$, the asset duration, which ensures that the value of the uncommitted surplus remains unchanged in case of interest rate changes, has to be smaller than the duration of the firm's liabilities.

4.4 The Value of the Company as a Going Concern

The economic balance sheet as described here does not reflect its ability to produce future profits from new and renewal business. Nevertheless, these future profits will be reflected in the price a purchaser would be willing to pay to acquire the company or in the company's stock. The sum of the company's *economic value*, which refers to future cash flows from business already in the books, and its *franchise value*, which refers to the cash flows from business it anticipates writing in the future, represents the company's *shareholder value* (or, in case of a mutual company, its policyholder value). In the following part, I will concentrate only on that part of the company's franchise value that reflects renewal of business already on the company's books. But the results can easily be generalized to include new business, too.

The present value of future retentions of existing business V(FR) is the difference between the present value of future premiums net of expenses V(FP), and the present value of future loss payments from those retentions V(FL): $V(FR) = V(FP) - V(FL)$.

A variable that has an influence on all these is the *persistency* p. It is the proportion of policies that is renewed from one year to the next. We assume that p is constant, i.e. p policies that the company has on its books at $t \quad 0$ will be renewed at $t = 1$, p^2 will be renewed the following period and so on. Since the same level cannot be maintained forever, we assume that p falls to zero after n years.

The fact that loss costs change over time may seems to complicate the prediction of future cash flows from future business. Loss cost changes have to be offset by corresponding premium changes, if a firm wants to keep its desired return on equity. In this analysis we assume that these changes occur simultaneously, although they do not in reality. Thus, the difference of the present value of expected future premiums and expected future losses is still correct, which is important for estimating the value of the company as a going concern, although each present value is not correctly approximated.

Since we still consider the same kind of policy, the first renewals of policies already on the books will occur in the next year, and the loss payment will therefore be expected to occur in $T+1$ years. For the following year, we expect p^2 policies to be renewed and the loss payment to occur at time $T+2$, and so on until $t=n$. Thus, we get the following formula for the *present value of future loss payments*:

$$V(FL) = \sum_{t=1}^{n} Le^{-sT} p^t e^{-st}$$

If we use $d = pe^{-s}$ and the geometric sum, we can simplify this to

$$V(FL) = Le^{-sT}(d + d^2 + \dots + d^n) = Le^{-sT} d(1 + d^1 + \dots + d^{n-1}) = Le^{-sT} d \frac{1-d^n}{1-d}$$

$$= Le^{-sT} pe^{-s} \frac{1-p^n e^{-ns}}{1-pe^{-s}} = Le^{-sT} \frac{p}{e^s(1-pe^{-s})} \frac{e^{ns}(1-p^n e^{-ns})}{e^{ns}}$$

$$= Le^{-sT} \frac{p}{e^s - p} \frac{e^{ns} - p^n}{e^{ns}}.$$

Note that $\frac{e^{ns}-p^n}{e^{ns}}$ approaches 1 as n approaches infinity and that Le^{-sT} is the discounted loss of a single policy. As the premiums for future business will be paid starting the following year and also depend on the persistency rate, the *present value of future premiums* can be derived analogously:

$$V(FP)=\sum_{t=1}^{n} Np^t e^{-st} = N\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}.$$

Hence, the *present value of future retentions* is

$$V(FR)=V(FP)-V(FL)=(N-Le^{-sT})\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}.$$

The first term represents the discounted cash flow from a single policy, and should therefore be positive for a profitable company. As e^s-p and $e^{ns}-p^n$ are either both negative or both positive, the product of the two ratios is never negative. Consequently, the present value of future retentions is a hidden asset, i.e. hidden by accounting conventions.

The company's shareholder value can then be expressed as

$$V(A)-(1+k)V(R)+V(FR)+V(NB),$$

where V(NB) represents the *present value of future business*. For simplification, we do not deal with V(NB), which reflects the policies sold to new clients, separately, but include it in the present value of future retentions by using a value of p that is greater than its true value. Therefore, V(FR) can be seen in the following chapters as the present value of future retentions that contains both retentions of existing business and newly written business.

