# COST OF CAPITAL: PROBLEMS \& DETAILED SOLUTIONS 

(copyright © 2020 Joseph W. Trefzger)

## Very Basic

1. Adams Associated Artisans, Boone Basic Industries, Calhoun Corporation, and DuPage Distributors are four very similar firms in the same industry (same general line of business activity), and all have the same capital structures (so the relative risks faced by each company's various groups of investors are the same). Adams has just borrowed money by issuing 30-year bonds, agreeing to pay a $7.95 \%$ annual interest rate. Boone has just issued preferred stock, with a $\$ 100$ per share par value, for which it agreed to pay $\$ 6.80$ per share in dividends every year. Calhoun has just issued common stock, which investors bought for $\$ 44$ per share; market observers expect Calhoun to pay $\$ 2.20$ in per-share dividends to its common stockholders in the coming year. Now the managers of DuPage are examining their own company's before-tax cost of debt $\left(k_{d}\right)$, cost of preferred stock $\left(k_{p}\right)$, and cost of common stock or equity $\left(\mathrm{k}_{\mathrm{e}}\right)$. What inferences can they draw by observing their competitors' recent financing activities?

Type: Definitional. In computing a company's weighted average cost of capital, and the costs of the debt, preferred stock, and common equity components that make up the weighted average, we are trying to "get inside the heads" of the investing public, asking ourselves: what would it cost the company to deliver the annual financial returns that informed investors would require for providing money? There are various ways to try to understand what motivates investors in a given economic environment. A very straightforward approach that can work in some cases is simply to observe what a firm's close competitors have experienced recently under similar circumstances. For example, if a similar company recently agreed to pay a $7.95 \%$ annual interest rate when it committed to borrowing for 30 years, then DuPage's managers would expect rational lenders to ask for $7.95 \%$ in annual interest if they lent for 30 years to DuPage; we would infer that $k_{d}$ for DuPage is $7.95 \%$. We can draw this inference because lenders get exactly what they have been promised, nothing more and nothing less (unless the borrowing firm declares bankruptcy). If DuPage gets its lenders from the same investing public that Adams borrows from, and if it is borrowing under similar terms, then it should expect to face the same annual interest rate cost.

In the same manner, if a similar company recently agreed to provide a yearly financial return equaling $6.80 \%$ of the amount it received (the par value) when it obtained money from preferred stockholders, then DuPage's managers would expect rational "middle of the line" investors to ask for a $6.80 \%$ annual dividend rate if they became preferred stockholders in DuPage; it would be reasonable to infer that $k_{p}$ for DuPage is $6.80 \%$. We can draw this inference because preferred stockholders typically get exactly what they have been told they will get, nothing more and nothing less (unless the firm faces some serious, unexpected financial difficulties). If potential DuPage preferred stockholders are part of the same investing public from which Boone got its preferred stockholders, then DuPage should expect to face that same yearly cost of compensating preferred stockholders with dividends if all important terms of the agreement are the same.

However, while it is appealing to say that $k_{e}$ for DuPage should be the same as $k_{e}$ for Calhoun, as long as the two companies are truly similar (in terms of the risks and the financial rewards that their common stockholders would expect to face), here we do not know what Calhoun's cost of common equity is. Recall that common stockholders are promised nothing in advance; they simply get the residual - the financial value that remains after all other parties with claims against the company (workers, material and service providers, lenders, government tax collectors, and preferred stockholders) have been fully compensated. And recall that common stockholders can be rewarded in two ways: dividends, and the reinvestment of earnings (which, if done wisely, strengthens the company, such that each share of common stock represents a proportional future claim on a bigger, stronger operation). Here we know only the $5 \%$ dividend return, or dividend yield, Trefzger/FIL 240 \& 404 Topic 5 Problems \& Solutions: Cost of Capital
that Calhoun common stockholders expect to receive in the coming year. We do not know whether these investors expect the annual dividend per share to increase (or decrease) as time passes, or to what degree they may expect reinvested earnings to increase the value of Calhoun's common stock.

