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# Children's Measurement Project: Illinois State Site (Grades 2-5) Mini-Center for Measurement at Michigan State Sept 17-19 2009

Jeffrey E. Barrett and Craig Cullen

Chepina Witkowski and Amanda Miller

Illinois State University

Douglas H. Clements and Julie Sarama

Doug Van Dine

University at Buffalo, State University of New York

Contact information: [jbarrett@ilstu.edu](mailto:jbarrett@ilstu.edu)

# Overview of Presentation

- Previous Studies: 1997, 2003, 2006, 2007
- Current work: Design of Children's Measurement Project
  - Growth Chart for Grades 2-3
  - Instructional Tasks at each of three levels
  - Current Session: Volume, Area and Length
- Next steps:
  - Follow Longitudinal cases into middle school;
  - Confirm Learning Trajectory and extend to more classroom testing
  - Revise Assessment items and grade level tests in cycles with Rasch Analysis, IRT; Task design cycle for instructional, diagnostic, and evaluation.

# Previous Studies

(Clements, Barrett, Sarama, Szilagyi, et al.)

- 3 different types of reasoning and strategy use in grade 3 (1997)
- 4 levels of thinking about length in a teaching experiment with Grade 4 (2003)
- Elaborate and verify fit for theory of levels across grades 2-11 (2006)
- PreK to Grade 3 Learning Trajectory for Length (2004)
- Validation of LT up to grade 2 (2007)

# Current Work:

## Definition of a Learning Trajectory (LT)

- A tool for assessment, for improving Learning Standards, and for differentiating instruction
- Learning trajectories incorporate developmental progressions into a larger framework (Clements & Sarama, 2004, 2007; Sarama & Clements, 2009; also Smith, Wiser, Anderson & Krajcik, 2006 )
  - **An educational goal** (concepts of length measurement);
  - **A developmental account of thinking and strategies** related to the goal;
  - **A sequence of instructional or assessment tasks** coordinated with the developmental account  
(cf. *Key Developmental Understandings*, Simon, 2007)

# Our Questions

- How do students develop coherent, integrated strategies for measurement from Pre-Kindergarten through Grade 5?
- What instructional tasks and intervention strategies promote learning and development through the levels in the Length, Area, and volume trajectories for children in Grades 2-5?

# Our study:

## Two Methods of Inquiry

Using a Teaching Experiment (Steffe & Thompson, 2000) to predict and check models of student learning and development through designed cycles of formative instruction.

- Two classes of children, beginning in Grade 2 in Spring 2008
- Focus Cases: 4-8 children
- Background Cases: 8 more children
- Ongoing Teaching Episodes complemented by Classroom Lessons and brief trial tasks to teachers

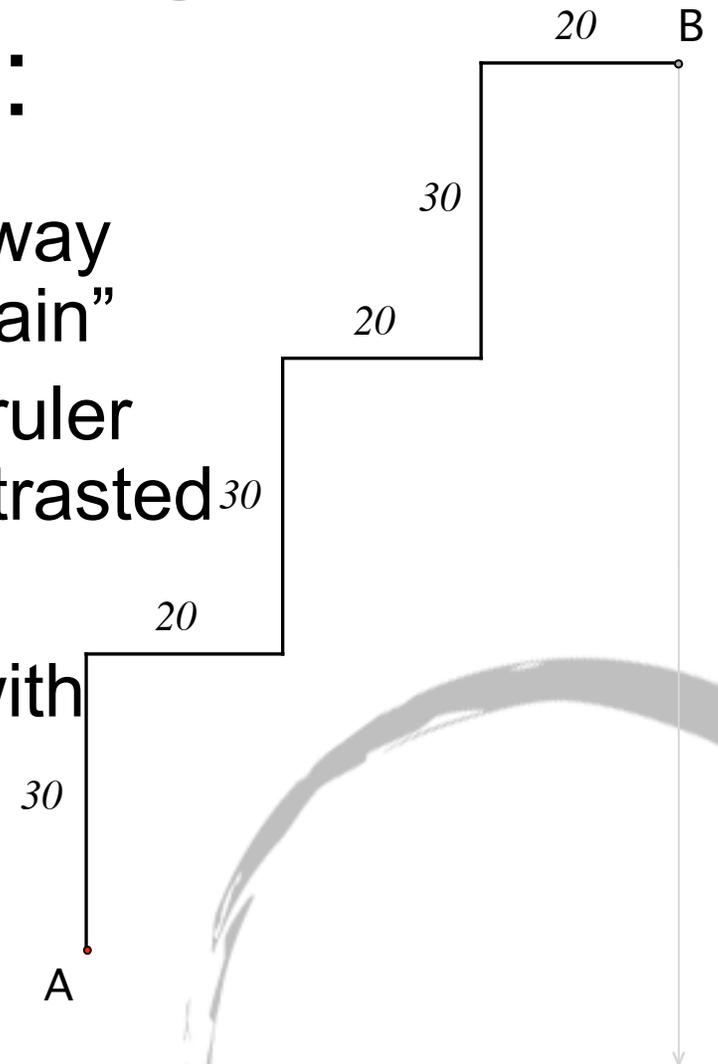
Using Assessments (pre/post) at all grades, both sites

