**ANALYZING PREFERRED AND COMMON STOCKS: PROBLEMS & SOLUTIONS**

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*This problem set covers all of our preferred and common stock valuation situations. The problems progress in a building-block fashion, with concepts presented in an order that provides a coherent coverage of the topics. Therefore some of the later problems may actually be easier, computationally, than some earlier problems.*

*But you are encouraged to work each problem in order before moving ahead, to facilitate understanding.*

1. Mercury Industries raised money several years ago by issuing preferred stock, with a $100 per share par value. Holders of these preferred shares receive an 8.24% stated annual dividend, paid in a single year-end installment. Based on the risks of providing money to Mercury, rational buyers of these preferred shares require a 9.5% effective annual rate (EAR) of return. What price should a well-informed investor pay for each share of Mercury preferred stock? What if the required effective annual rate of return were instead 11.5%, 7.5%, or 8.24%?

*Type: Preferred Stock Value; Annual Dividends.* As we saw in the earlier discussion of bonds, we can compute the value of any financial asset as the sum of the present values of all expected future periodic cash flows (CFs), discounted for the appropriate number of periods and by the required periodic rate of return. The most general way to state this relationship is

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

(Note that we really are just working with the Net Present Value equation:

NPV = CF0 $\left(\frac{1}{1 + r}\right)^{0}$ + CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

but solving for CF0 — the price we would expect to pay today — and with NPV set equal to $0, which defines the case in which the invested money earns a fair rate of return but nothing extra. (Recall that a $0 NPV outcome is to be expected under very competitive market conditions — and the securities markets in which bonds and shares of stock trade are very competitive, with buyers’ quest to pay as little as possible interacting with sellers’ quest to receive as much as possible to drive prices to a level that gives the buyer a fair expected rate of return, but nothing more.)

We could predict cash flows for the investment’s entire expected life (which, in some cases, could be infinite) and sum the discounted present values. But typically we work with a simplified version of this general equation. For example, when we value coupon-paying bonds the cash flows are the stream of interest payments (an annuity), plus a single amount to be received at the end of the investment period; in zero-coupon bond valuation the only cash flow is the par or other value to be received when the investment ends. Preferred stock, on the other hand, typically is seen as a perpetuity; if the issuing firm expects to pay the same dividend every period, and if it does not state a date when the dividend payments will end, we treat them as though they will be received forever.

Of course, companies can withhold expected preferred dividends if they face cash shortages (whereas interest promised to bond holders is a legally enforceable obligation). Thus while we could identify the computed theoretical value of preferred stock as VP , we also might show it as $\hat{P\_{0}}$, the price we think sensible investors should pay today, at time 0, for preferred (or common) stock, with the “hat” suggesting a guess or prediction made in the face of uncertainty regarding the dividends.

Because our preferred stock examples will involve equal periodic dividends (typically a preferred stockholder receives periodic dividends that are a fixed percentage of the stated par value per share, with no maturity date), we will think of preferred stock’s dividend stream as a perpetuity and will compute each share’s theoretical value with this perpetuity-based simplification of the general asset value equation:

VP = $\hat{P\_{0}}$ = $\frac{D}{r}$

In preferred stock problems we might be told the unchanging periodic per-share dividend D in dollar terms, or might have to compute it by multiplying the dividend percentage times the par value. Here we are told that the periodic (here, annual) dividend is 8.24% of the $100 par, which is $8.24 per year. (Preferred stock issued by U.S. companies often has a $100 par value per share, although other par values, such as $25 or $50, also can be seen.) If the required periodic (annual) return is 9.5%, the indicated value is

VP = $\hat{P\_{0}}$ = $\frac{\$8.24}{.095}$ = $86.74 per share

Preferred stock is, like a bond, a fixed-income security. Recall from our bond coverage that the value of a fixed-income security falls below (rises above) the par value if the required periodic rate of return rises above (falls below) the periodic cash flow percentage. In this example, the 9.5% required annual rate of return exceeds the 8.24% annual dividend rate, so the value of each preferred share is less than the $100 par value. (With dividends paid annually we do not have to distinguish between EARs and APRs.) If the required periodic return were an even higher 11.5%, the estimated value would be an even lower

VP = $\hat{P\_{0}}$ = $\frac{\$8.24}{.115}$ = $71.65 per share

If investors’ required periodic return were less than 8.24%, the value would be higher than the $100 par value; with a 7.5% required periodic (here, annual) return we compute

VP = $\hat{P\_{0}}$ = $\frac{\$8.24}{.075}$ = $109.87 per share

But with a required periodic return equal to the 8.24% periodic cash flow rate, the estimated value should equal the par value:

VP = $\hat{P\_{0}}$ = $\frac{\$8.24}{.0824}$ = $100.00 per share

2. Venus Corporation raised money several years ago by issuing preferred stock, with a $100 per share par value. Holders of these preferred shares receive an 8.24% stated annual dividend, paid in equal quarterly installments. Based on the risks of providing money to Venus, rational buyers of these preferred shares require a 9.9512% effective annual rate (EAR) of return. What price should a well-informed investor be willing to pay for each share of Venus preferred stock? What if the required EAR instead were 11.6792%, 7.3967%, or 8.4981%?

*Type: Preferred Stock Value; Quarterly Dividends.* Again referring back to our bond coverage, recall that the value of any financial asset can be computed as the sum of the present values of all expected subsequent periodic cash flows (CFs), discounted for the appropriate number of periods and by the required periodic rate of return. Again, our most general representation is

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

We could predict cash flows for the investment’s entire (possibly infinite) life and add together the discounted present values. But again we are dealing with preferred stock, with its perpetual stream of expected equal cash flows, and thus can use a perpetuity-based simplified form of the general equation. And as in the prior question, we know that firms can withhold expected preferred dividends, and thus we might identify the computed value of preferred stock either as VP or as $\hat{P\_{0}}$, the per-share price we think sensible investors should pay today for preferred (or common) stock, with the “hat” suggesting an estimate of what the theoretical value should be in the face of uncertainty regarding dividends. Recall that the perpetuity-based simple form of the general asset value equation we use in estimating preferred stock’s value is:

VP = $\hat{P\_{0}}$ = $\frac{D}{r}$

But now the expected dividends are to be paid quarterly (which generally is the actual payment frequency for dividends on preferred stock issued by US-based companies), so our estimate of value must be based on quarterly periodic cash flows and a required quarterly rate of return.

In preferred stock problems we might be told the unchanging periodic dividend D in dollar terms, or might have to compute it by multiplying the specified dividend percentage times the par value. Here the annual dividend is 8.24% of the $100 par, or $8.24 per year, but this total is broken into 4 quarterly payments of $8.24 ÷ 4 = $2.06 each. And here we have quarterly dividend payments, so we must compute the required quarterly periodic rate of return, which we do by “un-compounding” the 9.9512% effective annual rate (EAR) of return: $\sqrt[4]{1.099512}$ – 1 = .024, or 2.4%. With a $2.06 expected periodic (quarterly) dividend and a 2.4% required periodic (quarterly) rate of return, our value estimate for this perpetuity is

VP = $\hat{P\_{0}}$ = $\frac{\$2.06}{.024}$ = $85.83 per share

Because preferred stock is a fixed-income security, its value (like that of bonds) falls below (rises above) the par value if the required periodic rate of return rises above (falls below) the periodic cash flow percentage. In this example, the 2.4% required quarterly rate of return exceeds the 2.06% quarterly dividend rate, so the estimated value of each preferred share is less than the

$100 par value. (With a 2.4% required quarterly rate of return, the required annual percentage rate [APR] of return is .024 x 4 = 9.6%, while the required EAR is, of course, (1.024)4 – 1 = 9.9512%.) If the required quarterly periodic rate of return were an even higher $\sqrt[4]{1.116792}$ – 1 = .028, or 2.8%, the estimated value would be an even lower

VP = $\hat{P\_{0}}$ = $\frac{\$2.06}{.028}$ = $73.57 per share

If investors’ required quarterly periodic return were less than 2.06%, the estimated value would exceed the $100 par value; with a $\sqrt[4]{1.073967}$ – 1 = .018, or 1.8%, required quarterly periodic return we compute

VP = $\hat{P\_{0}}$ = $\frac{\$2.06}{.018}$ = $114.44 per share

But with a required quarterly periodic return of $\sqrt[4]{1.084981}$ – 1 = .0206, or 2.06%, which equals the 2.06% periodic cash flow rate, the estimated value should equal the par value:

VP = $\hat{P\_{0}}$ = $\frac{\$2.06}{.0206}$ = $100.00 per share

3. Earth Enterprises raised money several years ago by issuing preferred stock, with a $100 per share par value. Holders of these preferred shares get a 7.6% stated annual dividend, paid in four equal quarterly installments.

If the Earth preferred currently sells for $80.74 per share, what effective annual rate (EAR) of return would buyers

of the shares seem to be receiving? What if these buyers instead paid $134.99, or $100, per share?

*Type: Preferred Stock; Solving for Rate of Return.* When we estimate the value of preferred stock with an expected stream of unending equal dividend payments, the general asset valuation equation (summing PVs of all expected cash flows) simplifies to a perpetuity:

VP = $\hat{P\_{0}}$ = $\frac{D}{r}$

We can use this equation in solving for any possible unknown. Here we want to measure the effective annual rate of return received by an investor who pays a specified per-share price for preferred stock. The total annual dividend is 7.6% x $100 = $7.60, so with quarterly payments

the periodic cash flow is $7.60 ÷ 4 = $1.90. A buyer who pays $80.74 per share and expects to

get $1.90 in dividends every quarter expects a percentage return computed as follows:

VP = $\hat{P\_{0}}$ = $\frac{D}{r}$

$80.74 = $\frac{\$1.90}{r}$

r = $1.90 ÷ $80.74 = .023532

This computed r, of course, is the expected periodic (here, quarterly) rate of return. We annualize this quarterly return either by multiplying it times the number of periods in a year to get an annual percentage rate, or APR (here .023532 x 4 = 9.4130%, a non-compounded figure we were not asked to compute); or else by compounding out as (1.023532)4 – 1 = 9.75%. But a buyer paying a higher $134.99 per share and expecting $1.80 in dividends every quarter expects a lower percentage return of:

$134.99 = $\frac{\$1.90}{r}$

r = $1.90 ÷ $134.99 = .014075

Again this r is the expected periodic (here, quarterly) rate of return, which we annualize as an APR (.014075 x 4 = 5.6298%, which does not take into account intra-year compounding) or as an EAR of (1.014075)4 – 1 = 5.75% (a figure that does include intra-year compounding). Finally, someone paying $100 per share and expecting $1.90 in dividends every quarter expects a percentage return of:

$100 = $\frac{\$1.90}{r}$

r = $1.90 ÷ $100 = .019000

Again we annualize this expected quarterly periodic rate of return as an APR (.019000 x 4 = 7.6%) or as a compounded EAR of (1.019000)4 – 1 = 7.8194%. Note that in this final case a buyer who pays the $100 par value earns an APR equal to the 7.6% stated dividend percentage rate (but the EAR is higher, just like the EAR you earn on a savings account exceeds the stated APR if there is more than one interest compounding period per year).

4. Rational investors who buy shares of preferred stock issued by Mars Manufacturing ($100 per share par value) require a 7.1859% effective annual rate (EAR) of return. If the Mars preferred stock currently sells for $96.00

per share, what total annual dividend (paid in quarterly installments) would the company seem to be paying?

What if sensible investors instead required an 8.0311% EAR? What if the shares instead sold for $104.00?

*Type: Preferred Stock; Solving for Dividend.* Once again we use the simplified form of the general valuation equation that applies to preferred stock (or any investment) with perpetual payments:

VP = $\hat{P\_{0}}$ = $\frac{D}{r}$

This is an admittedly unusual problem, in that the unknown is the annual dividend (whereas in dealing with preferred stock, as with bonds, typically we compute either the appropriate price to pay or else the rate of return expected by someone who pays a specified price). But the valuation equation is our guide to solving for any unknown. Here we want to measure the number of dollars that represents a given periodic rate of return. We are told the effective annual rate (EAR) of return required by the buyer, but with dividends paid quarterly we must think in quarterly terms, “un-compounding” the EAR to get a quarterly periodic rate as our working number. A 7.1859% EAR corresponds to a quarterly periodic rate of $\sqrt[4]{1.071859}$ – 1 = .0175, or 1.75%.