Thus while we can draw inferences regarding a company's cost of debt $k_{d}$ and cost of preferred stock $k_{p}$ simply by looking at the recent experiences of similar companies, we need to use more systematic approaches when we try to "get inside the heads" of potential common stockholders and estimate $k_{e}$. Lenders and preferred stockholders do not share in any financial "leftovers," so what they pay for what they have been told they will get tells us the percentage returns they expect. But observing the prices paid by common stockholders tells us little about the percentage returns they expect each year, because the residual values they expect to receive, year-in and year-out forever into the future, can be very difficult for us to estimate.
2. Bureau Backpack's before-tax cost of debt financing, $\mathrm{k}_{\mathrm{d}}$, is $8.25 \%$. If Bureau's marginal income tax rate is a state-plus-federal combined $23 \%$, what is its after-tax cost of debt, $\mathrm{k}_{\mathrm{d}}(1-\mathrm{t})$ ? Carroll Camping Gear's after-tax cost of debt financing, $\mathrm{k}_{\mathrm{d}}(1-\mathrm{t})$, is $7.705 \%$. If Carroll pays state-plus-federal income tax at a $28 \%$ combined marginal rate, what is its before-tax cost of debt, $\mathrm{k}_{\mathrm{d}}$ ? In computing a weighted average cost of capital, are we more interested in the before- or after-tax cost of debt financing? Why do we not compute a before-tax and after-tax cost of equity?

Type: Cost of debt. The appropriate cost figure to use in computing a weighted average cost of capital is the after-tax cost, which is the cost to the company of delivering the appropriate annual rate of return to the money provider, after factoring in any special administrative costs or expected income tax savings. Recall the basic format of an income statement:

```
Sales
- Cost of Goods Sold
Operating Income or EBIT
- Interest to Lenders
Earnings Before Taxes (EBT)
- Income Taxes
Net Income (remaining for stockholders)
```

We can see that providing returns to stockholders (both preferred and common) brings about no income tax savings, because by the time we get down to net income, which is viewed legally as the source of returns to preferred and common stockholders, the income taxes have already been paid. Thus there is no basis for realizing any income tax savings in dealing with preferred or common stockholders; the before-tax and after-tax costs are the same for preferred stock and common equity. But interest payments cause the EBT figure to be smaller than it would otherwise be; thus paying interest to lenders leads to an income tax savings - meaning that debt financing costs less on an after-tax basis than it does on a before-tax basis.

Here Bureau would expect to promise new lenders an interest rate of $8.25 \%$, but every dollar paid as interest would save Bureau $23 \$$ in income tax within a few months (any money paid as interest does not remain as income to be taxed), so the after-tax cost of debt is $k_{d}(1-t)=.0825(1-.23)=$ $.0825(.77)=.0635$, or $6.35 \%$. In other words, if Bureau were to borrow money, it would expect to pay an $8.25 \%$ annual interest rate to its lenders, but the cost to Bureau of delivering that $8.25 \%$ return each year would be only $6.35 \%$ (with the difference being absorbed by the government in the form of a lower income tax bill for Bureau).

On the other hand, the annual cost to Carroll of delivering a fair interest rate to new lenders, after recognizing the income tax benefit (saving $28 \$$ in income tax every time it pays a dollar in interest), would be only $7.705 \%$. With $k_{d}(1-\dagger)=k_{d}(1-.28)=k_{d}(.72)=.07705$, it must be that $k_{d}=.07705 \div$
 $10.7 \%$ annual interest rate to its lenders, but the cost to Carroll of delivering that $10.7 \%$ return would be only $7.705 \%$ (with the difference coming in the form of a lower income tax bill).

