# End-of-period vs. Beginning-of-period Payments in Annuity Situations 

## Illustrated with Future Value of Annuity Problem

You want to compute how much will be in a bank account at the end of year 3 if $\$ 100$ is deposited each year over the 3-year period running Jan. 1, Yr. 1 until Dec. 31, Yr. 3. This is a Future Value of Annuity problem: a series of payments that are equally spaced apart in time and equal in amount relate to a large dollar amount that will not exist intact until a future date (the end of year 3).

The structure (equal payments relating to big future amount) tells us it is FV of Annuity; that is true regardless of whether the repeated payments occur at the end or start of each period. (And remember we care about the structure, not which side of the transaction a particular party might be on; the dollar amounts are exactly the same for the saver making deposits/receiving interest and the bank receiving deposits/paying interest.)

End $v s$. beginning-of-period payments relates only to the small administrative matter of whether interest occurs one more or fewer times over the plan's life.

End-of-period deposits involves 3 deposits but only 2 applications of interest:

$01-01-\mathrm{Yr} 1$ $\qquad$ 12-31-Yr 1 $\qquad$ 12-31-Yr 2 12-31-Yr 3 | $\uparrow$ |
| :---: |
| No Int. | in Yr. 1

Earn Int.
in Yr. 2 Deposit $\$ 100$


Beginning-of-period deposits means 3 deposits and 3 applications of interest (note that $12-31-$ Yr 1 instantaneously becomes $01-01-$ Yr 2 ; those two dates are the same):

01-01-Yr 1 $\qquad$ 01-01-Yr 2 Deposit $\$ 100$ Deposit \$100
$\qquad$ 01-01-Yr 3

| $\uparrow$ | Deposit \$100 | $\uparrow$Earn Int. <br> Earn Int. <br> in Yr. 2 |
| :---: | :---: | :---: |
| in Yr. 3 |  |  | 12-31-Yr 3

 Deposit $\$ 100$ Earn Int.
in Yr. 2 $\begin{array}{cc}\text { Deposit \$100 } & \uparrow \\ & \begin{array}{c}\text { Earn Int. } \\ \text { in Yr. } 3\end{array}\end{array}$ $\uparrow$
$\uparrow$
$\uparrow$
Total is $\$ 300$
+3 int. pmts.

