FIL 404: Working Topic 4 Basic Time Value of Money Problems

Always start with the question: is there a series of payments that are equally spaced apart in time and equal or related in amount? ("Related" would mean changing by a constant percentage from period to period.)

A) If there is no series of payments (or there is a series of unequal/unrelated payments), it is a non-annuity problem (or series of non-annuity problems), structured as

$$\text{BAMT} \times (1 + r)^n = \text{EAMT}$$

B) If there is series of equal or related payments it is annuity problem, structured as

$$\text{PMT} \times \text{FAC} = \text{TOT}$$

1) If TOT (large amount that corresponds to series of small PMTs) exists intact in the present, we have PV of Annuity problem and FAC should be PV of Annuity factor:

$$\text{PMT} \times \left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right) = \text{TOT} \qquad \text{OR} \qquad \text{PMT} \times \left(\frac{1-\left(\frac{1+g}{1+r}\right)^n}{r-g}\right) = \text{TOT}$$

Level — Constant % Change

2) If TOT will not exist intact until a future date, we have FV of Annuity problem and FAC should be FV of Annuity factor:

$$\text{PMT} \times \left(\frac{(1+r)^n-1}{r}\right) = \text{TOT} \qquad \text{OR} \qquad \text{PMT} \times \left(\frac{(1+r)^n-(1+g)^n}{r-g}\right) = \text{TOT}$$

Level — Constant % Change

Multiply either of the ordinary annuity factors shown above, which are consistent with end-of-period PMTs, by (1 + r) if there are beginning-of-period PMTs.

Computing PV of Annuity factor on scientific calculator without writing things down or using calculator memory. Consider PV of Annuity factor for 7%, 13 periods:

$$\left(\frac{1-\left(\frac{1}{1.07}\right)^{13}}{.07}\right) = 8.35765$$

a. Type 1.07, hit y^x key, type 13, and hit = key (should be 2.40985). Then hit 1/x key. Gives you right-hand side of factor's numerator (should be .41496).
b. Instead of subtracting it from 1, subtract 1 from it and then undo resulting negative value. Type – 1 = (should be -.58504), then hit +/– key. Now you have entire numerator of .58404.
c. Divide by typing ÷ .07 = (should get the factor's correct value: 8.35765).