In finding weighted average costs of capital for the two firms, the relevant after-tax cost of debt figures to use in the computations would be $6.35 \%$ for Bureau and $7.705 \%$ for Carroll.
3. The financial managers of Edwards Equipment, an Illinois-based manufacturing company, want to borrow $\$ 260,000,000$ to help pay for some needed new machinery. They plan to borrow this money by issuing bonds that will mature in 15 years (meaning that Edwards will pay interest to the lenders each year, but will not pay back the $\$ 260,000,000$ in borrowed principal for 15 years). Several years ago, Edwards borrowed money under a large bond issue and paid a $7.25 \%$ annual interest rate. A competing firm, Fayette Fabricating, very recently borrowed a large amount of money under a 15 -year bond issue, paying an $8.95 \%$ annual interest rate. What would Edwards's managers be likely to estimate their annual before-tax cost of debt, $\mathrm{k}_{\mathrm{d}}$, to be? If Edwards pays income tax at a $26 \%$ combined federal-plus-state marginal rate, what would be the estimate of its after-tax cost of debt, $\mathrm{k}_{\mathrm{d}}(1-\mathrm{t})$ ?

Type: Cost of debt. The interest rate a borrower expects to pay - whether that borrower is a family borrowing a small amount to buy a house, or a corporation borrowing a huge amount to pay for costly business assets - is likely to be approximately the rate that similar types of borrowers have paid when they borrowed similar amounts under similar conditions in the recent past. In other words, to "get inside the heads" of potential lenders, Edwards's managers can look to the behavior of those who have lent money to similar firm Fayette recently under similar terms (Edwards's "Wall Street" investment bankers would study the market and provide guidance in setting this rate). If objective lenders who analyzed the risks of lending to this type of company under current economic conditions have lent money recently to Fayette at an $8.95 \%$ annual interest rate, we'd expect other objective lenders to provide money to Edwards at $8.95 \% /$ year also, as long as the terms - amount of money, repayment schedule, and the risk (in the view of the investing public) of being repaid - were similar.

The 7.25\% annual interest rate that Edwards paid several years ago is irrelevant to the analysis. Over the time period since that earlier bond issue, two big things have (potentially) changed. One is the general level of interest rates across the economy. The other is the way that Edwards's ability to meet its debt obligations is perceived in the marketplace. Comparing Edwards to a similar firm under current market conditions is a much better indicator of its likely cost of borrowing today than comparing Edwards to itself under market conditions that no longer prevail.

So as long as lenders do not tend to feel they face more (or less) default or liquidity risk in lending for 15 years to Edwards than they did in lending for 15 years to Fayette, the Edwards managers should estimate the before-tax cost of debt, $\mathrm{k}_{\mathrm{d}}$, to be $8.95 \%$. That is the annual interest rate they would expect to pay lenders if Edwards borrowed under terms similar to those of Fayette's recent bond issue. However, because interest paid to lenders is a "tax-deductible" expense (it is subtracted from operating income, or EBIT, in reaching the pre-tax income, or EBT, on which income tax is paid), it does not cost $8.95 \%$ for Edwards to deliver an $8.95 \%$ annual interest return to new lenders. Because Edwards saves $26 \$$ in income tax every time it pays $\$ 1.00$ in interest, it would cost only $74 \%$ of $8.95 \%$ to deliver an $8.95 \%$ annual return to the lenders. In other words, the after-tax cost of debt financing for Edwards would be $8.95 \%$ minus the income tax savings, or $k_{d}(1-t)=.0895(1-.26)=.0895(.74)=.06623$, or $6.623 \%$.
Trefzger/FIL 240 \& 404
Topic 5 Problems \& Solutions: Cost of Capital

## Work These for Sure

4. Gallatin Garden Gizmos, Inc. wants to expand its capacity by building a new production facility. It will pay for the expansion by issuing new common stock, and by borrowing money through a 20 -year bond issue. It just so happens that five years ago Gallatin issued 25-year bonds, which carried an annual coupon interest rate of $7.5 \%$. When these bonds sell in market transactions today, they are priced to provide a yield to maturity of $9.5 \%$ (meaning that companies like Gallatin have borrowed money under similar conditions recently at a $9.5 \%$ annual interest rate). What is our estimate of Gallatin's after-tax cost of debt $\mathrm{k}_{\mathrm{d}}(1-\mathrm{t})$ if the firm's marginal state-plus-federal combined income tax rate is $45 \%$ ? What if it is $25 \%$ ? What if Gallatin does not pay income tax?