The quarterly dividend payment that gives a 1.75% quarterly periodic rate of return is:

VP = $\hat{P\_{0}}$ = $\frac{D}{r}$

$96.00 = $\frac{D}{.0175}$ ; so D = $96.00 x .0175 = $1.68

This quarterly dividend would result in a $1.68 x 4 = $6.72 total dividend each year. (If the par value is $100 per share, the stated annual dividend percentage, which is an APR measure, would be 6.72% of par.) The dividend per quarter that provides a higher quarterly periodic return of $\sqrt[4]{1.080311}$ – 1 = .0195, or 1.95% is:

$96.00 = $\frac{D}{.0195}$; so D = $96.00 x .0195 = $1.87 ,

or a $1.87 x 4 = $7.48 total dividend each year. (With a $100 par value, the stated annual dividend percentage would be an APR of 7.48% of par.) Finally, if the price were $104 rather than $96, a 7.1859% EAR (for a 1.75% quarterly periodic rate, as shown above) would require a dividend greater than $1.68 per quarter, specifically,

$104.00 = $\frac{D}{.0175}$; so D = $104.00 x .0175 = $1.82 ;

while at a $104 price an 8.0311% EAR (for a 1.95% quarterly periodic rate, as shown above) would require a dividend greater than $1.87 quarterly:

$104.00 = $\frac{D}{.0195}$; so D = $104.00 x .0195 = $2.03

The annual dividend totals would be $1.82 x 4 = $7.28 and $2.03 x 4 = $8.12, respectively.

5. Jupiter International’s business is based on older technologies that would not be cost-effective to update. The company is expected to go out of business 7 years from today. Jupiter should maintain a fairly steady revenue stream during its remaining years (largely from servicing equipment it has sold in the past); analysts project that common stockholders will receive regular cash dividends from operations of $3.85 per share each year, along with

a liquidating dividend of $13.00 per share at the end of year 7. If the risk of providing money to Jupiter as common stockholders causes rational investors to require a 10.5% effective annual rate (EAR) of return, what price should people be willing to pay for shares of Jupiter common stock? What if the required EAR were 7.5%, or 13.5%?

*Type: Common Stock Value; Finite Dividend Stream.* As we have seen repeatedly, the value of any financial asset can be computed as the sum of the present values of all expected future periodic cash flows, discounted for the appropriate number of periods and by the required periodic rate of return. Once again, the most general way to state this relationship is

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

but we typically work with a simplified version. In the example at hand, we have a finite number

of expected dividends (the case is unusual, in that the firm is expected to go out of business at

a specified point in time, whereas usually we think of companies as having infinite lives and thus common stockholders’ stream of expected dividend payments as being never-ending).

Of course, the dividends that common stockholders expect to receive may not actually be realized. Unlike with bonds or preferred stock, nothing is promised regarding the amount of cash that will be paid as dividends to common stockholders each period. Investors have expectations, and managers of the issuing firm might talk about the dividends they expect to pay, but what is paid as dividends to common stockholders ultimately rests on a range of issues: how much profit the firm ends up earning, the portion of that income management wants to retain for internal use, federal income

tax policies, company traditions, stockholders’ preferences as perceived by management, and management’s own preferences. Thus while we could identify the computed value of common stock as VS, we also might show it as $\hat{P\_{0}}$, the price we think investors ought to pay today, at time 0, for common (as with preferred) stock, with the “hat” suggesting a guess or estimate of the theoretically correct value made in the face of uncertainty regarding the future dividends.

With the finite stream of seven cash flows (here they are dividends, so we replace the general CF designation with the more specific D) expected — $3.85 regular dividends in each of years 1 through 6, and then $3.85 + the $13.00 liquidating dividend = $16.85 in year 7 — we could compute the price we feel investors should pay for Jupiter’s common shares as:

VS = $\hat{P\_{0}}$ = D1 $\left(\frac{1}{1 + r}\right)^{1}$ + D2 $\left(\frac{1}{1 + r}\right)^{2}$ + D3 $\left(\frac{1}{1 + r}\right)^{3}$ + D4 $\left(\frac{1}{1 + r}\right)^{4}$ + D5 $\left(\frac{1}{1 + r}\right)^{5}$ + D6 $\left(\frac{1}{1 + r}\right)^{6}$ + D7 $\left(\frac{1}{1 + r}\right)^{7}$

= $3.85 $\left(\frac{1}{1.105}\right)^{1}$ + $3.85 $\left(\frac{1}{1.105}\right)^{2}$ + $3.85 $\left(\frac{1}{1.105}\right)^{3}$ + $3.85 $\left(\frac{1}{1.105}\right)^{4}$ + $3.85 $\left(\frac{1}{1.105}\right)^{5}$ + $3.85 $\left(\frac{1}{1.105}\right)^{6}$

+ $16.85 $\left(\frac{1}{1.105}\right)^{7}$= $\frac{\$3.85}{\left(1.105\right)^{1}}$ + $\frac{\$3.85}{\left(1.105\right)^{2}}$ + $\frac{\$3.85}{\left(1.105\right)^{3}}$ + $\frac{\$3.85}{\left(1.105\right)^{4}}$ + $\frac{\$3.85}{\left(1.105\right)^{5}}$ + $\frac{\$3.85}{\left(1.105\right)^{6}}$ + $\frac{\$16.85}{\left(1.105\right)^{7}}$

= $3.48 + $3.15 + $2.85 + $2.58 + $2.34 + $2.11 + $ 8.38 = $24.89 per share

Recall that companies typically pay dividends to common (and preferred) stockholders quarterly. Yet we use the expected full-year dividend as the periodic cash flow and the full EAR as the periodic discount rate. Why? Computationally, we treat common stock as though it had annual dividend payments even though they actually are made quarterly, because future cash dividends for

common stockholders are so uncertain; it makes no sense to strive for computational precision when the input figures are subject to much guess work (whereas payments to bond and preferred stock investors are specified in advance). [Thus we need not distinguish between APRs and EARs when analyzing common stock.] In this problem, common stockholders expect payments that look a lot

like the payment pattern for bonds with annual coupon interest payments: equal amounts in years

1 – 6, and then a larger ending payment in year 7. We can get the answer shown above with an abbreviated version of our general valuation equation:

VS = $\hat{P\_{0}}$ = $3.85 $\left(\frac{1-\left(\frac{1}{1.105}\right)^{6}}{.105}\right)$ + $16.85 $\left(\frac{1}{1.105}\right)^{7}$

= $3.85 (4.292179) + $16.85 (.497123)

= $16.52 + $8.38 = $24.90 per share

(a 1¢ rounding difference from above). We also could group the last year’s expected regular dividend with the rest of the regular dividend stream and handle the $13.00 liquidating payment separately:

VS = $\hat{P\_{0}}$ = $3.85 $\left(\frac{1-\left(\frac{1}{1.105}\right)^{7}}{.105}\right)$ + $13.00 $\left(\frac{1}{1.105}\right)^{7}$

= $3.85 (4.789303) + $13.00 (.497123)

= $18.44 + $6.46 = $24.90 per share

As with all the cases we have seen before, investors pay a higher (lower) price to cause a given stream of expedcted payments to represent a lower (higher) periodic rate of return. If investors’ required periodic (for common stock, annual) rate of return were only 7.5%, then they would willingly pay a higher

VS = $\hat{P\_{0}}$ = $3.85 $\left(\frac{1-\left(\frac{1}{1.075}\right)^{7}}{.075}\right)$ + $13.00 $\left(\frac{1}{1.075}\right)^{7}$

= $20.39 + $7.84 = $28.23 per share

(a price not drastically higher than the $24.90 from the 11.5% case because there are only a few years of cash flows expected). Investors requiring a higher 13.5% annual return would insist on paying a lower

VS = $\hat{P\_{0}}$ = $3.85 $\left(\frac{1-\left(\frac{1}{1.135}\right)^{7}}{.135}\right)$ + $13.00 $\left(\frac{1}{1.135}\right)^{7}$

= $16.77 + $5.36 = $22.13 per share

6. Saturn Company, which has been in existence for several years, has never paid a cash dividend to its common stockholders, and the company managers have announced no plan to pay dividends in the future. How can this stock have any value as an investment? If Mr. and Mrs. Back O’Line believed that they could sell Saturn common stock in three years for a price of $136 per share, and if they require a 9.45% effective annual rate (EAR) of return based on the risk of providing money to Saturn, what should they be willing to pay today to buy the often-purchased “round lot” of 100 shares? What if they instead required an EAR of 8.35%, or 11.25%, or 12.75%, or 6.15%? o

*Type: Common Stock Value; Zero Dividends Expected.* Once again we can base our analysis on an abbreviated version of the general valuation equation

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

Here the computational portion of the analysis involves just the present value of a single dollar amount, as with a zero-coupon bond or a collectible item. Someone who expected to collect a $136 cash flow (in the form of a resale price) in 3 years, and required that amount to represent a 9.45% effective annual rate of return, should be willing to pay a price no higher than

VS = $\hat{P\_{0}}$ = $136 $\left(\frac{1}{1.0945}\right)^{3}$ = $103.73 per share, or $10,373 for 100 shares

A lower required annual rate of return, such as 8.35%, would justify a higher price:

VS = $\hat{P\_{0}}$ = $136 $\left(\frac{1}{1.0835}\right)^{3}$ = $106.92 per share, or $10,692 for 100 shares

A higher required annual rate of return, such as 11.25%, would prompt offering a lower price:

VS = $\hat{P\_{0}}$ = $136 $\left(\frac{1}{1.1125}\right)^{3}$ = $98.77 per share, or $9,877 for 100 shares

Even more extreme high or low effective annual returns of 12.75% and 6.15% would be consistent with respective values of

VS = $\hat{P\_{0}}$ = $136 $\left(\frac{1}{1.1275}\right)^{3}$ = $94.88 per share, or $9,488 for 100 shares and

VS = $\hat{P\_{0}}$ = $136 $\left(\frac{1}{1.0615}\right)^{3}$ = $113.70 per share, or $11,370 for 100 shares

Here the computations are simple, but what about the underlying concept: how could stock have a positive value if its holders never expected to receive cash dividends? The answer is that it would not; someone has to expect to receive dividends sometime. Perhaps positive dividends are not expected to begin until far in the future; such a situation would not be inconsistent with our general valuation equation (projected cash flows would simply be $0 until some distant future date):

VS = $\hat{P\_{0}}$ = $0 $\left(\frac{1}{1 + r}\right)^{1}$ + $0 $\left(\frac{1}{1 + r}\right)^{2}$ + $0 $\left(\frac{1}{1 + r}\right)^{3}$ + … + D25 $\left(\frac{1}{1 + r}\right)^{25}$ + D26 $\left(\frac{1}{1 + r}\right)^{26}$ + … + Dn $\left(\frac{1}{1 + r}\right)^{n}$

So a particular investor might not expect to receive dividends during his/her own holding period, but the price that prevails in the market at a given time should be the PV, at that date, of all expected subsequent dividend payments — even if the first nonzero dividend is not expected to be paid until many years later. Here the stock can be expected to sell for $136 in three years only if $136 will be the present value, three years from now, of all dividends expected to be received sometime after the end of year 3. To find the value today of a series of payments that will have a $136 PV three periods (here, years) from now, we discount that $136 amount for 3 periods (years), at a discount rate that reflects investors’ required periodic (here, effective annual) rate of return.

7. Uranus, Ltd., which has been in existence for several years, has never paid a cash dividend to its common stockholders. However, the company’s management team has announced that it intends to begin paying dividends in year 6, after the major start-up costs have been paid for and operations have stabilized. Assume that the risk of providing money to Uranus as a common stockholder calls for a 10.65% effective annual rate (EAR) of return. If the dividends are projected to be $.65 per share each quarter ($2.60 per year) in each of years 6 through infinity, what should rational investors be willing to pay today to buy 100 shares (the often-purchased “round lot” trading unit)? What if the dividends were expected to start at $2.60 in year 6, and then grow by a rate approximating a constant 4.5% forever into the future? o

*Type: Common Stock Value; Deferred Dividend Stream.* Again we can compute value as the sum of the present values of all expected future periodic cash flows, discounted for the right number of periods and by the required periodic rate of return. As typically is the case, we work with a simplified version of the general valuation equation

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

Common (or preferred) stock’s cash flows are dividends, so we replace the general CF notation in the equation with the more specific D. And as noted earlier, common stockholders typically get cash dividends quarterly, but in our analysis we combine 4 quarterly payments for an annual total because the uncertain estimates do not merit greater computational precision. If dividends are expected to be paid forever, in theory we would have to estimate the expected dollar amounts of an endless stream of payments. In practice we could project the magnitudes of a large finite number of dividend payments, such as 10,000 (the PV of anything to be received 10,000 — or even 100 — years into the future tends to be so small that ignoring it has little impact on our final estimate). So now we can think of the general form of the common stock valuation equation as

VS = $\hat{P\_{0}}$ = D1 $\left(\frac{1}{1 + r}\right)^{1}$ + D2 $\left(\frac{1}{1 + r}\right)^{2}$ + D3 $\left(\frac{1}{1 + r}\right)^{3}$ + D4 $\left(\frac{1}{1 + r}\right)^{4}$ + D5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + D10,000 $\left(\frac{1}{1 + r}\right)^{10,000}$

And because multiplying by $\left(\frac{1}{1 + r}\right)^{n}$ is the same as dividing by (1 + r)n, a more efficient way to state the general common stock valuation equation is

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

If dividends were expected to be received forever, and were not expected to follow a convenient pattern, we would have to use this form of the general value equation. But a projected pattern means that an even more abbreviated form of the general value equation can be used. Whereas in problem 6 we expected future dividends after a deferral period but had no specific way to quantify them (we knew only their expected present value 3 years from today), here we expect a specific dividend stream following a five-year deferral period. Let’s begin with the case of expected dividends of $2.60 per share in years 6 through infinity and a 10.65% required annual return. We could plug the expected dividend and return values into the above equation and find