# Our trajectory for this range of grade levels: 2-4

<i>Level</i>	<i>Thinking &amp; Actions</i>	<i>Tasks</i>
<b>Indirect Length Comparer</b>	Can use a third object to compare the length of two objects.	
<b>End-to-End</b>	Expects that length is quantifiable as a composition of shorter lengths. Compares an end-to-end train of countable objects to the linear extent of an object.	Fewer objects than needed to fill the space.  Exaggerated mistakes for gapping and overlapping.
<b>Unit Repeater &amp; Relater</b>	Is able to iterate a unit along an object to find length.	Contrasting a tick mark tool and an interval tool. Diminishing line segments. Broken ruler tasks.
<b>Length Measurer</b>	Can compose and partition length units. Can think of the length of a bent path as the sum of its parts. Mentally iterates a unit and sub units (internalized ruler).	Predict with mental iterations and check with a tool.
<b>Conceptual Ruler Measuring</b>	Operates mentally with units and composite units. Can mentally project a known length along an object to measure or partition an unknown length.	
<b>Integrated Conceptual Path Measuring</b>	Processes perimeter as an integration of sets of units and as a flexibly wrapped single collection of units. Treats unit selection as a variable and anticipates the effects of changes in unit selection when relating sizes.	
<b>Coordinated, Integrated Abstract Measures with Derived Units</b>	Level of argumentation and verification of claims. Coordinates and operates on collections of units, collections of units of units, and on collections of entire paths complex, bent paths. Students may address rates derived units as support for arguments about complex interactions.	

# Illustrate our LT for length with examples:

- E to E: instructional task- take away some blocks from a complete “train”
- URR: instructional task- broken ruler and collections of unit strips contrasted
- LM: instructional task- 218 units, or counting sides inconsistently with entire perimeter.



# Growth Chart

<i>Child's name</i>	Spring Semester, 2008	Grade 2 :				Grade 3 :
	Initial assessment Feb 08	follow-up Mar 08	TE 1 Apr 08	TE 2 May 08	TE 3 May 08	TE 4 Oct 2008
<i>Sara</i>	ILC	EE	EE	EE	EE	(URR)
<i>Anselm</i>	EE	URR	URR	URR	URR	URR
<i>Danny</i>	EE	URR	(LM)	URR	URR	LM
<i>Allie</i>	URR	URR	URR	(LM)	LM	LM
<i>Ollie</i>	URR	URR	URR	URR	(LM)	LM
<i>Rick</i>	URR	URR	LM	LM	LM	LM
<i>David</i>	URR	URR	LM	LM	(LM)	LM
<i>Annette</i>	URR	URR	LM	LM	LM	(CM)

## Situating Eight Grade 2-3 Students within the Trajectory

- End to End up into Unit Repeater  
(student Sara) (May 08 to Oct 08)
- Unit Repeater  
(student Anselm) (Jan 08 to Oct 08)
- Unit Repeater up into Length Measurer  
(six other students) (Feb 08 to May 08)