4.5 The Interest-Rate Sensitivity of Future Business

If we assume that our objective is to protect the company's shareholder value from interest rate risk, then we have to consider future retentions in our analysis.

Using the concept of immunization, the company's shareholder value will not change if a small change in interest rate occurs, if

$$\frac{d}{ds}[V(A)-(1+k)V(R)+V(FR)]=\frac{d}{ds}V(A)-(1+k)\frac{d}{ds}V(R)+\frac{d}{ds}V(FR)=0\,.$$

Therefore, the following condition must be satisfied:

$$V(A)\cdot D(A)+V(FR)\cdot D(FR)=(1+k)V(R)\cdot D(R)\,,$$

where D(FR) is the *duration of future retentions*. This means the duration of the assets has to be

$$D(A)=\frac{(1+k)V(R)\cdot D(R)}{V(A)}-\frac{V(FR)\cdot D(FR)}{V(A)}\,.$$

Since we already know that V(FR) is positive for a profitable firm, and that V(A) is as well, it is now important to determine the duration of the hidden asset, D(FR). If D(FR) is positive, the risk-minimizing strategy will be to invest in assets with lower duration than the duration suggested when the hidden asset was omitted.

To obtain the duration of future retentions, we have to calculate the duration of both future losses and future premiums, and then combine these results. We assume that p>0 and n>0 because in the case that one of these variables is zero, the present value of future premiums, losses and retentions is zero as well.

The *duration of future losses* is

$$D(FL) = -\frac{1}{V(FL)}\frac{d}{ds}V(FL) = -\frac{1}{Le^{-sT}}\frac{e^s - p}{p}\frac{e^{ns}}{e^{ns} - p^n}Lp\frac{d}{ds}\left[\frac{e^{-sT}}{e^s - p}\frac{e^{ns} - p^n}{e^{ns}}\right]$$

$$= -\frac{e^s - p}{e^{-sT}}\frac{e^{ns}}{e^{ns} - p^n}\left[\frac{-(e^s - p)Te^{-sT} - e^{-sT}e^s}{(e^s - p)^2}\frac{e^{ns} - p^n}{e^{ns}} + \frac{e^{-sT}}{e^s - p}\frac{e^{ns}ne^{ns} - ne^{ns}(e^{ns} - p^n)}{(e^{ns})^2}\right]$$

$$= -\frac{1}{e^{ns} - p^n}\left[\frac{-(e^s - p)T - e^s}{e^s - p}\frac{e^{ns} - p^n}{1} + \frac{ne^{ns} - n(e^{ns} - p^n)}{1}\right]$$

$$= \frac{(e^s - p)T(e^{ns} - p^n)}{(e^{ns} - p^n)(e^s - p)} + \frac{e^s(e^{ns} - p^n)}{(e^{ns} - p^n)(e^s - p)} - \frac{np^n}{e^{ns} - p^n}$$

$$= T + \frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n}$$

Note, that the last term approaches zero as n approaches infinity.

It is more difficult to find the duration of future premiums because the cash flows from future premiums may themselves vary with interest rates. If interest rates decrease, premium cash flows generally increase. If interest rates rise, premium cash flows usually fall. Additionally, the extent, to what premium cash flows react to interest rate changes, is determined by the company's own pricing strategy. Their pricing strategy mainly depends on that of their competitors and regulators.

To consider the fact of different possibilities to react to changes in interest rates, we divide the premium into two components. We assume that the fixed premium N_f does not react to changes in interest rates, and that the variable part of the premium N_v responds to changes in interest rates, i.e.

$$N = (1 - v)N_f + vN_v.$$

The *percentage of the company's premium that varies with interest rates,* v, $0 \leq v \leq 1$, determines the pricing strategies of a company. Differences in pricing strategies are represented by different values of v.

In order to determine the duration of overall future premiums, we first have to find the duration of its components. Note, that prior to any change in s, N_f and N_v are initially equal. The term "fixed" only refers to premium changes caused by interest rate changes, not to loss costs, as discussed earlier, or other factors.