VS = $\hat{P\_{0}}$ = $\frac{\$0}{\left(1.1065\right)^{1}}$ + $\frac{\$0}{\left(1.1065\right)^{2}}$ + … + $\frac{\$0}{\left(1.1065\right)^{5}}$ + $\frac{\$2.60}{\left(1.1065\right)^{6}}$ + $\frac{\$2.60}{\left(1.1065\right)^{7}}$ + … + $\frac{\$2.60}{\left(1.1065\right)^{10,000}}$

Of course, what we ultimately have is a series of equal $2.60 dividends. If the first were expected at the end of the current period, the series would have a simple perpetuity and we could compute the present value as:

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$ = $\frac{\$2.60}{.1065}$ = $24.41

But the series is not expected to begin until year 6 (after five years have passed), so it is worth less than if it were expected to begin in the current year. As with any other financial value that will not exist for five years, we find its value today by discounting for five years:

VS = $\hat{P\_{0}}$ = $\left(\frac{D}{r}\right)\left(\frac{1}{1 + r}\right)^{n}$ = $\left(\frac{\$2.60}{.1065}\right)\left(\frac{1}{1.1065}\right)^{5}$ = $14.72 per share

or $1,472 for 100 shares. (This technique is akin to the “discount the annuity” approach for solving deferred annuity problems.) What if the dividends are expected to start at $2.60 in year 6 and then grow in a manner that could be modeled as a 4.5% constant annual rate? We could plug the expected dividend and return values into the general valuation equation and find

VS = $\hat{P\_{0}}$ = $\frac{\$0}{\left(1.1065\right)^{1}}$ + $\frac{\$0}{\left(1.1065\right)^{2}}$ + … + $\frac{\$0}{\left(1.1065\right)^{5}}$ + $\frac{\$2.60}{\left(1.1065\right)^{6}}$ + $\frac{\$2.60(1.045)}{\left(1.1065\right)^{7}}$ + … + $\frac{\$2.60(1.045)^{9,994}}{\left(1.1065\right)^{10,000}}$

Of course, what we have here ultimately is a perpetuity with cash flows changing by a constant percentage rate. If dividends were expected to begin in the current year (such that we would combine the year’s four expected quarterly dividends into an annual total and treat that total as though expected at the end of this year), we could compute the perpetuity’s value with the equation

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{\$2.60}{.1065 - .045}$ = $\frac{\$2.60}{.0615}$ = $42.28

[This “constant dividend growth model” is a widely used financial formula. In our weighted average cost of capital discussion we measured the cost of providing fair returns to common stockholders, in a situation with dividends expected to grow by a constant percentage rate per period, as

kS (or r) = $\frac{D\_{1}}{P\_{0}}$ + g

If we rearrange this relationship algebraically to solve instead for P0, we find

r = $\frac{D\_{1}}{P\_{0}}$ + g ⇒ r - g = $\frac{D\_{1}}{P\_{0}}$ ⇒ $P\_{0 }$(r – g) = $D\_{1 }$ ⇒ $P\_{0 }$= $\frac{D\_{1}}{r - g}$ .]

But the series is not expected to begin until year 6 (after five years have passed), so its present value is lower than if the series were expected to begin this year. As with any other financial value that will not exist for five years, we find its magnitude today by discounting for five years:

VS = $\hat{P\_{0}}$ = $\left(\frac{D\_{1}}{r - g}\right)\left(\frac{1}{1 + r}\right)^{n}$ = $\left(\frac{\$2.60}{.1065 - .045}\right)\left(\frac{1}{1.1065}\right)^{5}$ = $\left(\frac{\$2.60}{.0615}\right)\left(.602897\right)$ = $25.49 per share,

or $2,549 for 100 shares. Note that the price we think investors should willingly pay for a firm’s common stock increases as a greater stream of dividends is predicted (e.g., growing, as in the second part of the problem [$25.49], instead of level, as in the first part [$14.72]). Note also that no analyst would likely expect dividends to stay level or change in a constant manner until infinity; the constant dividend growth model is an approximation used for computational convenience.

***Questions 8 through 13 are based on the following information*.** Sally Ride, senior investment manager for the Neptune Mutual Fund, is trying to decide whether to purchase shares of Pluto Company’s common stock for the Neptune portfolio. She assigns six analysts to estimate a fair price to pay for a share of the stock. Pluto paid its common stockholders a quarterly per-share dividend of $1.14 over the past year (so we treat D0 as $1.14 x 4 = $4.56). All analysts at Neptune agree that an appropriate effective average annual rate of return for holding Pluto common stock is 12.25%. However, each of the six analysts has a different view of future dividend payments.

8. Analyst Neil Armstrong does not think Pluto’s per-share dividend will change in the foreseeable future by even one small step; in other words, he feels it will remain at something close to the recent $4.56 level far enough into the future that he will model the expected dividend stream as a perpetuity. What is the highest price Armstrong would recommend paying for each share of Pluto Company common stock? What if the required effective annual rate of return instead were 10.25%?

*Type: Common Stock Value; Constant Dividend Payment.* We should by now be comfortable with the general equation for computing/estimating the values of financial assets that provide expected cash flow streams:

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

Because common stock’s expected cash flows are dividends, we can replace the CF notation with the more specific D, and can approximate the present value of an infinite expected stream by going out a long time into the distant future, perhaps 10,000 years:

VS = $\hat{P\_{0}}$ = D1 $\left(\frac{1}{1 + r}\right)^{1}$ + D2 $\left(\frac{1}{1 + r}\right)^{2}$ + D3 $\left(\frac{1}{1 + r}\right)^{3}$ + D4 $\left(\frac{1}{1 + r}\right)^{4}$ + D5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + D10,000 $\left(\frac{1}{1 + r}\right)^{10,000}$

or, with less clutter:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

So we could predict cash flows essentially for the investment’s entire life and sum their discounted present values. If we truly expected an infinite stream, we could sum the present values of the predicted dividends going out several thousand years as an approximation, since the present value of anything to be received in the far, far distant future is so close to $0 that we can ignore it without materially affecting our value estimate. For example, here our analyst could compute

VS = $\hat{P\_{0}}$ = $4.56 $\left(\frac{1}{1.1225}\right)^{1}$ + $4.56 $\left(\frac{1}{1.1225}\right)^{2}$ + $4.56 $\left(\frac{1}{1.1225}\right)^{3}$ + … + $4.56 $\left(\frac{1}{1.1225}\right)^{10,000}$

= $\frac{\$4.56}{\left(1.1225\right)^{1}}$ + $\frac{\$4.56}{\left(1.1225\right)^{2}}$ + $\frac{\$4.56}{\left(1.1225\right)^{3}}$ + $\frac{\$4.56}{\left(1.1225\right)^{4}}$ + $\frac{\$4.56}{\left(1.1225\right)^{5}}$ + $\frac{\$4.56}{\left(1.1225\right)^{6}}$ + … + $\frac{\$4.56}{\left(1.1225\right)^{10,000}}$

Indeed, if the cash flows are not expected to follow a nice pattern, we have no choice but to use this general equation. But if we can work with a convenient pattern, then an abbreviated form of the general formula becomes useful. A stream of anticipated dividends expected to remain at the same level for a prolonged period can be modeled as a perpetuity, and we can achieve the result of the general asset valuation formula by computing:

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$ = $\frac{\$4.56}{.1225}$ = $37.22 per share

Armstrong is unlikely to believe that the annual dividend per share will never, ever change; he simply thinks they will stay at the most recent level for a long time and thus finds it practical to compute with the perpetuity formula. Thus he feels the Neptune fund should willingly pay a price no higher than $37.22 per share for Pluto common stock. Here we identify the dividend simply as D, without a time-period subscript, because we compute as though the yearly dividend were never expected to change from the most recently observed $4.56 level. Note also that in estimating common stock value we discount full-year dividend totals by an effective annual rate of return, rather than using the quarterly cash flow and rate of return figures. Since projecting dividends for common stockholders is subject to so much error, it makes no sense to strive for greater accuracy in “crunching the numbers” (whereas with bonds and preferred stock the interest/dividend payments are specified in advance, so with greater confidence in the cash flow estimates it makes sense to discount on a more accurate semiannual or quarterly basis).

If the expected average annual return were only 10.25%, Armstrong would be comfortable buying the long-term expected stream of $4.56 dividend payments he foresees for a price as high as:

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$ = $\frac{\$4.56}{.1025}$ = $44.49 per share

9. Analyst Leia Vader-Solo feels that some unseen force will cause Pluto’s annual per-share dividend to rise fairly steadily into the distant future, by a magnitude she finds it practical to model as a constant 2.5% per year. What is the highest price Vader-Solo would recommend paying for each share of Pluto Company common stock? What if the required effective average annual rate of return instead were 10.25%? What if instead she predicted a reasonably steady dividend growth rate that could be modeled as a constant 14% annually?

*Type: Common Stock Value; Constant Positive Rate of Dividend Growth.* It is unlikely that any analyst would expect the dividends paid to some company’s common stockholders to grow by exactly 2.5% per year forever. But this analyst expects them to grow into the foreseeable future in a manner that can be modeled, for practical computational purposes, as a 2.5% constant annual rate (noting, as we did in the previous problem, that dividends expected far into the distant future have present values so small that the dollar amounts we assign them will have no meaningful impact on the per-share values computed). So we could predict the yearly dividends based on that 2.5% growth pattern for a long period into the future and then sum the present values. Here we must use time period subscripts to denote the periods to which we treat various dividends as applying:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

= $\frac{\$4.67}{\left(1.1225\right)^{1}}$ + $\frac{\$4.79}{\left(1.1225\right)^{2}}$ + $\frac{\$4.91}{\left(1.1225\right)^{3}}$ + $\frac{\$5.03}{\left(1.1225\right)^{4}}$ + $\frac{\$5.16}{\left(1.1225\right)^{5}}$ + $\frac{\$5.29}{\left(1.1225\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1.1225\right)^{10,000}}$

(with D0 = $4.56, we estimate D1 as D0 x 1.025 = $4.56 x 1.025 = $4.67; D2 is D0 x 1.0252 = $4.56 x 1.050625 = $4.79; D10,000 would be $4.56 x 1.02510,000, a huge value). Indeed, if the cash flows were not expected to follow any type of pattern we would have no choice but to use this general equation. But with an expected dividend stream that can be modeled/approximated as a convenient pattern, we can use an abbreviated version of the general formula. Here the analyst plans to compute by treating dividends as though they will rise at a 2.5% constant annual rate forever, beginning with the coming year. A shortened form of the general asset value equation that can be used with a constantly changing perpetual cash flow stream is

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1.025)}{.1225 - .025}$ = $\frac{\$4.67}{.0975}$ = $47.90 per share

(this equation is explained in question 7 above). Note that D1 is our needed working number

for the equation’s numerator. In some cases a D1 estimate might be given, but in other cases (as here) we must compute a D1 estimate by increasing a known/verifiable D0 figure by the percentage treated as the constant expected growth rate. Vader-Solo, who is more optimistic about Pluto’s future than is Armstrong, would willingly pay a higher price per share ($47.90 vs. $37.22). If Vader-Solo felt Pluto common stockholders required only a 10.25% average annual return, she would be comfortable seeing the fund pay a higher price for the growing dividend stream, up to

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1.025)}{.1025 - .025}$ = $\frac{\$4.67}{.0775}$ = $60.26 per share

It should make sense that a given stream of dividends would represent a lower rate of return if

a higher price were paid to receive the stream (or that those content with a lower return would willingly pay a higher price for the same expected stream). But if expected growth were modeled as a 14% constant annual rate, we would get a nonsense answer for either the r = .1225 or r = .1025 case:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1.14)}{.1225 - .14}$ = $\frac{\$5.20}{-.0175}$ = -$297.14 per share or

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1.14)}{.1025 - .14}$ = $\frac{\$5.20}{-.0375}$ = -$138.66 per share

If each year dividends grew proportionally by more than the expected average annual rate of return, then growth alone would provide more than the stockholder requires, and no price would be too high to pay (thus the negative value of the answer). We know that such an outcome could not be true. The problem here is that growth should make up part of the expected/required rate of return, so the expected periodic rate of return must be greater than the long-term expected average periodic growth rate. The expected dividend growth rate could exceed the expected long-term rate of return only for a small number of years (as we will see in a later problem).

10. Analyst Will Robinson fears there is a danger that Pluto’s annual per-share dividend will *decline* fairly steadily in the coming years. If he models the change’s magnitude as a constant -1.5% annually forever into the future, what is the highest price Robinson would recommend paying for each share of Pluto Company common stock? What if instead he predicted a dividend change that could be modeled as a constant -6.5% annually forever? What if the required average effective annual rate of return instead were 10.25%?