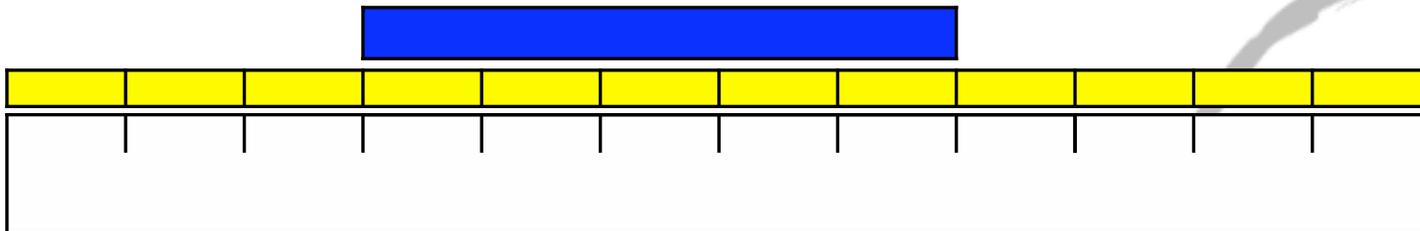
# Finding: LT emphasis on Units supported procedural work

- Focus on unit concept in the Learning Trajectory supported procedural development
- Anselm at URR
- (strips he labeled 4, 3 and 1)



## Finding: LT supported integrating number line and measures

- Abby: For 3 sessions since Jan 08 she has struggled to coordinate point counting and interval counting. In May 08, a breakthrough:



# Finding: The LT met criteria for a theory of levels

- **Four Criteria** (Clements & Battista, 1992):
  - The levels provide adequate distinctions over time (KDU, Simon, 2006)
  - The sequence is invariant and stepwise
  - Each level is integrative of prior levels
  - Each level is coherent
- **Analysis of our Growth Chart**
  - Our accounts of 8 students show distinctions, growth along steps. This finding generally confirms the stepwise progress with short term regressions but long term stability.

# Finding: Prompting Growth

- coordinate continuous quantity and counting in discrete quantity (number sequence)
- Challenge false dichotomy between counting intervals and counting tick marks (endpoints):
  - Decreasing sequence of segments (3, 2 and 1 unit in length) used to prompt ratio and unit image. The ordinal sequence contrasts with cardinality.
  - Juxtaposing interval strip with line of tick marks promoted unit integration.
  - Find length of a strip that obscures a mid-portion of an extended ruler (80 inches) to restore use of number labels at unit endpoints.

# Current Snapshot of Case Students in Grade 4

- Volume: a scaffold, of a partially complete layer in a box, with a single row showing the box height (1560 cubes fill the box) (kids: 60, 300, 700, 1400, 1550) (see Abby case)
- Area: A long tile is used to fill a tabbed rectangular shape (47 long tiles) (kids: 15, 37, 47)
- Length: imposing an estimation with reasoning from inches to feet and (84 inches)
- Path length: finish a partial path to 218, given a broken path up to 150 so far (in parts). (Arielle)

# Research Issues to pursue

- More work to assess children's grasp of continuous and discrete quantity
- Tasks requiring coordination of ordinal and cardinal values on measures support comparative reasoning and ratio concept
- Prompting children to use close transitional forms of rulers bridging from interval sets to tick mark sequences prompted more abstracted unit concepts (units of units)



# Next Steps:

- Extend to Grades 6-8 and follow the Longitudinal sample of students.
- Revise Assessments: diagnosis, instruction and evaluation purposes for tasks and items. Use IRT modeling to examine the LTs on Length, Area, Volume.
- Design instructional tasks to prompt children to integrate measurement and arithmetic knowledge.
- Revisit LT critical levels with classrooms and groups of students, widen the sampling.
- Engage Teachers in Lesson Study analysis of student thinking and strategy growth.

# Further notes...

- We expect that our work on a task sequence will allow students to settle within a scheme for working on a class of tasks, and thus reflect, and hopefully curtail their work.
- By curtailing less sophisticated processes for measuring, or finding a quantity, the students are hypothesized to internalize, and forge more structured, conceptual advances. This can be brought out into the written notations and abstractions.

# and

- We work to promote scientific reasoning (Lehrer):
  - 1. identify variance and invariance under wide ranging cases. We also pay close attention to closely related cases (continuity)
  - 2. checking boundary cases, especially extreme values and simple cases (e.g., 0 and 1)
  - 3. identify dependence and independence within a logical system, including causal relations (so, what is arbitrary and therefore independent, and what is not?)
  - 4. characterize structures based on invariant features (and employ structure to anticipate further structures and abstractions about the structural relations among quantities).



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*Thank you!*

Contact information:

Jeff Barrett

[jbarrett@ilstu.edu](mailto:jbarrett@ilstu.edu)

<http://gse.buffalo.edu/org/triad/nsf>