Since the cash flows from fixed premiums do not change in response to interest rate fluctuations, the present value of the fixed future premiums is

$$V(N_f) = N \frac{p}{e^s - p} \frac{e^{ns} - p^n}{e^{ns}},$$

where $N = Le^{-sT}[1 - k + ke^{(r-s)T}]$, as defined in the beginning.

The *duration of fixed future premiums* is

$$D(N_f) = -\frac{1}{V(N_f)} \frac{d}{ds} V(N_f) = -\frac{1}{N} \frac{e^s - p}{p} \frac{e^{ns}}{e^{ns} - p^n} N \frac{d}{ds}\left[\frac{p}{e^s - p} \frac{e^{ns} - p^n}{e^{ns}}\right]$$

$$= -\frac{e^s - p}{p} \frac{e^{ns}}{e^{ns} - p^n}\left[\frac{(e^s - p)0 - pe^s}{(e^s - p)^2} \frac{e^{ns} - p^n}{e^{ns}} + \frac{p}{e^s - p} \frac{e^{ns} ne^{ns} - (e^{ns} - p^n)ne^{ns}}{(e^{ns})^2}\right]$$

$$= -\frac{1}{e^{ns} - p^n}\left[\frac{-e^s}{e^s - p} \frac{e^{ns} - p^n}{1} + \frac{p^n ne^{ns}}{e^{ns}}\right]$$

$$= \frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n}$$

Since a company's required return on equity may itself change with interest rates for variable premiums, we assume the company's required pre-tax return on surplus $r = a + bs$ is determined by the company's choice of parameters a and b, or by its chosen capital structure. As a consequence, the variable premiums are of the form

$$N_v = Le^{-sT}[1 - k + ke^{(r-s)T}] = Le^{-sT}[1 - k + ke^{aT + (b-1)sT}],$$

and the present value of the variable future premiums is

$$V(N_v)=\sum_{t=1}^{n} N_v p^t e^{-st} = Le^{-sT}[1-k+ke^{aT+(b-1)sT}]\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}.$$

The *duration of variable future premiums* therefore has to be

$$D(N_v)=-\frac{1}{V(N_v)}\frac{d}{ds}V(N_v)$$

$$=-\frac{1}{Le^{-sT}[1-k+ke^{aT+(b-1)sT}]}\frac{e^s-p}{p}\frac{e^{ns}}{e^{ns}-p^n}Lp\frac{d}{ds}\left[\frac{e^{-sT}[1-k+ke^{aT+(b-1)sT}]}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}\right]$$

$$=-\frac{e^s-p}{e^{-sT}[1-k+ke^{aT+(b-1)sT}]}\frac{e^{ns}}{e^{ns}-p^n}\left[\frac{e^{-sT}[1-k+ke^{aT+(b-1)sT}]}{e^s-p}\frac{e^{ns}ne^{ns}-(e^{ns}-p^n)ne^{ns}}{(e^{ns})^2}+\right.$$

$$\left.+\frac{e^{ns}-p^n}{e^{ns}}\frac{(e^s-p)[e^{-sT}k(b-1)Te^{aT+(b-1)sT}-(1-k+ke^{aT+(b-1)sT})Te^{-sT}]-e^{-sT}(1-k+ke^{aT+(b-1)sT})e^s}{(e^s-p)^2}\right]$$

$$=-\frac{1}{[1-k+ke^{aT+(b-1)sT}]}\frac{1}{e^{ns}-p^n}\left[\frac{[1-k+ke^{aT+(b-1)sT}]}{1}\frac{p^n ne^{ns}}{e^{ns}}+\right.$$

$$\left.+\frac{e^{ns}-p^n}{1}\frac{(e^s-p)[k(b-1)Te^{aT+(b-1)sT}-(1-k+ke^{aT+(b-1)sT})T]-(1-k+ke^{aT+(b-1)sT})e^s}{e^s-p}\right]$$

$$=-\frac{np^n}{e^{ns}-p^n}-\frac{k(b-1)Te^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}+T+\frac{e^s}{e^s-p}$$

$$=\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+T\left[1+\frac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}\right].$$