*Type: Common Stock Value; Constant Negative Rate of Dividend Growth.* Again we know no analyst would be likely to foresee dividends truly changing by a constant annual rate forever into the future, but this analyst expects change over a fairly long period that can be modeled, for practical purposes in computing, as a constant annual –1.5% percentage rate. Again we could predict yearly dividends for a long period into the future and then sum the present values, and would have to use time period subscripts to denote the periods to which various dividends are treated as applying:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

= $\frac{\$4.49}{\left(1.1225\right)^{1}}$ + $\frac{\$4.42}{\left(1.1225\right)^{2}}$ + $\frac{\$4.36}{\left(1.1225\right)^{3}}$ + $\frac{\$4.29}{\left(1.1225\right)^{4}}$ + $\frac{\$4.23}{\left(1.1225\right)^{5}}$ + $\frac{\$4.16}{\left(1.1225\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1.1225\right)^{10,000}}$

(with D0 = $4.56 we find D1 as D0 x .985 = $4.56 x .985 = $4.49; then D2 is D0 x .9852 = $4.56 x .970225 = $4.42; D10,000 would be $4.56 x .98510,000, an extremely small value). Indeed, if we could not treat the cash flows as following a nice pattern, we would have to use this general equation, but approximating the expected dividend change as a 1.5% constant annual decline forever, beginning with the coming year, we can obtain the same result with a shortened form of the general formula. Using the constant dividend growth model we compute

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1-.015)}{.1225 - (-.015)}$ = $\frac{\$4.56(.985)}{.1225 + .015}$ = $\frac{\$4.49}{.1375}$ = $32.65 per share

Robinson, who is less optimistic about Pluto’s future than is Armstrong, would insist on paying a lower price per share ($32.65 vs. $37.22). If Robinson were even more pessimistic, expecting dividends to decline by a higher magnitude that could be modeled for practical purposes as a constant 6.5% per year, then the price he would willingly pay would be a much lower

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1-.065)}{.1225 - (-.065)}$ = $\frac{\$4.56(.935)}{.1225 + .065}$ = $\frac{\$4.26}{.1875}$ = $22.72 per share

Stock with a declining expected dividend stream is not worthless; the investor still expects to get dividends — just not as high as with a stable or growing expected stream. The more pessimistic the analyst is regarding the future dividend stream, the less he or she would recommend paying for each share ($22.72 vs. $32.65 above). A practical problem is that dividends declining by a constant percentage rate eventually get so small that it might be awkward to pay dividends (with a constant 6.5% annual decline, the dividend 150 years from now would be $4.56 x .935150 = about 1/50 of a cent per share per year), though fractional cents could be credited to someone’s brokerage account.

If Pluto common stockholders required only a 10.25% average annual return, Robinson would be comfortable seeing the fund pay a slightly higher price for the declining dividend stream, up to

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1-.015)}{.1025 - (-.015)}$ = $\frac{\$4.56(.985)}{.1025 + .015}$ = $\frac{\$4.49}{.1175}$ = $38.21 per share or

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1-.065)}{.1025 - (-.065)}$ = $\frac{\$4.56(.935)}{.1025 + .065}$ = $\frac{\$4.26}{.1675}$ = $25.43 per share

depending on whether he modeled the dividend growth rate to be a constant –1.5% or –6.5%.

11. Analyst Judy Jetson foresees a slightly rosier future for Pluto than Armstrong or Robinson, predicting that the annual per-share dividend will remain at $4.56 for three years, and then increase forever into the future in a fairly steady manner that can be modeled as a constant 2.5% per year. What is the highest price Jetson would recommend paying for each share of Pluto Company common stock? What if the required effective annual rate of return were instead to average 10.25%?

*Type: Common Stock Value; Long Period of Constant Growth Following Short Period of Zero Growth.* Now our analyst expects a level per-share dividend payment for a few years, followed by growth at a fairly steady rate subsequently into the future. As always, we could use our less cluttered common stock version of the general asset valuation equation:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

Here dividends are expected to remain unchanged from their recent level for 3 more years, and then grow in a manner Jetson models as a constant 2.5% per year thereafter. Thus she computes based on this dividend stream: D1 of D0 x 1.00 = $4.56 x 1.00 = $4.56; D2 of D1 x 1.00 = $4.56 x 1.00 = $4.56; D3 of $4.56 x 1.00 = $4.56; D4 of $4.56 x 1.025 = $4.67; D5 of $4.67 x 1.025 = $4.79; D6 would be 2.5% greater than D5 and D7 2.5% greater than D6; D10,000 would be $4.56 x (1.025)9,997, an extremely large value. Plugging into the general valuation formula, we would try to compute

VS = $\hat{P\_{0}}$ = $\frac{\$4.56}{\left(1.1225\right)^{1}}$ + $\frac{\$4.56}{\left(1.1225\right)^{2}}$ + $\frac{\$4.56}{\left(1.1225\right)^{3}}$ + $\frac{\$4.67}{\left(1.1225\right)^{4}}$ + $\frac{\$4.79}{\left(1.1225\right)^{5}}$ + … + $\frac{D\_{10,000}}{\left(1.1225\right)^{10,000}}$

Indeed, if cash flows were not expected to follow any kind of pattern, we would have to use this general equation, a very long and tedious process. Here we might want to treat the first three years as though they have no pattern, computing the total present value of D1 through D3 as

$\frac{\$4.56}{\left(1.1225\right)^{1}}$ + $\frac{\$4.56}{\left(1.1225\right)^{2}}$ + $\frac{\$4.56}{\left(1.1225\right)^{3}}$ = $4.06 + $3.62 + $3.22 = $10.90

Of course, because these three equal predicted payments constitute a level annuity we could more quickly find their combined PV as

PMT x FAC = TOT

$4.56 $\left(\frac{1-\left(\frac{1}{1.1225}\right)^{3}}{.1225}\right)$ = $10.90

So $10.90 is the value today of the right to collect $4.56 x 3 = $13.68 in expected dividends over years 1 — 3. What about the value today of the right to collect expected dividends in years 4 to infinity? Again, we could sum the present values of individual predicted dividends shown from year 4 through, say, 10,000 (with the present value of any dividend expected after year 10,000 being

so close to zero that we can ignore it without serious impact on our value estimate). But if the dividends are modeled to follow a convenient expected pattern starting in year 4, we can get the same result with a variation on the constant dividend growth model.

Let’s move forward in time to the day when Jetson treats constant dividend growth as setting in, the start of year 4 (which is 3 years away). The value she views the stock as having in 3 years, when the first subsequent dividend to be received will be D4 = $4.67 (which then will be seen as D1, and which includes the 2.5% growth rate) is

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$4.56(1.025)}{.1225 - .025}$ = $\frac{\$4.67}{.0975}$ = $47.90

Our analyst predicts that 3 years from now the right to collect dividends 4 through infinity will be worth $47.90. What is it worth today to have something (the right to collect dividends 4 through infinity) that will be worth $47.90 in 3 years? Like anything else that will not be received for 3 years, we compute its present value by discounting back for 3 years at the appropriate annual discount rate:

$\left(\frac{\$4.67}{.0975}\right)\left(\frac{1}{1.1225}\right)^{3}$ = ($47.90)(.707035) = $33.87

Thus Jetson computes the stock’s value — the present value of all expected future dividends — as $10.90 (the value today of the right to collect D1, D2, and D3) + $33.87 (the value today of the right to collect D4 to D∞) = $44.77. It stands to reason that Jetson, who is more optimistic about Pluto’s future (growth in dividends eventually expected) than Armstrong (no growth) or Robinson (negative growth), would be willing to pay a price higher than their respective $37.22 or $32.65, but a price lower than the even more optimistic (growth starting immediately) Vader-Solo’s $47.90. Tying it all together, we can compute the value in a situation modeled as having long-term constant percentage growth following a shorter period without that constant rate of growth as

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + … + $\frac{D\_{C-1}}{\left(1 + r\right)^{C-1}}$ + $\left(\frac{D\_{C}}{\left(r – g\right)}\right)$ $\left(\frac{1}{1 + r}\right)^{C-1}$

with C representing the first year when expected dividends can be modeled with a long-term constant growth rate (C ‒ 1 is the final year of the period without long-term constant growth). Note that DC = DC – 1 (1 + g), which is $4.56 (1.025) = $4.67 in the case at hand. Thus we compute

VS = $\hat{P\_{0}}$ = $\frac{\$4.56}{\left(1.1225\right)^{1}}$ + $\frac{\$4.56}{\left(1.1225\right)^{2}}$ + $\frac{\$4.56}{\left(1.1225\right)^{3}}$ + $\left(\frac{\$4.67}{\left(.1225 - .025\right)}\right)\left(\frac{1}{1.1225}\right)^{3}$

= $4.06 + $3.62 + $3.22 + ($47.90)(.707035)

= $4.06 + $3.62 + $3.22 + $33.87 = $44.77 per share

as found when we walked more deliberately through all of the steps above. This more efficient approach is the one we should become comfortable working with. Note that if Jetson thought Pluto’s common equity investors would be happy with only a 10.25% average annual rate of return, she would encourage the Neptune fund manager to pay a per-share price as high as

VS = $\hat{P\_{0}}$ = $\frac{\$4.56}{\left(1.1025\right)^{1}}$ + $\frac{\$4.56}{\left(1.1025\right)^{2}}$ + $\frac{\$4.56}{\left(1.1025\right)^{3}}$ + $\left(\frac{\$4.67}{\left(.1025 - .025\right)}\right)\left(\frac{1}{1.1025}\right)^{3}$

= $4.14 + $3.75 + $3.40 + ($60.26)(.746215)

= $4.14 + $3.75 + $3.40 + $44.97 = $56.26 per share

12. Although Sigourney Ripley is reluctant to alienate her fellow analysts, she expresses the even more optimistic view that the per-share dividend Pluto pays to its common stockholders will grow by approximately 15% per year for three years, and then by a magnitude that can be modeled as a constant 2.5% per year forever into the future. What is the highest price Ripley would recommend that the Neptune Mutual Fund pay for each share of Pluto Company common stock? What if the required effective average annual rate of return instead were a lower 10.25%?

*Type: Common Stock Value; Long Period of Constant Growth Following Short Period of Constant Growth.*

This analyst also expects dividends to start growing in year 4 at a rate that can be modeled as a constant 2.5% forever, while expecting a different (and higher) rate of approximately constant growth in years 1 to 3 as well. Ripley perhaps feels Pluto is well situated within its industry to realize strong growth in sales and profits (and ability to pay dividends) over the next few years, but knows any such advantage is likely to be short-lived as competitors figure out the same secrets to success. [And recall from question 9 above that periodic growth (e.g., 15%) could be expected to exceed the required average periodic rate of return (e.g., 12.25%) only for a short time.] Once again, we could use a version of the general asset valuation equation:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

Ripley models the expected dividend stream as follows: D1 is D0 x 1.15 = $4.56 x 1.15 = $5.24; D2 is D1 x 1.15 = $5.24 x 1.15 = $6.03; D3 is $6.03 x 1.15 = $6.94; D4 is $6.94 x 1.025 = $7.11; D5 is $7.11 x 1.025 = $7.29; D6 would be 2.5% greater than D5 and D7 2.5% greater than D6; D10,000 would be $4.56 x (1.15)3 x (1.025)9,997, an extremely large value. (Note that the year 3 dividend is expected to be approximately 15% greater than the year 2 dividend, but year 4’s dividend is expected to be only about 2.5% above year 3’s, since 2.5% constant annual growth is treated as setting in starting with year 4.) Plugging into the general valuation formula, we would try to compute

VS = $\hat{P\_{0}}$ = $\frac{\$5.24}{\left(1.1225\right)^{1}}$ + $\frac{\$6.03}{\left(1.1225\right)^{2}}$ + $\frac{\$6.94}{\left(1.1225\right)^{3}}$ + $\frac{\$7.11}{\left(1.1225\right)^{4}}$ + $\frac{\$7.29}{\left(1.1225\right)^{5}}$ + … + $\frac{D\_{10,000}}{\left(1.1225\right)^{10,000}}$

And if cash flows were not expected to follow any kind of pattern, we would have to use this general equation, a very long and tedious process. In fact, we might want to compute for the first three years as though they have no pattern, finding the present value of D1 through D3 as

$\frac{\$5.24}{\left(1.1225\right)^{1}}$ + $\frac{\$6.03}{\left(1.1225\right)^{2}}$ + $\frac{\$6.94}{\left(1.1225\right)^{3}}$ = $4.67 + $4.79 + $4.91 = $14.37

(much higher than Jetson’s $10.90 estimated value of the right to collect D1 through D3, since Ripley treats the dividends as growing substantially during that period while Jetson treats them as remaining level at $4.56). [FIL 404 students might note that these three payments constitute a changing annuity, whose combined present value we could more quickly find, though with a slight rounding difference from above, as

PMT x FAC = TOT

$5.24 $\left(\frac{1-\left(\frac{1.15}{1.1225}\right)^{3}}{.1225 - .15}\right)$ = $14.35 .]

