The Distributive Property

The *distributive property* enters time value of money analysis in situations that, in our coverage, we characterize as "annuities" (payments equally spaced in time, equal/related in amount). Per the distributive property,

$$y(a) + y(b) + y(c) = y\,(a + b + c), \quad \textit{e.g.},$$

$$10(2) + 10(4) + 10(7) = 10\,(2 + 4 + 7) = 10\,(13) = 130$$

But $10(2) + 3(4) + 12(7)$ cannot be combined/simplified.

And the distributive property works only with amounts we multiply by, not with those by which we divide:

$$12/6 + 12/3 + 12/4 \text{ [which is } 2 + 4 + 3 = 9]$$

$$\neq 12/(6 + 3 + 4) \text{ [which is } 12/13 = .923]$$

<u>Cash Flow Series Equal to a Big Future Amount</u>

In a "non-annuity" sequence, we must deal with each cash flow separately. If you deposit $100 at the end of year 1, $800 at the end of year 2, and $300 at the end of year 3, how much will you have by the end of year 3 if your account balance earns a 4% annual rate of return?

$$\$100\,(1.04)^2 + \$800\,(1.04)^1 + \$300\,(1.04)^0$$
$$= \$100\,(108.16) + \$800\,(1.04) + \$300\,(1.00)$$
$$= \$108.16 + \$832.00 + \$300.00 = \$\underline{\underline{1,240.16}}$$

What if you instead deposit $100 at the end of years 1 through 3 and the account balance earns a 4% annual rate of return? We can compute this FV of Annuity case as we did for the "non-annuity" series above:

$$\$100\,(1.04)^2 + \$100\,(1.04)^1 + \$100\,(1.04)^0$$
$$= \$100\,(108.16) + \$100\,(1.04) + \$100\,(1.00)$$
$$= \$108.16 + \$104.00 + \$100.00 = \$\underline{\underline{312.16}}$$

But in an annuity case we can also treat the cash flows as a team:

$$\$100\,(1.04)^2 + \$100\,(1.04)^1 + \$100\,(1.04)^0$$

$$= \$100\,[(1.04)^2 + (1.04)^1 + (1.04)^0] = \$100\left(\frac{(1.04)^3 - 1}{.04}\right)$$

$$= \$100\,(3.1216) = \$\underline{\underline{312.16}}$$

- The FV of Annuity factor is just the sum of the so-called FV of $1 factors for the same discount rate and number of cash flow periods.
- An annuity problem is just a group of "non-annuity" problems we can treat as a team when the cash flows are equal or related, because then we can make use of the distributive property.

Cash Flow Series Equal to a Big Present Amount

In a "non-annuity" sequence, we must deal with each CF separately. How much must you have in your account today if you want to withdraw \$200 at the end of year 1, \$700 at the end of year 2, and \$400 at the end of year 3 if your account balance earns a 6% annual rate of return?

$$\$200 \div (1.06)^1 + \$700 \div (1.06)^2 + \$400 \div (1.06)^3$$

$$= \$200\left(\frac{1}{1.06}\right)^1 + \$700\left(\frac{1}{1.06}\right)^2 + \$400\left(\frac{1}{1.06}\right)^3$$

$$= \$200\ (.9434) + \$700\ (.8900) + \$400\ (.8396)$$
$$= \$188.68 + \$623.00 + \$335.85 = \$\underline{\underline{1,147.53}}$$

What if you instead want to withdraw \$200 at the end of each of the three years and the account balance earns a 4% annual rate of return? We can compute this PV of Annuity case as we did with the "non-annuity" payment series above:

$$\$200\left(\frac{1}{1.06}\right)^1 + \$200\left(\frac{1}{1.06}\right)^2 + \$200\left(\frac{1}{1.06}\right)^3$$

$$= \$200\ (.9434) + \$200\ (.8900) + \$200\ (.8396)$$
$$= \$188.68 + \$178.00 + \$167.92 = \$\underline{\underline{534.60}}$$

But in annuity case we can also treat the cash flows as a team:

$$\$200\left(\frac{1}{1.06}\right)^1 + \$200\left(\frac{1}{1.06}\right)^2 + \$200\left(\frac{1}{1.06}\right)^3$$

$$= \$200\left[\left(\frac{1}{1.06}\right)^1 + \left(\frac{1}{1.06}\right)^2 + \left(\frac{1}{1.06}\right)^3\right] = \$200\left(\frac{1-\left(\frac{1}{1.06}\right)^3}{.06}\right)$$

$$= \$200\ (2.6730) = \$\underline{\underline{534.60}}$$

- The PV of Annuity factor is just the sum of the so-called PV of \$1 factors for the same discount rate and number of cash flow periods.
- As stated above, an annuity problem is just a group of "non-annuity" problems we can treat as a team when the cash flows are equal or related, because then we can make use of the distributive property.