Bubbles and Rationality in Bitcoin

George A. Waters*
Department of Economics
Campus Box 4200
Illinois State University
Normal, IL 61790-4200

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Abstract

Periodically collapsing rational bubbles model speculative demand in asset markets. The price and quantity of bitcoin are integrated of different orders, which is evidence of a bubble. Cointegration tests that allow for the potential presence of such bubbles with alternative proxies for fundamentals cannot reject a bubble in bitcoin.

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*gawater@ilstu.edu

On top of this fundamental demand, we can add a speculative demand. Suppose you know or you think you know that Bitcoin will go up some more before its inevitable crash. In order to speculate on Bitcoin, you have to buy some bitcoins. John Cochrane, "The Bitcoin market isn't irrational" (2018)

Observing bubble-like behavior in the bitcoin data does not require any deep insight, though not everyone agrees on the correct interpretation. Cochrane (2018) argues against a rational bubble explanation, but his description of the behavior of investors reflects precisely that. Furthermore, there is evidence supporting the presence of rational bubbles in the bitcoin market.

With stock market data, cointegration, meaning the presence of a common trend, between prices and dividends is evidence against the presence of bubbles. Prices should represent the future flow of profits or dividends, so they should have a long run relationship. Diba and Grossman (1988) develop statistics to test for cointegration in such an environment, which is a test of the stationarity of the residuals from the least squares regression of prices on dividends. However, Evans (1991) presents a model of a class of periodically collapsing rational bubbles (PCRB) that cannot be detected by such tests. The primary tool for this study of the bitcoin market is the cointegration test of Taylor and Peel (1998), which allows for skewness and excess kurtosis and is a robust test in the presence of such bubbles.

The "speculative demand" for bitcoin that Cochrane (2018) cites to argue against the presence of bubbles, is actually a good description of the behavior in the PCRB model. For an asset price determined by fundamentals f_t and a bubble component b_t such that $p_t = f_t + b_t$, the bubble term in the PCRB model is as follows.

$$\begin{aligned} b_t &= \rho^{-1} b_{t-1} v_t \qquad \text{if } b_t \leq \alpha \\ b_t &= \left[\delta + \pi^{-1}\rho^{-1}\psi_t \left(b_{t-1} - (1+r)^{-1}\delta\right)\right] v_t \qquad \text{if } b_t > \alpha \end{aligned}$$

The parameter ρ represents the discount factor where $0 < \rho < 1$ and v_t is a stochastic variable with mean one. The stochastic term ψ_t is a Bernoulli process such that it equals 1 with probability π and 0 with probability $1 - \pi$. The parameters δ and α are both positive and satisfy the condition $\delta < (1 + r)\,\alpha$.

The PCRB process can switch between two regimes depending on the threshold parameter $\alpha > 0$. As long as b_t remains below α, it grows at mean rate ρ^{-1} but if b_t rises above α it grows at the faster mean rate $\rho^{-1}\pi^{-1}$ as long as ψ_t is 1. When ψ_t is 0, the bubble collapses and falls to δ in expectation.

The PCRB model satisfies rational expectations, meaning the bubble component b_t is unforecastable. If dividends are also unforecastable, a common assumption, the asset price is as well and thereby satisfies the weak version of the efficient markets hypothesis. The rational expectations property $E_{t-1}(b_t) = \rho^{-1} b_{t-1}$ is satisfied in both regimes, though b_t could grow at a rate faster than ρ^{-1} for an extended length of time. As in the above description of speculative demand, it is rational to hold an asset in the explosive regime ($b_t > \alpha$) even if there is a possibility of collapse since there is also a chance the price will rise unusually quickly in the near future. Furthermore, a PCRB process does not violate a transversality condition, which is a common critique of rational bubbles. The transversality condition requires that the price does not diverge, which is satisfied for the PCRB model since such bubbles do collapse, eventually.

Standard cointegration tests on the price p_t and fundamentals f_t, which are typically earnings or dividends, would have difficulty detecting PCRB, since the maintained hypothesis for these tests is a linear process, either autoregressive or explosive. Even though it is explosive at times, the PCRB could appear to be a persistent autoregressive process.

For the bitcoin market, the issue of the fundamental value of the asset is unclear so we use multiple approaches to test for a bubble. One could focus on the cost of mining. Since there is increasing marginal cost in the mining of bitcoins, the quantity of bitcoin and the price of bitcoin should increase together. Alternatively, bitcoin's value as a medium of exchange, the "convenience yield" in Cochrane's (2018) terminology, is a candidate for the fundamental value.

<Table 1 here>

To test for cointegration, one must first demonstrate that the variables are integrated of the same order. All series are daily for the sample 7/18/2010-2/27/2018[1]. Table 1 reports results for the Augmented Dickey-Fuller test on the bitcoin price p, the quantity of bitcoin outstanding q, the difficulty of mining $diff$[2], the price of gold p_G, and log transformations of these variables.

There is strong evidence that the bitcoin price is integrated of order one in both level and log. However, the result of the test rejects the null of a unit root in the level and log of the quantity q, meaning it is integrated of order zero That the price and quantity are integrated of different orders shows the lack of a long run relationship and is evidence of a bubble in itself. Mining difficulty $diff$ is a related candidate for fundamental value, and the evidence suggest its is stationary as well. Liu and Tsyvinski (2018)[3] use the number of bitcoin wallets as a funtdamental value. Though the available sample for this data is shorter than that used here, the number of wallets is also integrated of order zero.