As the actual future premiums are a combination of fixed and variable premiums, the *duration of overall future premiums* is the weighted average of the durations of the components:

$$D(FP)=(1-v)D(N_f)+vD(N_v)$$

$$= (1-v)\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}\right]+v\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+T\left[1+\frac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}\right]\right]$$

$$= \frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+vT\left[1+\frac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}\right]$$

If we denote the term in brackets by M, which is equal to 1 when b is equal to 1, we obtain

$$D(FP) = \frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+vTM.$$

Since we determined the duration of future losses and future premiums, we can now combine the results. The present value of future retentions is defined as $V(FR) = V(FP) - V(FL)$. It follows that

$$\frac{d}{ds}V(FR) = \frac{d}{ds}V(FP) - \frac{d}{ds}V(FL),$$

and consequently $D(FR)V(FR) = D(FP)V(FP) - D(FL)V(FL)$.

Thus, the *duration of future retentions* is

$$D(FR) = \frac{D(FP)V(FP) - D(FL)V(FL)}{V(FR)}$$

$$= \frac{N\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+vTM\right]-Le^{-sT}\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}\left[T+\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}\right]}{(N-Le^{-sT})\frac{p}{e^s-p}\frac{e^{ns}-p^n}{e^{ns}}}$$

$$= \frac{N\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+vTM\right]-Le^{-sT}\left[T+\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}\right]}{(N-Le^{-sT})}$$

$$= \frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n} + T\left[\frac{vNM - Le^{-sT}}{N - Le^{-sT}}\right],$$

where $M = 1 + \frac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}$. The middle term of the final formula for D(FR) approaches zero as n approaches infinity. The term in brackets is positive, when v is sufficiently large, but it can also be negative. Hence, the duration of future retentions depends to a large extent on the company's pricing strategy, represented by v. Actually, the dependence on v is so strong, that the company can determine the duration of future retentions to be positive or negative.

Recall the formula for the risk-minimizing duration of the assets:

$$D(A) = \frac{(1+k)V(R)\cdot D(R)}{V(A)} - \frac{V(FR)\cdot D(FR)}{V(A)}.$$

When future premiums change with interest rates, i.e. v is equal to 1, future retentions are an asset with positive duration. If future premiums are variable, premiums generally rise when interest rates fall, which means premium cash flows are inversely sensitive to interest rate changes. In order to protect shareholder value from interest rate risks, the investment strategy of the assets has to be modified. As mentioned earlier, V(FR) and V(A) are positive, and thus the invested assets must have a lower duration than would be the case if future business is ignored. The extent to which the duration has to be reduced depends on the present value of future retentions. The greater the present value, the lower has to be the duration of invested assets.

However, when future premiums are fixed, i.e. v is equal to 0, future retentions are an asset with negative duration, because the premium cash flows are received before losses are paid, and therefore the duration of future losses exceeds the duration of fixed future premiums. To compensate for this, the duration of invested assets has to be increased by an amount that again depends on the present value of future retentions.

The conclusion we can draw is that a company's investment strategy and its pricing strategy cannot be managed independently of one another. If the shareholder value should be protected against interest rate changes, those two strategies have to be coordinated.

4.6 The Impact of Competition

In the preceding chapters, we assumed that the company's growth rate p is constant, although it is not in reality. The retention rate of a company and its ability to create future business may decrease, if the company increases its prices or if its competitors lower their prices. As a result, changes in interest rates may have an indirect effect on a company's shareholder value. In the previous chapters we only considered the direct effects of changes in interest rates, but now we also want to take the indirect effects into account.

We assume that the company and its main competitors have the same loss payout pattern (T and L), surplus requirement (k), and profit objectives (a, b and r). We suppose they only differ in their pricing strategies. Let w represent the pricing strategies of the company's competitors, the same one for all of them, and v still be the company's own pricing strategy. Additionally, we assume that the company and its competitors charge the same price before any interest rate change occurs.