So $14.37 is Ripley’s working estimate of the value today of the right to collect the year 1 — 3 dividend stream, D1 — D3. What about the value today of the right to collect expected dividends D4 through D∞? Again, we could sum the present values of individual dividends modeled as applying to years 4 through, say, 10,000. But with long-term annual growth treated as a constant percentage rate expected to begin in year 4, we can get the same result with a variation on the constant dividend growth model. Moving ahead in time to the day when constant dividend growth is treated as setting in (the start of year 4, which is 3 years away), we find what Ripley thinks the stock will be worth in 3 years, when she feels the next dividend to be received (D4) will be approximately $7.11 (which then will be thought of as D1, and which includes the 2.5% growth rate):

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$6.94(1.025)}{.1225 - .025}$ = $\frac{\$7.11}{.0975}$ = $72.92

(much higher than Jetson’s $47.90 estimated value of the right to collect D4 — D∞, since Ripley expects a higher D3 to serve as the basis for the modeled 2.5% growth beginning in year 4). So Ripley predicts that 3 years from now the right to collect dividends 4 — ∞ will be worth $72.92. What is it worth today to have something (the right to collect D4 — D∞) that will be worth $72.92 in 3 years? Like anything else that will not be received for 3 years, we get its present value by discounting back for 3 years at the appropriate annual discount rate:

$\left(\frac{\$7.11}{.0975}\right)\left(\frac{1}{1.1225}\right)^{3}$ = ($72.92)(.707035) = $51.56

Thus Ripley finds the stock’s value — the present value of all expected future dividends — as $14.37 (the value today of the right to collect D1 — D3) + $51.56 (the value today of the right to collect D4 — D∞) = $65.93. It stands to reason that Ripley, who is more optimistic about Pluto’s future than any of her fellow analysts yet considered, would be content to see Neptune pay a price higher than any of these colleagues would find acceptable (in the $32.65 — $47.90 range). As in question 10, we can tie it all together and compute the value in a situation modeled with long-term constant percentage growth, following a shorter period without that constant rate of growth, as

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + … + $\frac{D\_{C-1}}{\left(1 + r\right)^{C-1}}$ + $\left(\frac{D\_{C}}{\left(r – g\right)}\right)$ $\left(\frac{1}{1 + r}\right)^{C-1}$

with C representing the first year when dividends are modeled as growing as a constant long-term rate, and C – 1 representing the final year of the period without expected long-term constant growth. Note that DC = DC – 1 (1 + g). In the case at hand, we compute

VS = $\hat{P\_{0}}$ = $\frac{\$5.24}{\left(1.1225\right)^{1}}$ + $\frac{\$6.03}{\left(1.1225\right)^{2}}$ + $\frac{\$6.94}{\left(1.1225\right)^{3}}$ + $\left(\frac{\$7.11}{\left(.1225 - .025\right)}\right)\left(\frac{1}{1.1225}\right)^{3}$

= $4.67 + $4.79 + $4.91 + ($72.92)(.707035)

= $4.67 + $4.79 + $4.91 + $51.56 = $65.93 per share,

just as we found by moving more deliberately through all the steps above. If Ripley thought Pluto’s common stockholders would be happy with a lower 10.25% average annual rate of return, she would encourage Neptune’s portfolio manager to pay a price as high as

VS = $\hat{P\_{0}}$ = $\frac{\$5.24}{\left(1.1025\right)^{1}}$ + $\frac{\$6.03}{\left(1.1025\right)^{2}}$ + $\frac{\$6.94}{\left(1.1025\right)^{3}}$ + $\left(\frac{\$7.11}{\left(.1025 - .025\right)}\right)\left(\frac{1}{1.1025}\right)^{3}$

= $4.75 + $4.96 + $5.18 + ($91.74)(.746215)

= $4.75 + $4.96 + $5.18 + $68.46 = $83.35 per share

13. Finally, analyst James T. Kirk is even more optimistic than Ripley, expecting Pluto Company to boldly raise

its dividend where no similar firm has gone before. Specifically, he predicts that the annual per-share dividend will grow by approximately 45% in year 1, 35% in year 2, 25% in year 3, and 15% in year 4, and then subsequently by a fairly steady rate that can be modeled as a constant 2.5% per year forever into the future. What is the highest price Kirk would recommend paying for each share of Pluto Company common stock? What if instead he expected growth to be approximately 8% in year 5, before leveling off to a figure that could be modeled as a constant 2.5% starting in year 6? What if the required effective average annual rate of return instead were 10.25%?

*Type: Common Stock Value; Long Period of Constant Growth Following Period of Non-Constant Growth.* Now the analyst foresees dividend growth that can be modeled as a constant 2.5% rate per year starting in year 5 (or perhaps 6), following a series of higher, declining non-constant growth rates. Kirk definitely sees Pluto as well positioned to realize tremendous growth in sales and profits (and ability to pay dividends) over the next few years but, like Ripley, knows that any such advantage is likely to be short-lived as competitors copy Pluto’s successful practices. [And again recall from question 9 above that yearly growth (e.g., 45%) could be expected to exceed the required average annual rate of return (e.g., 12.25%) only for a short time.] As always, we could use a form of the general asset valuation equation:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

In his initial projections, Kirk expects dividends to be approximately as follows: D1 is D0 x 1.45 = $4.56 x 1.45 = $6.61; D2 is D1 x 1.35 = $6.61 x 1.35 = $8.93; D3 is $8.93 x 1.25 = $11.16; D4 is $11.16 x 1.15 = $12.83; D5 is $12.83 x 1.025 = $13.15; D6 would be 2.5% greater than D5 and D7 2.5% greater than D6; D10,000 would be ($4.56 x 1.45 x 1.35 x 1.25 x 1.15) x 1.0259,996, an extremely large value. Plugging into the general valuation formula, we would try to compute

VS = $\hat{P\_{0}}$ = $\frac{\$6.61}{\left(1.1225\right)^{1}}$ + $\frac{\$8.93}{\left(1.1225\right)^{2}}$ + $\frac{\$11.16}{\left(1.1225\right)^{3}}$ + $\frac{\$12.83}{\left(1.1225\right)^{4}}$ + $\frac{\$13.15}{\left(1.1225\right)^{5}}$ + … + $\frac{D\_{10,000}}{\left(1.1225\right)^{10,000}}$

Indeed, if cash flows were not expected to follow any kind of pattern, we would have to use this long and tedious general equation. Here, in Kirk’s model the first four years clearly follow no convenient pattern, so we must compute the PV of D1 — D4 with our general valuation formula, as

$\frac{\$6.61}{\left(1.1225\right)^{1}}$ + $\frac{\$8.93}{\left(1.1225\right)^{2}}$ + $\frac{\$11.16}{\left(1.1225\right)^{3}}$ + $\frac{\$12.83}{\left(1.1225\right)^{4}}$ = $5.89 + $7.09 + $7.89 + $8.08 = $28.95

So $28.95 is the value today of the right to collect projected dividends D1 — D4. What about the value today of the right to collect expected D5 — D∞? Again, we could sum the present values of individual dividends expected from year 5 through, say, 10,000. But with expected long-term annual growth modeled at a constant percentage rate beginning in year 5, we get the same result with a variation on the constant dividend growth model. Moving ahead in time to the day when Kirk treats constant dividend growth as setting in (the start of year 5, which is 4 years away), we find what he thinks the stock will be worth in 4 years, when he feels the first dividend to be received will be D5 = $13.15 or so (which then will be thought of as D1, and which includes the 2.5% growth rate):

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$12.83(1.025)}{.1225 - .025}$ = $\frac{\$13.15}{.0975}$ = $134.87

Kirk predicts that 4 years from now the right to collect dividends 5 — ∞ will be worth $134.87. What is the value today of something (the right to collect D5 — D∞) that we expect will be worth $134.87 in 4 years? Like anything else that will not be received for 4 years, we find its present value by discounting back for 4 years at the appropriate annual discount rate:

$\left(\frac{\$13.15}{.0975}\right)\left(\frac{1}{1.1225}\right)^{4}$ = ($134.87)(.629875) = $84.95

Thus Kirk estimates the stock’s value — the present value of all expected future dividends — as $28.95 (the value today of the right to collect D1 — D3) + $84.95 (the value today of the right to collect D4 — D∞) = $113.90. It should seem logical that Kirk, who is far more optimistic about Pluto’s future than are any of his fellow analysts (including Ripley), would encourage Neptune to buy Pluto common stock even at a price far higher than the $65.91 that Ripley would willingly pay. And as before, we can tie it all together and compute the value in a situation modeled with long-term constant percentage growth, following a shorter period without constant growth, as

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + … + $\frac{D\_{C-1}}{\left(1 + r\right)^{C-1}}$ + $\left(\frac{D\_{C}}{\left(r – g\right)}\right)$ $\left(\frac{1}{1 + r}\right)^{C-1}$

with C representing the first year when dividends are treated for computational convenience as growing at a long-term constant rate, C ‒ 1 representing the final year of the period without long-term expected constant dividend growth, and DC computed as DC – 1 (1 + g). In the case at hand, we find

VS = $\hat{P\_{0}}$ = $\frac{\$6.61}{\left(1.1225\right)^{1}}$ + $\frac{\$8.93}{\left(1.1225\right)^{2}}$ + $\frac{\$11.16}{\left(1.1225\right)^{3}}$ + $\frac{\$12.83}{\left(1.1225\right)^{4}}$ + $\left(\frac{\$13.15}{\left(.1225 - .025\right)}\right)\left(\frac{1}{1.1225}\right)^{4}$

= $5.89 + $7.09 + $7.89 + $8.08 + ($134.87)(.629875)

= $5.89 + $7.09 + $7.89 + $8.08 + $84.95 = $113.90 per share,

the same theoretical value we found by moving through all the steps above.

If Kirk thought Pluto’s dividend instead would grow by approximately 8% in year 5 before leveling off to a figure that could be modeled, for practical computational purposes, as 2.5% forever starting in year 6, then he would compute D1 as $4.56 x 1.45 = $6.61; D2 as $6.61 x 1.35 = $8.93; D3 as $8.93 x 1.25 = $11.16; D4 as $11.16 x 1.15 = $12.83; D5 as $12.83 x 1.08 = $13.86; D6 as $13.86 x 1.025 = $14.20; D7 would be 2.5% greater than D6 and D8 2.5% greater than D7; D10,000 would be computed as ($4.56 x 1.45 x 1.35 x 1.25 x 1.15 x 1.08) x 1.0259,995, an extremely large value. Going directly to our least time-consuming approach, we would compute

VS = $\hat{P\_{0}}$ = $\frac{\$6.61}{\left(1.1225\right)^{1}}$ + $\frac{\$8.93}{\left(1.1225\right)^{2}}$ + $\frac{\$11.16}{\left(1.1225\right)^{3}}$ + $\frac{\$12.83}{\left(1.1225\right)^{4}}$ + $\frac{\$13.86}{\left(1.1225\right)^{5}}$ + $\left(\frac{\$14.20}{\left(.1225 - .025\right)}\right)\left(\frac{1}{1.1225}\right)^{5}$

= $5.89 + $7.09 + $7.89 + $8.08 + $7.78 + ($145.64)(.561136)

= $5.89 + $7.09 + $7.89 + $8.08 + $7.78 + $81.72 = $118.45 per share,

a slightly higher indicated value than if expected dividend growth were only about 2.5% in year 5. Note the exponent pattern: the last exponent before we switch to the constant dividend growth model, and the exponent used with the constant growth term, are the same (a common error would be to set it up (the 6 is shown with strikethrough so you will not think it is correct) as

VS = $\hat{P\_{0}}$ = $\frac{\$6.61}{\left(1.1225\right)^{1}}$ + $\frac{\$8.93}{\left(1.1225\right)^{2}}$ + $\frac{\$11.16}{\left(1.1225\right)^{3}}$ + $\frac{\$12.83}{\left(1.1225\right)^{4}}$ + $\frac{\$13.86}{\left(1.1225\right)^{5}}$ + $\left(\frac{\$14.20}{\left(.1225 - .025\right)}\right)\left(\frac{1}{1.1225}\right)^{6}$

Why is the exponent on the last term not a 6 (or a 5 in the prior example)? One way to deal with the issue is simply to memorize the fact that the last exponent before constant growth sets in and the exponent on the final (constant growth) term are equal. But it would be better to see why this relationship holds. Note in the example directly above that it will be 5 years before we expect $145.64 to be a combined value of all subsequent dividend payments. If that is the case, what is it worth today to have a claim (the right to collect D6 — D∞) that is expected to be worth $145.64 in 5 years? We find the answer by discounting that amount back for 5 years.