Next, we examine the cointegration of the bitcoin price with the other independent variables as its fundamental value A standard approach is to conduct the same ADF test used for Table 1 on the residuals

[1] The sample has 2782 observations. All data is taken from coinmarketcap.com with the exception of the mining difficulty series, which comes from data.bitcoin.org.

[2] The hash rate would be another measure of the cost of mining, but the data is too limited for the tests reported here.

[3] Other references include Borri and Shakhnov (2018) who study bitcoin price differences across different exchanges and currency pairs, and Borri (2018) who finds that bitcoin prices are exposed to crash-risk in other cryptocurrencies, but not in other standard assets, including equities and commodities.

of the initial least squares regression. However, Taylor and Peel (1998) show that the significance of such tests is biased in the presence of PCRB. Therefore, they develop a test that controls for the skewness and excess kurtosis that could arise. For the cointegration test, the estimation equation is

$$\Delta u_t = \beta u_{t-1} + \gamma_1 w_{1.t} + \gamma_2 w_{2.t},$$

where the term u_t is the residual of the initial linear regression, and the terms $w_{1.t}$ and $w_{2.t}$ are transformations of the skewness and excess kurtosis of u_t.

<Table 2 here>

Table 2 reports results for both the Dickey-Fuller test that excludes the w terms, and the Taylor and Peel test that includes them. For both the estimated parameter is the $\widehat{\beta}$, while the statistics DF[4] and CR test the null of non-cointegration $\widehat{\beta} = 0$. The significance probabilities are determined by Monte Carlo experiments[5] for the present sample analogous to those in Taylor and Peel (1998).

The presence of a bubble cannot be rejected. Stationarity tests on the residuals of three different linear regressions are reported: the price against *i)* a constant, ii) a constant and a time trend and *iii)* the price of gold. The constant with or without the time trend in *i)* and *ii)* represents the value of bitcoin as a medium of exchange, which should be stable. The intuition behind *iii)* is that bitcoin and gold are competing stores of value that do not depend on government behavior. Hence, both values should move with savers preference for such an investment.

One cannot reject the null of non-stationarity according the CR test, which is robust to the potential presence of bubbles, at any reasonable level of significance. Though the DF test is not robust to the presence of PCRBs, the resulting p-values are not close to standard thresholds for significance. For the more reliable CR test, the p-values are very high, the lowest being 0.8572, pointing up the difference in the two tests and the possibility of bubbles in the bitcoin market. Note that in the Taylor and Peel (1998) paper, the test did reject non-cointegration in aggregate prices versus dividends for the S&P 500 over more than a century. The p-values for the test including a time trend or with the price of gold are even higher.

The most appropriate version of the test uses the log of the prices, as demonstrated in Waters (2009), and those are reported in Table 2. As a robustness check, the tests were also run in levels. For all such tests, the estimate of $\widehat{\beta}$ is positive, indicating divergent or explosive behavior.

[4] Note that the ADF tests in Tables 1 and 2 differ since there are no lags $u_{t-1}, u_{t-2}, \ldots$ included in the test reported in the latter.

[5] The DF and CR tests are computed with 20,000 simulations of a unit root with drift for the price and dividend (if necessary) using coefficients estimated with the bitcoin price data.

The tests reported here that center on the cointegration test of Taylor and Peel offer evidence of a bubble in the bitcoin price. The price is not stationary for any robust test, though its cost of production is, nor is it cointegrated with alternative fundamental values. Arguments in favor of rationality do not imply that bubbles are not present. Though the market may be rational, note the R in PCRB, a bubble in bitcoin cannot be rejected.

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Table 1

	ADF	p-value	lags
p	-0.7663	0.8278	27
Δp	-8.7729	0.0000	27
$\log p$	-2.2162	0.2006	27
$\Delta \log p$	-20.8092	0.0000	27
q	-4.7854	0.0001	27
Δq	-2.3167	0.1667	27
$\log q$	-5.1116	0.0000	27
$\Delta \log q$	-4.6050	0.0001	27
p_G	-1.8940	0.3354	25
Δp_G	-43.0045	0.0000	25
$\log p_G$	-1.8093	0.3763	25
$\Delta \log p_G$	-43.0045	0.0000	25
$\log diff$	-3.3190	0.0142	27
$\Delta \log diff$	-7.4994	0.0000	27

Table 1 shows result for the Augmented Dickey-Fuller Test on the null of non-stationarity. The number of lags is chosen to maximizes the Schwartz information criterion.

Table 2

price	indep. var.	DF stats	p-value	CR stats	p-value
$\log p$	c	-0.00089		-0.00041	
		(-2.6065)	0.2805	(-1.4085)	0.8572
$\log p$	c, t	-0.0022		-0.00088	
		(-2.5362)	0.3126	(-1.2088)	0.9070
$\log p$	$\log p_G$	-0.0034		-0.0030	
		(-2.7885)	0.3076	(-2.5089)	0.3265

Table 2 shows results for the test on the null of non-stationarity of the residuals of the least squares test with the variables in the first two columns. Columns 3 and 4 show results for the Dickey-Fuller test (with no lags), and columns 5 and 6 show results for the CR statistics developed in Taylor and Peel (1998).