Let $V(N) = V(vN_v + (1-v)N_f)$ be the present value of future premiums of the company and consider it the price the company charges for all future policies together. Let $V(N_c) = V(wN_v + (1-w)N_f)$ be the present value of future premiums of the competitors and consider it the price the competitors charge. The price V(N) charged by the company will change in the following percentage, if a change in the spot rate s occurs:

$$\frac{1}{V(N)}\frac{d}{ds}V(N) = -D(N) = -\frac{e^s}{e^s - p} + \frac{np^n}{e^{ns} - p^n} - vTM,$$

where $M = 1 + \dfrac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}$.

The corresponding percentage change in the price $V(N_f)$ charged by its competitors is

$$\frac{1}{V(N_c)}\frac{d}{ds}V(N_c) = -D(N_c) = -\frac{e^s}{e^s - p} + \frac{np^n}{e^{ns} - p^n} - wTM .$$

We assume the company and its competitors charged the same price, i.e. $N = N_c$, in the beginning, before any change in the spot rate occurred. Therefore, the *company's percentage price change relative to its competitors* is

$$\frac{1}{V(N)}\frac{dV(N_{rel})}{ds} = \frac{1}{V(N)}\left[\frac{dV(N)}{ds} - \frac{dV(N_c)}{ds}\right] = (w - v)TM .$$

If the company has fixed premiums, i.e. $v = 0$, and the competitors have variable premiums, i.e. $w = 1$, then an increase in interest rates, i.e. $ds > 0$, will force the competitors to lower their premiums. Thus, the result is a positive price change relative to competitors, i.e. $dV(N_{rel}) > 0$.

If q denotes the *elasticity of the company's growth rate p with respect to changes in its relative pricing*, then it is

$$q \equiv -\frac{dp}{p}\frac{V(N)}{dV(N_{rel})} .$$

The value of q should be positive, because both V(N) and p are positive, and because a positive price change relative to its competitors, i.e. $dV(N_{rel}) > 0$, should make dp negative.

Next, the proportional sensitivity of the present value of the company's future business to changes in its premium growth rate p is defined as

$$\frac{1}{V(FR)}\frac{dV(FR)}{dp} = \frac{1}{(N - Le^{-sT})}\frac{e^s - p}{p}\frac{e^{ns}}{e^{ns} - p^n}\frac{d}{ds}\left[(N - Le^{-sT})\frac{p}{e^s - p}\frac{e^{ns} - p^n}{e^{ns}}\right]$$

$$= \frac{1}{(N - Le^{-sT})}\frac{e^s - p}{p}\frac{e^{ns}}{e^{ns} - p^n}\frac{N - Le^{-sT}}{e^{ns}}\frac{d}{ds}\left[\frac{p}{e^s - p}\frac{e^{ns} - p^n}{1}\right]$$

$$= \frac{e^s - p}{p}\frac{1}{e^{ns} - p^n}\left[\frac{pe^{ns} - p^{n+1}}{e^s - p}\right]$$

$$= \frac{e^s - p}{p}\frac{1}{e^{ns} - p^n}\left[\frac{(e^s - p)(e^{ns} - (n+1)p^n) + (pe^{ns} - p^{n+1})}{(e^s - p)^2}\right]$$

$$= \frac{e^s - p}{p}\frac{1}{e^{ns} - p^n}\left[\frac{e^{ns} - np^n - p^n}{e^s - p} + \frac{p(e^{ns} - p^n)}{(e^s - p)^2}\right]$$

$$= \frac{1}{p} - \frac{np^n}{p(e^{ns} - p^n)} + \frac{1}{e^s - p} = \frac{e^s}{p(e^s - p)} - \frac{np^n}{p(e^{ns} - p^n)}$$

This should always be positive, since higher growth, i.e. $dp > 0$, adds value, and we assume that the business added is profitable, i.e. $V(FR) > 0$.