Another way to think about it might be: when we model constant growth as setting in immediately (as in question 9 above), we set up the constant dividend growth model based on D1, and then do not discount additionally (or we could say that we discount for 0 periods). In the example just above

we base the constant dividend growth computation on D6, and then discount for 5 periods. So think generally of setting up the constant dividend growth model based on DC, and then of discounting for C ‒ 1 periods. Finally, if Kirk believed that Pluto common stockholders would expect only a 10.25% effective average annual rate of return, then he would compute values of

VS = $\hat{P\_{0}}$ = $\frac{\$6.61}{\left(1.1025\right)^{1}}$ + $\frac{\$8.93}{\left(1.1025\right)^{2}}$ + $\frac{\$11.16}{\left(1.1025\right)^{3}}$ + $\frac{\$12.83}{\left(1.1025\right)^{4}}$ + $\left(\frac{\$13.15}{\left(.1025 - .025\right)}\right)\left(\frac{1}{1.1025}\right)^{4}$

= $6.00 + $7.35 + $8.33 + $8.68 + ($169.68)(.676839)

= $6.00 + $7.35 + $8.33 + $8.68 + $114.85 = $145.21 per share, and

VS = $\hat{P\_{0}}$ = $\frac{\$6.61}{\left(1.1025\right)^{1}}$ + $\frac{\$8.93}{\left(1.1025\right)^{2}}$ + $\frac{\$11.16}{\left(1.1025\right)^{3}}$ + $\frac{\$12.83}{\left(1.1025\right)^{4}}$ + $\frac{\$13.86}{\left(1.1025\right)^{5}}$ + $\left(\frac{\$14.20}{\left(.1025 - .025\right)}\right)\left(\frac{1}{1.1025}\right)^{5}$

= $6.00 + $7.35 + $8.33 + $8.68 + $8.51 + ($183.23)(.613913)

= $6.00 + $7.35 + $8.33 + $8.68 + $8.51 + $112.49 = $151.36 per share

(values higher than the $113.96 and $118.45 computed above as the present values of the same expected cash flow/dividend streams when discounted at a higher required annual rate of return).

14. Dune Industries paid its common stockholders a quarterly per-share dividend of 57¢ over the past year (so D0 is $.57 x 4 = $2.28). If stockholders’ required effective annual rate of return is 8.15%, and if the dividend is expected to remain at approximately that same dollar level for many years into the future such that it is practical to model the dividend stream as a perpetuity, what would we expect Dune’s common stock to be worth today, in 5 years, and in 20 years? What if the dividends instead were expected to rise in a fairly steady manner that could be modeled as a 3% constant annual rate? What if instead they were modeled as a constant annual decline of 2%?

*Type: Common Stock Value Expected at a Future Date.* The purpose for this seemingly strange problem is to help explain/reinforce some of the more difficult aspects of problems like 11 — 13 above. We compute any financial asset’s value as the present value, at the time measured, of all cash flows expected to be received after that date. Common stockholders’ cash flows are dividends. [They typically are paid quarterly, but for computing common stock values we usually combine a year’s four dividends into an annual total. We discount annual dividends using a required effective annual rate of return because the uncertainty with regard to the dividends common stockholders will receive makes it unproductive to do super-precise number crunching — vs. the bond and preferred stock cases, in which greater certainty regarding the cash flows merits more precise semiannual or quarterly computing.] The slightly simplified form of the general valuation equation we use is

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

but typically we obtain the result suggested by the tedious general equation approach through

a shorter computational technique (which we can do if the cash flows are expected to follow a path that can be modeled, for practical purposes, as a convenient pattern — and fortunately, when we predict the distant future we are likely to assume that some type of pattern ultimately will set in). For example, if dividends are expected to remain at approximately the same level for many years into the future, such that it is reasonable to model them as a perpetuity, we achieve the same result as with general valuation equation simply by computing

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$

and if dividends are expected to change in such a way that we can model the rate of change as a constant positive or negative percentage every year forever, we achieve the general valuation equation’s result simply by computing

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$

So if we can treat the dividends as being expected to remain at something close to $2.28 per share annually for many years into the distant future, the value today is estimated to be

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$ = $\frac{\$2.28}{.0815}$ = $27.98 per share

What about 5 years from now? At that time, the typical investor will say, “If you buy this stock you will get a stream of regular dividend payments that we treat, for practical computational purposes, as $2.28 each year forever.” If an 8.15% effective average annual rate of return is expected for an indefinitely long investment, the value measured five years from now should be

VS = $\hat{P\_{5}}$ = $\frac{D}{r}$ = $\frac{\$2.28}{.0815}$ = $27.98 per share

And 20 years from now the typical investor will say, “If you buy this stock you will get a stream of regular dividend payments that we treat, for practical computational purposes, as $2.28 each year forever.” So if an 8.15% average annual rate of return is expected, the value measured twenty years from now also should be

VS = $\hat{P\_{20}}$ = $\frac{D}{r}$ = $\frac{\$2.28}{.0815}$ = $27.98 per share

If dividends are expected to rise in a fairly steady manner that can be modeled as 3% per year forever, the value today (at the end of year 0) is estimated to be

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$2.28(1.03)}{.0815 - .03}$ = $\frac{\$2.35}{.0515}$ = $45.60 per share

If dividends do grow in approximately that manner, then five years from now the typical investor will say, “If you buy a share of this stock your first dividend, in the coming year, will be $2.72” [note that D6 = $2.28 (1.03)6 = $2.72]. With a required 8.15% effective average annual rate of return, the value measured five years from now should be

VS = $\hat{P\_{5}}$ = $\frac{D\_{6}}{r - g}$ = $\frac{D\_{0}\left(1 + g\right)^{6}}{r - g}$ = $\frac{\$2.28\left(1.03\right)^{6}}{.0815 - .03}$ = $\frac{\$2.72}{.0515}$ = $52.86 per share

And if dividends do grow in approximately that manner, then twenty years from now the typical investor will say, “If you buy this stock your first dividend, in the coming year, will be $4.24” [note that D21 = $2.28 (1.03)21 = $4.24]. With a required 8.15% average annual rate of return, the value measured twenty years from now should be

VS = $\hat{P\_{20}}$ = $\frac{D\_{21}}{r - g}$ = $\frac{D\_{0}\left(1 + g\right)^{21}}{r - g}$ = $\frac{\$2.28\left(1.03\right)^{21}}{.0815 - .03}$ = $\frac{\$4.24}{.0515}$ = $82.36 per share

What we see from these numbers is that one reason a so-called “growth” stock sells for a higher price, relative to a stock whose dividend stream is not expected to grow, is that the buyer expects to be able to sell the growth stock at a later date for a higher price than is paid today. If dividends are expected to decline (here, in a manner modeled as declining by 2% per year) over time, the effect is just the opposite, with the values as of today, 5 years from now, and 20 years from now estimated as:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$2.28(1 - .02)}{.0815 - (-.02)}$ = $\frac{\$2.28(.98)}{.0815 + .02}$ = $\frac{\$2.23}{.1015}$ = $22.01 per share,

VS = $\hat{P\_{5}}$ = $\frac{D\_{6}}{r - g}$ = $\frac{D\_{0}\left(1 + g\right)^{6}}{r - g}$ = $\frac{\$2.28\left(1 - .02\right)^{6}}{.0815 - (-.02)}$ = $\frac{\$2.28\left(.98\right)^{6}}{.0815 + .02}$ = $\frac{\$2.02}{.1015}$ = $19.90 per share, and

VS = $\hat{P\_{20}}$ = $\frac{D\_{21}}{r - g}$ = $\frac{D\_{0}\left(1 + g\right)^{21}}{r - g}$ = $\frac{\$2.28\left(1 - .02\right)^{21}}{.0815 - (-.02)}$ = $\frac{\$2.28\left(.98\right)^{21}}{.0815 + .02}$ = $\frac{\$1.49}{.1015}$ = $14.70 per share

Thus one reason a stock with declining expected dividends sells for a lower price, relative to a stock whose dividend stream is expected to stay level (or certainly relative to a growth stock), is that the buyer expects to have to sell it at a later date for less than the price paid today, such that there is an expected capital loss.

Of course, the investor does not have to sell the stock for that higher (lower) price to realize the economic benefit (cost) of growth; she can simply chose to hold on to it, secure (unhappy) in the knowledge that it has become worth more (less). In fact, in problems like numbers 11 — 13 above, with constant growth expected to set in after a few years, the portion of the overall value estimate based on

$$\frac{D\_{C}}{r - g}$$

(with C representing the first year of expected constant dividend growth) can be thought of either as the price the investor expects to be able to sell for at the start of year C, or merely as the expected present value, at the start of year C, of the right to collect dividends DC through D∞ (the present value of the remaining expected dividends is what the stock is worth to the investor/what she would expect to be able to sell it for).

15. Investment analyst Perry White thinks Krypton Corporation’s common stockholders will receive dividends of approximately $3.50 per share each year for fifty years, and then approximately $4.00 per share each subsequent year into the distant future thereafter. Analysts Lois Lane and Jimmy Olson agree that dividends can be modeled as $3.50 per share each year for fifty years, but Lane feels they will then increase to something close to $40.00 per share in each subsequent year for many years, while Olson insists they will then rise to approximately $400.00 per share in each subsequent year for a long period. If the three analysts agree that Krypton common stockholders require an 11.35% effective annual rate of return, what are their respective estimates of the stock’s value? Why are these estimates not farther apart than they are? What if expected per-share dividends instead were approximately $40.00 for fifty years and then $3.50 yearly into the distant future thereafter?

*Type: Common Stock Valuation; Value of Distant Future Dividends.* As when analyzing any financial asset, we think in terms of the general asset valuation equation:

VAsset (= CF0) = CF1 $\left(\frac{1}{1 + r}\right)^{1}$ + CF2 $\left(\frac{1}{1 + r}\right)^{2}$ + CF3 $\left(\frac{1}{1 + r}\right)^{3}$ + CF4 $\left(\frac{1}{1 + r}\right)^{4}$ + CF5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + CFn $\left(\frac{1}{1 + r}\right)^{n}$

For common stock valuation we can denote the cash flows more specifically as expected dividends:

VS = $\hat{P\_{0}}$ = D1 $\left(\frac{1}{1 + r}\right)^{1}$ + D2 $\left(\frac{1}{1 + r}\right)^{2}$ + D3 $\left(\frac{1}{1 + r}\right)^{3}$ + D4 $\left(\frac{1}{1 + r}\right)^{4}$ + D5 $\left(\frac{1}{1 + r}\right)^{5}$ + … + Dn $\left(\frac{1}{1 + r}\right)^{n}$

and can show that same general common stock valuation equation in a slightly streamlined version:

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{\left(1 + r\right)^{1}}$ + $\frac{D\_{2}}{\left(1 + r\right)^{2}}$ + $\frac{D\_{3}}{\left(1 + r\right)^{3}}$ + $\frac{D\_{4}}{\left(1 + r\right)^{4}}$ + $\frac{D\_{5}}{\left(1 + r\right)^{5}}$ + $\frac{D\_{6}}{\left(1 + r\right)^{6}}$ + … + $\frac{D\_{10,000}}{\left(1 + r\right)^{10,000}}$

If a finite stream of n unchanging dividends is expected, this general formula simplifies to

VS = $\hat{P\_{0}}$ = D $\left(\frac{1-\left(\frac{1}{1 + r}\right)^{n}}{r}\right)$

If expected dividends can be modeled as a perpetual unchanging stream, this general formula simplifies to

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$

And if expected dividends can be modeled as a perpetual unchanging stream after some number n of periods has passed, we measure the stream’s value today as

VS = $\hat{P\_{0}}$ = $\left(\frac{D}{r}\right)\left(\frac{1}{1 + r}\right)^{n}$

Here we begin with a stream of annual dividends expected to remain at approximately $3.50 per share (let’s call this DFirst) for a finite number of years (50), followed by annual dividends expected to be approximately $4 per share (let’s call this DSecond) into the far distant future thereafter. Combining our formulas from above, we find White’s value estimate as

VS = $\hat{P\_{0}}$ = DFirst $\left(\frac{1-\left(\frac{1}{1 + r}\right)^{n}}{r}\right)$ + $\left(\frac{D\_{Second}}{r}\right)\left(\frac{1}{1 + r}\right)^{n}$

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{50}}{.1135}\right)$ + $\left(\frac{\$4.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{50}$

= $30.694259 + $.163137 = $30.86 per share

However, if the stream of annual dividends designated as DSecond instead were expected to be a close to a much higher $40 per share each, Lane’s computed value would be only a tiny bit higher at

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{50}}{.1135}\right)$ + $\left(\frac{\$40.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{50}$

= $30.694259 + $1.631372 = $32.33 per share

[If, on the other hand, the per-share annual dividend expectation were reversed, with $40 expected for 50 years and $3.50 thereafter, the estimated value would be a much higher

VS = $\hat{P\_{0}}$ = $40.00 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{50}}{.1135}\right)$ + $\left(\frac{\$3.50}{.1135}\right)\left(\frac{1}{1.1135}\right)^{50}$

= $350.791535 + $.142745 = $350.93 per share.]

But even if the latter stream of per-share dividends were expected to be much, much higher at something close to $400 per year, Olson’s computed value would be only somewhat higher at

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{50}}{.1135}\right)$ + $\left(\frac{\$400.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{50}$

= $30.694259 + $16.313722 = $47.01 per share

The point here is that even though common stock’s per-share value estimate is based on an infinite stream of expected dividends, it is the first several years’ worth of expected dividends that account for most of the value. We could expect D51 — D∞ to be only $4, or could project them to be 1,000% as high at $40 per year or 10,000% as high at $400 per year, and our value estimates still will not be far apart in dollar or percentage terms ($30.86 is 95.5% of $32.33, and 65.6% of $47.01).