In order to get the indirect impact of changes in the interest rate s, which results from changes in p, on the present value of future retentions, we combine all the results obtained. What we are looking for is the *partial duration of future retentions with respect to p,* which is

$$D_p(FR) \equiv -\frac{1}{V(FR)}\frac{\partial V(FR)}{\partial s}$$

$$= -\frac{1}{V(FR)}\frac{dV(FR)}{dp}\frac{dp}{p}p\frac{V(N)}{dV(N_{rel})}\frac{1}{V(N)}\frac{dV(N_{rel})}{ds}$$

$$= p\left[\frac{1}{V(FR)}\frac{dV(FR)}{dp}\right]\left[-\frac{dp}{p}\frac{V(N)}{dV(N_{rel})}\right]\left[\frac{1}{V(N)}\frac{dV(N_{rel})}{ds}\right]$$

$$= p\left[\frac{1}{V(FR)}\frac{dV(FR)}{dp}\right]q\left[\frac{1}{V(N)}\frac{dV(N_{rel})}{ds}\right]$$

$$= p\left[\frac{e^s}{p(e^s - p)} - \frac{np^n}{p(e^{ns} - p^n)}\right]q\left[(w - v)TM\right]$$

$$= q(w - v)TM\left[\frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n}\right],$$

where $M = 1 + \frac{(1-b)ke^{aT+(b-1)sT}}{1-k+ke^{aT+(b-1)sT}}$. Both the term in brackets and q are positive. If the company has fixed premiums, i.e. $v = 0$, whereas its competitors have variable premiums, i.e. $w = 1$, the indirect effect of a change in interest rates will be to increase the duration of future business. To compensate, the duration of invested assets has to be decreased. If the company has a variable-premium strategy, i.e. $v = 1$, while its competitors have fixed premiums, i.e. $w = 0$, the indirect effect of a change in interest rates will be to decrease the duration of future business. The duration of the assets has to be decreased to compensate. Thus, if the pricing strategy of the company is significantly different from the pricing strategy of its competitors, the investment strategy of the company has to be modified.

Recall the formula for the duration of future retentions, that takes only the direct impact of interest rate changes into account and holds p constant. Let us now denote it by

$$D_s(FR) = D(FR) = \frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n} + T\left[\frac{vNM - Le^{-sT}}{N - Le^{-sT}}\right].$$

Combing all the results we obtained, we get the *total duration of future retentions*:

$$D_{tot}(FR) = D_s(FR) + D_p(FR)$$

$$= \frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n} + T\left[\frac{vNM - Le^{-sT}}{N - Le^{-sT}}\right] + q(w - v)TM\left[\frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n}\right],$$

where $M = 1 + \frac{(1-b)ke^{aT+(b-1)sT}}{1 - k + ke^{aT+(b-1)sT}}$.

In order to protect shareholder value, i.e. the economic value of business already in the books and the franchise value of future business, from interest rate risk, a company's investment strategy and its pricing strategy cannot be determined independently of one another. To find the risk minimizing investment strategy, the investment and pricing strategy have to be coordinated. But additionally the pricing strategy of the company's main competitors and the sensitivity of customers to price differences between the company and its competitors have to be taken into account.

The following table summarizes the conclusions we drew.

Table 3: Summary (Panning, 1993)

		Additional implications of competitors' pricing strategy	
Company's pricing strategy	Implications for A/L Management of future business	Premiums **are not** responsive, w=0	Premiums **are** responsive, w=1
Premiums **are not** responsive to changes in interest rates, v=0	Duration of **future business** is *negative*. Duration of **invested assets** must be *increased* to compensate.	No additional effect.	Duration of **future business** is *increased*. Duration of **invested assets** must be *decreased* to compensate.
Premiums **are** responsive to changes in interest rates, v=1	Duration of **future business** is *positive*. Duration of **invested assets** must be *reduced* to compensate.	Duration of **future business** is *reduced*. Duration of **invested assets** must be *increased* to compensate.	No additional effect.