So no matter what how we model far distant expected future dividends for computational purposes, analysts’ value estimate do not differ all that much, as long as there is agreement on what the near-term dividends will be. (This observation is comforting in light of the great difficulty of predicting what will happen way off in the future. The answer, to some extent, is that it does not matter what we predict; regardless of what we think D51 — D∞ will be, their combined PV is quite small.) And note what happens to the estimates of per-share value if four analysts agree that dividends will be about $3.50 per share for 100 years but then expect $4 vs. $40 vs. $400 vs. $4,000 thereafter:

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{100}}{.1135}\right)$ + $\left(\frac{\$4.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{100}$ = $30.8363 + $.000755 = $30.837;

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{100}}{.1135}\right)$ + $\left(\frac{\$40.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{100}$ = $30.8363 + $.00755 = $30.844;

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{100}}{.1135}\right)$ + $\left(\frac{\$400.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{100}$ = $30.8363 + $.0755 = $30.91; and

VS = $\hat{P\_{0}}$ = $3.50 $\left(\frac{1-\left(\frac{1}{1.1135}\right)^{100}}{.1135}\right)$ + $\left(\frac{\$4,000.00}{.1135}\right)\left(\frac{1}{1.1135}\right)^{100}$ = $30.8363 + $.755 = $31.59

So whereas earlier we suggested projecting dividends out for 10,000 years to approximate the value of the theoretically infinite dividend stream, here we can see that even by going out just 100 years we would likely be accounting for almost all of a stock’s theoretical value.

16. Romulus Company’s common stock currently sells for $106.26 per share. Romulus stockholders expect a 9.85% effective annual rate of return. If knowledgeable analysts expect the company’s stream of earnings and dividends to grow in a manner that can be modeled, for practical purposes, as a 4.65% constant annual rate forever into the future, what quarterly dividend must have been paid during the most recent year? What if a growth instead could be modeled as approximately a 2.58% constant annual rate?

*Type: Common Stock Valuation; Dividend Unknown.* Typically we would know the most recent total of four quarterly dividends (D0), so this is an admittedly unusual case. But once we are comfortable estimating stock values based on discounting expected future dividends, we should be able to solve easily for any possible unknown. Here we are dealing with expected dividend growth modeled as a constant expected annual rate (in sales, earnings, dividends, and the stock’s price), so we solve for D0 in the

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$

shortened form of the general asset valuation equation. Plugging in the known values, we have

$106.26 = $\frac{D\_{1}}{.0985 - .0465}$ = $\frac{D\_{0}(1.0465)}{.0985 - .0465}$ = $\frac{D\_{0}(1.0465)}{.052}$ = D0 $\left(\frac{1.0465}{.052}\right)$ = D0 (20.125)

D0 = $106.26 ÷ 20.125 = $5.28

If the last year’s total of quarterly dividends was $5.28, each quarterly dividend (if all four payments were equal, which we assume they were) must have been $5.28 ÷ 4 = $1.32. And the expected total dividend for the coming year, D1, has to be ($1.32 x 4) x 1.0465 = $5.52552:

VS = $\hat{P\_{0}}$ = $\frac{\$5.52552}{.0985 - .0465}$ = = $\frac{\$5.52552}{.052}$ = $106.26 ✓

If expected growth instead were modeled as a 2.58% constant annual rate, we would conclude that the most recent year’s total dividend was

$106.26 = $\frac{D\_{1}}{.0985 - .0248}$ = $\frac{D\_{0}(1.0248)}{.0985 - .0248}$ = $\frac{D\_{0}(1.0248)}{.0737}$ = D0 $\left(\frac{1.0248}{.0737}\right)$ = D0 (13.905020)

D0 = $106.26 ÷ 13.905020 = $7.64

With a prior year’s annual dividend total of $7.64, each of the quarterly dividends must have been $7.64 ÷ 4 = $1.91 (if people were willing to pay $106.26 while expecting only 2.48% growth, they had to be getting a higher dividend to begin with). The expected total dividend D1 for the coming year under these conditions would be ($1.91 x 4) x 1.0248 = $7.83, such that

VS = $\hat{P\_{0}}$ = $\frac{\$7.83}{.0985 - .0248}$ = $\frac{\$7.83}{.0737}$ = $106.26 ✓

17. Klingon, Inc. common stock currently sells for $57.79/share. The firm’s stockholders expect a 10.95% effective annual rate of return on their common equity investment. If the past year’s total of four quarterly dividends was D0 = $4.16, and if earnings and dividends have been modeled by analysts as growing by approximately a constant annual percentage, what is that rate? What if the stock instead were currently selling for $48.59 per share?

*Type: Common Stock Valuation; Growth Rate Unknown.* Typically we would know the growth rate (in sales, earnings, dividends, and the stock’s price) that analysts are building into their valuation models, so this is an admittedly unusual situation. But once we are comfortable estimating stock values based on discounting expected future dividends, we should be able to solve easily for any possible unknown. Here we solve for g in the

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$

constant growth form of the general asset valuation equation. Plugging in the known values, we have

$57.79 = $\frac{D\_{1}}{.1095 - g}$ = $\frac{\$4.16(1 + g)}{.1095 - g}$

$57.79 (.1095 ‒ g) = $4.16 (1 + g)

6.328005 ‒ 57.79g = 4.16 + 4.16g

2.168005 = 61.95g

g = 2.168005 ÷ 61.95 = .035, or 3.5%

Double-check: VS = $\hat{P\_{0}}$ = $\frac{\$4.16(1.035)}{.1095 - .035}$ = $\frac{\$4.3056}{.0745}$ = $57.79 ✓

If the stock instead were currently selling for $48.59 per share, we would find

$48.59 = $\frac{D\_{1}}{.1095 - g}$ = $\frac{\$4.16(1 + g)}{.1095 - g}$

$48.59 (.1095 ‒ g) = $4.16 (1 + g)

5.320605 ‒ 48.59g = 4.16 + 4.16g

1.160605 = 52.75g

g = 1.160605 ÷ 52.75 = .022, or 2.2%

Double-check: VS = $\hat{P\_{0}}$ = $\frac{\$4.16(1.022)}{.1095 - .022}$ = $\frac{\$4.2515}{.0875}$ = $48.59 ✓

It should make sense that Klingon’s common stock would sell for a lower per-share price if the investing public did not expect dividends to grow by as high a yearly rate.

18. Tatooine Industries common stock currently sells for $10.92 per share. The quarterly dividend the company has paid to its common stockholders has been $.37 per share for the past several years. If the dividend is expected to remain approximately at that level far into the distant future, such that it is practical to model the stream as a perpetuity, what effective annual rate of return do buyers of the Tatooine stock expect? What if the stock instead were currently selling for $18.59 per share?

*Type: Common Stock Valuation; Rate of Return Unknown.* Though often we would know stockholders’ expected/required effective annual rate of return, this r value might be an unknown to solve for in some situations, such as computing a company’s weighted average cost of capital. Here, with the dividend stream expected to stay essentially unchanged for a prolonged period, we solve for r in the

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$

perpetuity-based shortened form of the general asset valuation equation. Note that the annual dividend total has been, and is expected to continue to be, $.37 x 4 = $1.48. Plugging in the known values, we have

$10.92 = $\frac{\$1.48}{r}$

$10.92 (r) = $1.48

r = $1.48 ÷ $10.92 = 13.55% (such that $1.48 ÷ .1355 = $10.92)

If the stock instead were currently selling for $18.59 per share, we would find

$18.59 = $\frac{\$1.48}{r}$

$18.59 (r) = $1.48

r = $1.48 ÷ $18.59 = 7.96% (such that $1.48 ÷ .0796 = $18.59)

It should make sense that investors paying a higher price for a given expected dividend stream are expecting (and willing) to earn a lower effective annual rate of return.

19. Algernon Corporation common stock currently sells for $27.71 per share. The annual dividend (total of four quarterly) the company paid to its common stockholders over the past year was D0 = $1.88/share. If analysts expect this dividend to grow in a manner that is practical to model as a constant rate of 2.75% per year forever into the future, what effective average annual rate of return do they expect holders of the Algernon common stock to earn? What if they instead modeled the expected growth rate to be a constant 4.95% per year, or -3.85% per year?

*Type: Common Stock Valuation; Rate of Return Unknown.* Here we are trying to predict what common stockholders’ annual rate of return r will be under various circumstances. In our discussion of weighted average cost of capital, we tried to predict the annual rate of return that common stockholders would require (i.e., expect to receive as fair compensation for the risks they have taken, and thus the rate the company managers must deliver to keep them happy). Here, with dividends modeled as growing at a constant annual rate forever into the future, we want to solve for r in the

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$

shortened (for use with constant periodic change) form of the general asset valuation equation. But recall that this representation is simply an algebraic rearrangement of the cost of common equity formula used in our cost of capital discussion when constant annual dividend growth was expected:

$\hat{k\_{S}}$ (or r) = $\frac{D\_{1}}{P\_{0}}$ + g = $\frac{D\_{0}(1 + g)}{P\_{0}}$ + g

So here, with D0 = $1.88, P0 = $27.71 and g = 2.75%, we compute an expected average annual return of

r = $\frac{\$1.88(1.0275)}{\$27.71}$ + .0275 = $\frac{\$1.9317}{\$27.71}$ + .0275

= .069711 + .0275 = .097211, or 9.7211%

A year’s dividend as a proportion of the price that prevailed at the beginning of the year, which here is $1.9317/$27.71 = 6.9711%, is known as the dividend yield. The dividend yield relates to the cash flow the common stockholder receives every year. But the dividend yield is only one component of the return received. Common stockholders also get the benefit of growth in the stock’s value (which would be likely to occur if the managers retained earnings and reinvested them wisely in new assets the company needs, for the stockholders’ benefit).

If fairly steady growth in sales, earnings, earnings retention, and dividends sere expected, the per-share value of the stock would be expected to increase by the same annual rate. This increase would result in a capital gain for the investor, so we can think of a constant dividend growth rate as also being a component of the stockholder’s annual rate of return, a capital gain percentage. The overall rate of return (whether measured after the fact or anticipated before the fact) is the sum of the dividend yield and the capital gain percentage.

If annual growth instead turned out to be g = 4.95%, the stockholder’s expected average annual return would be

r = $\frac{\$1.88(1.0495)}{\$27.71}$ + .0495 = $\frac{\$1.97306}{\$27.71}$ + .0495

= .071204 (dividend yield) + .0495 (capital gain percentage) = .120704, or 12.0704%

And if annual growth instead turned out to be g = -3.85%, investors’ expected average annual return would be

r = $\frac{\$1.88(1 - .0385)}{\$27.71}$ + (‒.0385) = $\frac{\$1.88(.9615)}{\$27.71}$ ‒ .0385 = $\frac{\$1.80762}{\$27.71}$ ‒ .0385

= .065233 (dividend yield) ‒ .0385 (capital loss percentage) = .026733 or 2.6733%

It is unlikely that anyone would willingly invest as a common stockholder if the rate of return were expected to be as low as 2.67% annually. But someone might buy stock expecting dividends to rise in a manner approximating 4.95% steady annual growth (for a 12.07% expected annual rate of return), and end up seeing the disappointing result of dividend “growth” approximating a negative 3.85% annual rate (for a 2.67% after-the-fact actual annual rate of return). Note that someone who required a 12.07% effective average annual rate of return but expected dividends to decline by a constant 3.85% annual rate would insist on paying a price no higher than

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$1.88(1 - .0385)}{.1207 - (-.0385)}$ = $\frac{\$1.88(.9615)}{.1207 + .0385}$ = $\frac{\$1.81}{.1592}$ = $11.37 per share

20. Analyst E.T. Spock just estimated the theoretical value of Vulcan International’s common stock to be almost $42 per share. But he knows that the current market price is less than $30 per share. Spock based his value estimate on the constant dividend growth model, using 6% as the average annual dividend growth rate and 13.5% (a figure on which investors and analysts widely agree) as the required effective annual rate of return. The annual dividend (total of four quarterly figures) paid by Vulcan to its common stockholders over the past dozen years has been as follows:

Year 1: $1.56 Year 4: $1.32 Year 7: $2.56 Year 10: $2.76

Year 2: $1.48 Year 5: $2.40 Year 8: $2.64 Year 11: $2.88

Year 3: $1.36 Year 6: $2.48 Year 9: $2.72 Year 12: $2.96

How did Spock compute the average annual growth rate in dividends? Has he found a tremendous bargain, or can you find a flaw in his logic?

*Type: Common Stock Valuation; Computing Growth Rate.* A common method for computing annualized growth is to find the rate at which the first value would compound to reach the last. Here a $1.56 annual dividend grew to $2.96 over eleven years (end of year 1 through end of year 12), such that Spock computed:

 BAMT (1 + r)n = EAMT

$1.56 (1 + r)11 = $2.96

(1 + r)11 = 1.897436

$\sqrt[11]{\left(1 + r\right)^{11}}$ = $\sqrt[11]{1.897436}$

1 + r = 1.8974361/11 = 1.897436.090909 = 1.06 , so r = 6%.