4.7 Example

In order to make the conclusions clearer, I want to present a numerical example for the strategies presented in this chapter. Consider a company that sells policies expecting a loss payment $L = 1000$ to occur two periods later, i.e. $T = 2$. We assume a 2-period spot rate $s = 3\%$, and a required pre-tax return on surplus $r = a + bs = 13\%$, where the parameters chosen are $a = 0.10$ and $b = 1$. Let the surplus required for each dollar of the ultimate loss payment be $k = 0.25$.

The premium for this policy then has to be

$$N = Le^{-sT}[1-k+ke^{(r-s)T}] = 1000e^{-0.03 \cdot 2}[1-0.25+0.25e^{(0.13-0.03)2}] = 993.89\,.$$

If future business and the effects of competition are neglected, the risk-minimizing duration of the company's assets should be chosen

$$D(A) = \frac{(1+k)V(R)D(R)}{V(A)} = \frac{(1+k)\left[Le^{-sT}\dfrac{e^{sT}-1}{e^{s}-1}\right]\left[-\dfrac{T}{e^{sT}-1}+\dfrac{e^{s}}{e^{s}-1}\right]}{LT(1+k)}$$

$$= \frac{(1+0.25)\cdot\left[1000e^{-0.03 \cdot 2}\dfrac{e^{0.03 \cdot 2}-1}{e^{0.03}-1}\right]\left[-\dfrac{2}{e^{0.03 \cdot 2}-1}+\dfrac{e^{0.03}}{e^{0.03}-1}\right]}{1000 \cdot 2(1+0.25)}$$

$$= \frac{1.25 \cdot 1912.21 \cdot 1.49}{2500} = 1.42\,.$$

We now consider future business when determining the optimal investment strategy for the company. We assume that constant retention rates $p = 0.9$ or $p = 1.1$, respectively, are maintained for $n = 15$ years. Further, we distinguish between a variable premium strategy, i.e. $v = 1$, and a fixed premium strategy, i.e. $v = 0$.

The duration of the assets now has to be

$$D(A) = \frac{(1+k)V(R)D(R)}{V(A)} - \frac{V(FR)D(FR)}{V(A)} = 1.42 - \frac{V(FR)D(FR)}{2500},$$

where

$$V(FR) = (N - Le^{-sT})\frac{p}{e^s - p}\frac{e^{ns} - p^n}{e^{ns}}$$

$$= (993.89 - 1000e^{-0.03 \cdot 2})\frac{p}{e^{0.03} - p}\frac{e^{15 \cdot 0.03} - p^{15}}{e^{15 \cdot 0.03}} = \begin{cases} 312.40 \text{ for } p = 0.9 \\ 1371.53 \text{ for } p = 1.1 \end{cases}$$

and

$$D(FR) = \frac{e^s}{e^s - p} - \frac{np^n}{e^{ns} - p^n} + T\left[\frac{vNM - Le^{-sT}}{N - Le^{-sT}}\right]$$

$$= \frac{e^{0.03}}{e^{0.03} - p} - \frac{15 \cdot p^{15}}{e^{15 \cdot 0.03} - p^{15}} + 2\left[\frac{v \cdot 993.89 \cdot 1 - 1000e^{-0.03 \cdot 2}}{993.89 - 1000e^{-0.03 \cdot 2}}\right].$$

The results we obtain for D(FR) and D(A) for the different cases are listed in the following table.

Table 4: Duration Considering Retentions

	p = 0.9		p = 1.1	
	v = 0	v = 1	v = 0	v = 1
D(FR)	-30.5	7.63	-26.93	11.2
D(A)	5.23	0.47	16.19	-4.72

(Source: Author's Calculation.)

If $v=0$, the duration of future retentions is negative. To protect shareholder value from interest rate risk, the duration of invested assets must be increased to compensate. In case of $v=1$, the duration of future retentions is positive, and therefore the duration of the assets has to be reduced.