That 6% is the compounded annual rate by which $1.56 would grow to $2.96 over 11 years. But to use the constant dividend growth model confidently, we have to expect future dividends to grow in a manner that reasonably could be modeled as a fairly constant annual rate. And to expect fairly constant future growth, we might want to see fairly constant growth over the past several years. Here the yearly growth from year 1 — 12 has been anything but constant; in the early years the annual dividend actually fell, but then the total took a huge jump in year 5. So based on recent history Spock may have little justification for projecting 6% constant annual dividend growth into the future, or for estimating a per-share theoretical value of

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$2.96(1.06)}{.135 - .06}$ = $\frac{\$3.14}{.075}$ = $41.87

Note that for the 7-year period involving years 5 through 12, the compounded annual growth rate is computed to be

 $2.40 (1 + r)7 = $2.96

 (1 + r)7 = 1.233333

$\sqrt[7]{\left(1 + r\right)^{7}}$ = $\sqrt[7]{1.233333}$

1 + r = 1.2333331/7 = 1.233333.142857 = 1.03 , so r = 3%,

a figure very consistent with actual year-to-year movements in recent years. Using this figure as g in the constant dividend growth model, we would compute

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$2.96(1.03)}{.135 - .03}$ = $\frac{\$3.05}{.105}$ = $29.05 ,

a per-share theoretical value more in line with the current market price. Even using 3% would not be justified if recent events in the firm did not indicate growth likely to continue in a manner that reasonably might be modeled as a 3% constant rate, but at least this figure is supported by recent trends. Note that an unrealistically high growth rate assumption will lead to an overestimate of the stock’s theoretical value — by an analyst who is careless, or one who wants to convince someone that an unrealistically high price is justified.

21. Many American investors hold shares of common stock issued by Melmac Manufacturing Company. The company is located in the country of Musialistan, and all of its business dealings are conducted in Musialistan’s currency, the Manna. The stock currently sells for 1,951 Mannas per share. The annual dividend the company paid to its common stockholders over the past year was 78 Mannas per share. If the dividend is expected to grow in a manner that could be modeled as a constant 3.31% per year forever into the future, what annual rate of return do buyers of the Melmac stock expect? If the rate of exchange quoted in foreign currency markets is 475 Mannas to the dollar, how many dollars would it take for an American investor to purchase 100 shares of Melmac common stock?

*Type: Common Stock Valuation; Rate of Return and Foreign Currency.* Here we have dividends that are expected to grow by a fairly constant annual percentage rate, and are trying to measure the annual rate of return those who pay today’s price are expecting to receive. Again we are dealing with the

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$

abbreviated form of the general asset valuation equation, which we can use in situations with sales, earnings, dividends, and the stock’s price expected to grow in a manner that reasonably could be modeled as approximately a constant annual percentage. Rearranging algebraically to find the rate of return (as we do in estimating the cost of common equity in our weighted average cost of capital discussion), we find that investors who pay 1,951 Mannas today expect

r = $\frac{D\_{0}(1 + g)}{P\_{0}}$ + g = $\frac{78(1.0331)}{1,951}$ + .0331 = .041303 + .0331 = .074403, or 7.4403%

as their total effective annual rate of return (4.1303% expected dividend yield + 3.31% expected capital gain percentage). The fact that we are dealing with a foreign currency does not change the structure of the computations. However, if an American buyer wanted to buy some Melmac shares, he would have to first convert his dollars to Mannas. If $1.00 can be exchanged for 475 Mannas, then the dollar total needed to buy one Melmac share would be 1,951 ÷ 475 = $4.107368. A more systematic way to demonstrate this relationship would be

1,951 Mannas ÷ $\frac{ 475 Mannas}{\$1.00}$ = 1,951 Mannas x $\frac{ \$1.00}{475 Mannas} $= $\frac{ 1,951}{475}$ x $1.00 = $4.107368

 Thus to buy 100 shares, the American investor would need $4.107368 x 100 = $410.74.

22. There are 7,700,000 outstanding (*i.e*., in existence and able to be bought or sold) shares of Remulac, Ltd. common stock. The stock’s current market price is $43.95 per share. Directors are elected to Remulac’s board through competitive, cumulative voting, with all ten director spots filled in an election held at each year’s annual meeting of shareholders. A relatively small group of unhappy shareholders would like to combine their votes and elect someone who shares their viewpoint, Ms. Primat, to the Remulac board. How many total shares must they control to be assured of getting their candidate elected? What if instead there were only six director positions? How much money would a group with no current holdings have to invest in Remulac’s common stock to be sure they could elect someone to the board?

*Type: Number of shares needed to assure a board seat.* Under cumulative voting, the number of votes each shareholder is entitled to cast is the number of shares held times the number of directors to be elected. Thus if ten director spots are up for election in a competitive election, someone with 100 shares is able to cast 100 x 10 = 1,000 votes. Those 1,000 votes can be spread however the voter wishes, across any number of candidates — including the casting of all 1,000 votes for one candidate. This procedure allows a relatively small group of stockholders to combine and concentrate their voting power toward getting one representative on the board (who can not carry the board vote on issues of concern to these stockholders, but who at least can express the group’s views at the company’s highest policy-making level).

The number of shares a group needs to own so they can be assured of electing one candidate under cumulative voting is $\left[\left(\frac{1}{n + 1}\right)SO\right]$ + 1, with n representing the number of board seats to be filled in the election and SO representing the number of shares outstanding. So if 10 board spots are to be filled, the number of shares the small group must own is $\left[\left(\frac{1}{11}\right)7,700,000\right]$+ 1 = 700,001.

Why is that number needed? Assume that our target group (the “good guys”) hopes to elect Ms. Primat, and that the larger group of other stockholders (the “bad guys”) will try to elect their own candidates to all ten positions. If the bad guys controlled 10/11 of the shares (10/11 x 7,700,000 = 7,000,000 shares), and they split their 7,000,000 x 10 = 70,000,000 votes equally over 10 chosen candidates, then each candidate would receive 7,000,000 votes. With the good guys holding the other 1/11 (= 700,000 shares), concentrating their 700,000 x 10 = 7,000,000 votes all in Ms. Primat, there would be an 11-way tie. Thus if the good guys had just one additional share, giving Ms. Primat 7,000,010 votes out of the 7,700,000 x 10 = 77,000,000 total to be cast, she would be assured of coming in no lower than 10th place in the total voting. For the bad guys to give 7,000,011 votes to each of 9 candidates, they would have only [77,000,000 - (7,000,011 x 9) – 7,000,010] = 6,999,891 votes to give their 10th candidate, so Ms. Primat would come in 10th and get a board seat. The total amount of money the good guys (or an outside group with no current holdings) would need to have invested in Remulac stock to be assured of winning a board seat would be 700,001 x $43.95 = $30,765,043.95.

If only 6 board spots are to be filled, the small group must own $\left[\left(\frac{1}{7}\right)7,700,000\right]$+ 1 = 1,100,001 shares. Again assume the “bad guys” will try to elect candidates to all six positions. With 6/7 of the shares (6/7 x 7,700,000 = 6,600,000) controlled, if they split their 6.6million x 6 = 39,600,000 votes equally over 6 chosen candidates, then each candidate would receive 6,600,000 votes. With the good guys holding the other 1/7 (= 1,100,000 shares), concentrating their votes all in Ms. Primat, then there would be a 7-way tie with 6,600,000 votes each. Thus if the good guys had just one additional share, giving Ms. Primat 6,600,006 votes out of the 7,700,000 x 6 = 46,200,000 total to be cast, she would be assured of coming in no lower than 6th place in the total voting. If the bad guys were to give 6,600,007 votes to each of their top 5 candidates, they would have only [46,200,000 ‒ (6,600,007 x 5) ‒ 6,600,006] = 6,599,959 votes to give their 6th candidate, so Ms. Primat would come in 6th and get a board seat. The amount of money the good guys (or an outside group with no current holdings) would need to have invested to be assured of winning a seat on the board of directors under these conditions would be 1,100,001 x $43.95 = $48,345,043.95.

23. (This problem offers a quick review of the important computational ideas covered in problems 8, 9, 10, and 13 above. A more detailed explanation of the steps shown in the solution that follows is contained in the solutions to those earlier problems.) Galileo Hubble, the chief investment officer for Galaxy Mutual Funds, has been thinking about buying shares of Milky Way Corporation’s common stock for the Galaxy portfolio. Shares can be purchased today at the current market price of $106.72 each. Hubble asks his four most experienced analysts to estimate the theoretically correct price to pay, based on each analyst’s own independent judgment. Milky Way paid a quarterly dividend to its common stockholders of $1.73 per share over the most recent year (so we treat D0 as $1.73 x 4 = $6.92). All four of the analysts believe that the average effective annual rate of return a Milky Way common stockholder should expect to earn is 8.86%, but they disagree on the dividend stream that should be expected.

Ms. Ordinary Spiral feels that the dividends will remain at something very close to the most recent year’s level for many years into the future, such that it is sensible to model the expected future dividend stream as a perpetuity. Mr. Barred Spiral is a bit more optimistic, expecting the dividends to grow in a manner that can be modeled, for practical purposes, as constant growth of 2.18% per year into the distant future. The pessimistic Ms. Elliptical believes that the dividends will be declining in future years in a way she can sensibly model as a constant change rate of negative 1.65% per year indefinitely. Finally, Mr. Irregular is very optimistic, believing that Milky Way’s advantages over its competitors will allow the managers to pay dividends that exceed the most recent year’s figure by 22% in coming year 1, then 17% above the year 1 figure in year 2, then 12% above the year 2 figure in year 3, then 9% above the year 3 figure in year 4, then 6% above the year 4 figure in year 5, before growth levels off to an additional 2.18% annually in each of years 6 – ∞. What is the highest price each analyst feels the Galaxy funds should be willing to pay per share for Milky Way Corp. common stock?

*Type: Common stock valuation.* The first analyst, Ms. Ordinary Spiral, would feel that a buyer is justified in paying a price of up to

VS = $\hat{P\_{0}}$ = $\frac{D}{r}$ = $\frac{\$6.92}{.0886}$ = $78.10 per share

(Recall that when we are computing the theoretical value of common stock we group four quarterly expected dividend payments into a yearly total and compute as though the payments occurred annually; more precise computations are not merited in light of the uncertainty of the dividends to be paid to a company’s common stockholders.) The more optimistic (“bullish”) second analyst, Mr. Barred Spiral, believes that a sensible buyer would be justified paying a price per share as high as

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$6.92(1.0218)}{.0886 - .0218}$ = $\frac{\$7.070856}{.0668}$ = $105.85 per share

The more pessimistic (“bearish) third analyst, Ms. Elliptical, computes the highest price per share that the Galaxy Funds or any sensible buyer should be willing to pay as

VS = $\hat{P\_{0}}$ = $\frac{D\_{1}}{r - g}$ = $\frac{D\_{0}(1 + g)}{r - g}$ = $\frac{\$6.92(1-.0165)}{.0886 - (-.0165)}$ = $\frac{\$6.92(.9835)}{.0886 + .0165}$ = $\frac{\$6.805820}{.1051}$ = $64.76 per share

Finally, super-optimistic analyst Mr. Irregular projects per-share dividends to be approximately as follows: D1 is $6.92 x 1.22 = $8.4424; D2 is $8.4424 x 1.17 = $9.877608; D3 is $9.877608 x 1.12 = $11.062921; D4 is $11.062921 x 1.09 = $12.058584; D5 is $12.058584 x 1.06 = $12.782099; D6 is $12.782099 x 1.0218 = $13.060749; D7 would be 2.18% greater than D6 and D8 2.18% greater than D7, etc. Mr. Irregular would compute

VS = $\hat{P\_{0}}$ = $\frac{\$8.4424}{\left(1.0886\right)^{1}}$ + $\frac{\$9.877608}{\left(1.0886\right)^{2}}$ + $\frac{\$11.062921}{\left(1.0886\right)^{3}}$ + $\frac{\$12.058584}{\left(1.0886\right)^{4}}$

+ $\frac{\$12.782099}{\left(1.0886\right)^{5}}$ + $\left(\frac{\$13.060749}{\left(.0886 - .0218\right)}\right)\left(\frac{1}{1.0886}\right)^{5}$

= $7.755282 + $8.335183 + $8.575606 + $8.586635 + $8.361044 + ($195.520189)(.654121)

= $7.755282 + $8.335183 + $8.575606 + $8.586635 + $8.361044 + $127.893938

= $169.51 per share

With the current market price at $106.72 Ms. Elliptical (with theoretical value estimate of $64.76) and Ms. Ordinary Spiral (with theoretical value estimate of $78.10) both believe that Milky Way common shares are significantly overpriced/too expensive to justify buying; Mr. Barred Spiral (with theoretical value estimate of $105.85) feels that the shares may be a bit overpriced, and Mr. Irregular (with $169.51 theoretical value estimate) finds the shares to be far cheaper than the price that the Galaxy Funds or any sensible buyer would be justified in paying, such that he encourages fund manager Hubbel to purchase a large number of shares. Hubbel would be likely to confer with each analyst and listen to each one’s reasoning before deciding whose recommendation to accept.