However, the magnitude of these effects is reduced or even reversed, if we also take the impact of competition into consideration. The duration of future retentions is modified if the company's pricing strategy differs from that of its competitors, i.e. when $v=0$ and $w=1$, or $v=1$ and $w=0$. If $v=w$, $D_p(FR)=0$ and there is no additional effect. If we choose $q=1$, i.e. a 10 percent relative price increase results in a 10 percent decrease in the retention rate, and use

$$D_{tot}(FR)=D_s(FR)+D_p(FR)=D(FR)+D_p(FR)$$

$$=\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}+T\left[\frac{vNM-Le^{-sT}}{N-Le^{-sT}}\right]+q(w-v)TM\left[\frac{e^s}{e^s-p}-\frac{np^n}{e^{ns}-p^n}\right]$$

$$= \frac{e^{0.03}}{e^{0.03} - p} - \frac{15p^{15}}{e^{15 \cdot 0.03} - p^{15}} + 2\left[\frac{v993.89 - 1000e^{-0.03 \cdot 2}}{993.89 - 1000e^{-0.03 \cdot 2}}\right] + (w - v)2\left[\frac{e^{0.03}}{e^{0.03} - p} - \frac{15p^{15}}{e^{15 \cdot 0.03} - p^{15}}\right]$$

we get the following results for the total duration of future retentions and the risk-minimizing duration of the invested assets.

Table 5: Duration Considering Retentions and Competition

	p = 0.9		p = 1.1	
	v = 0 and w = 1	v = 1 and w = 0	v = 0 and w = 1	v = 1 and w = 0
D_p(FR)	11.26	-11.26	18.4	-18.4
D_{tot}(FR)	-19.24	-3.63	-8.53	-7.2
D(A)	3.82	2.83	-8.67	11.51

(Source: Author's Calculation.)

We can clearly see that for $v = 0$ the total duration of future retentions is greater than the duration of future business we obtained just considering future business, and the duration of the assets has to be decreased to compensate. The extent to the duration of the assets has to be modified is determined by the present value of future retentions. For $v = 1$ the total duration of future retentions is less than the duration of future business in table 4, and therefore the duration of the invested assets has to be increased. Note that for $p = 0.9$ the duration of the assets we obtained when future retentions and the effects of competition were considered, are greater than the duration of the assets we got, when these two factors were ignored.

CHAPTER V

SUMMARY

Asset-liability management gained importance in the United States in the 1970s when interest rates increased sharply, became more volatile than ever before, and options embedded in the assets and liabilities were exercised more frequently. Asset-liability management primarily intended to eliminate this interest rate risk, basically by applying Redington's main idea of an equal and parallel treatment in the valuation of assets and liabilities, and his concepts of duration and immunization. The strength and the weakness of the traditional immunization strategy resulted in the development of all modern techniques of asset-liability management. We can classify them into static techniques and dynamic strategies. The static techniques focus on a complete match between assets and liabilities. But they fail to consider the trade-off between risk and return. The value driven dynamic strategies generally modify Redington's concept of immunization to protect a company's surplus from interest rate risk. The return driven strategies mainly focus on returns earned by company. All these methods have their potential advantages and disadvantages, and focusing on one risk might increase other kinds of risk. Therefore, several of these methods should be used simultaneously and an optimal trade-off between risk and return has to be determined in order to reduce not only interest rate risk, but the overall risk exposure of a company.

Statutory, GAP and tax accounting fail to recognize changes in the present value of cash flows according to changes in interest rate, embedded options in assets and liabilities, and possible future profits generated by existing and future policies. But when conducting asset-liability management, it is important to consider future business and see the company as an ongoing concern. Assets and liabilities that refer to future business not yet written have to be taken into account, because they are real and reflected in the market value of the company. In order to protect the shareholder value of a company, i.e. find the risk-minimizing investment strategy, a company's investment and pricing strategy have to be coordinated. But the pricing strategy of the company's main competitors and the sensitivity of customers to price differences between the company and its competitors also have to be taken into account.

Hence, asset-liability management can be seen as an ongoing process of formulating, checking and revising strategies associated with assets and liabilities both from existing and future business in order to attain a company's financial objectives, given the company's risk tolerances and other constraints. Asset-liability management has to control the interest rate risk without neglecting the asset default risk, the product pricing risk and other uncertainties.

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