Type: Cost of debt. This question addresses two important points. The first is that we can draw inferences on the interest rate a company would pay if it created new spots at the "front of the line" (i.e., if it borrowed new money) by looking at the rate of return bond buyers insist on receiving when they take existing spots at the "front of the line" (i.e., when people buy existing bonds). This rate is known as the yield to maturity. [In previous problems we used a different technique for "getting inside the heads" of potential lenders: looking at the interest rates paid on similar recent bond issues by firms similar to the one we are examining.] Bonds issued by US-based companies typically sell initially for a $\$ 1,000$ "par value," but the prices they command later adjust to reflect the prevailing interest rate environment. What has actually happened here - you were not given all these details - is that each bond (which will mature in 20 years, at which point the $\$ 1,000$ originally lent will be returned) was issued with a $7.5 \%$ annual interest rate (for a $.075 \times \$ 1,000=\$ 75$ interest payment every year), and that each sells today for $\$ 823.75$. If we used trial and error to compute the annual rate of return inherent in these values, we would find it to be $9.5 \%$ (as in the previous question, the annual interest rate Gallatin paid a few years ago does not provide us with an estimate of the rate it would pay today); double check to assure that $9.5 \%$ is correct:

$$
\begin{aligned}
& \$ 75\left(\frac{1-\left(\frac{1}{1+r}\right)^{20}}{r}\right)+\$ 1,000\left(\frac{1}{1+r}\right)^{20} \Rightarrow \text { solve for } r \text { with trial and error } \\
& \$ 75\left(\frac{1-\left(\frac{1}{1.095}\right)^{20}}{.09}\right)+\$ 1,000\left(\frac{1}{1.095}\right)^{20} \\
& =\$ 75(8.812382)+\$ 1,000(.162824)=\$ 660.93+\$ 162.82=\$ 823.75 .
\end{aligned}
$$

Solving this equation with trial and error is how we would measure the yield to maturity if we had to compute it (for this chapter's exam purposes you would be given the yield to maturity and would simply have to know that it represents the interest rate the company would expect to pay on new borrowings; we will study yield to maturity in more detail in our bond unit). So we know that people who have recently examined the risk of becoming 20-year lenders to Gallatin have offered prices that would provide them with $9.5 \%$ annual rates of return. Therefore, unless we possessed strong evidence to the contrary, we would conclude that new 20-year lenders would also expect $9.5 \%$ annual rates of return, and thus the company should expect to pay a $9.5 \%$ annual interest rate on new 20-year borrowings.

The second important point here is that interest paid to lenders is subtracted from operating income (EBIT) in the computation of taxable income (EBT), so the price of borrowing is lower on an after-tax basis than on a before-tax basis. (On the other hand, if a company raises money from common stockholders, or even preferred stockholders, it pays dividends or retains earnings after income taxes have been paid, and thus there is no income tax benefit for financing with equity). Further, the cost of debt financing becomes increasingly low if the marginal income tax rate is higher, because then the income tax savings from paying interest becomes more substantial.

Here we compute the $k_{d}(1-t)$ component of Gallatin's weighted average cost of capital as:

$$
\begin{aligned}
& .095(1-.45)=5.225 \% \\
& \text { if the marginal income tax rate is } 45 \% \text {, a higher } \\
& .095(1-.25)=\underline{7.125 \%} \text { if the marginal income tax rate is a lower } 25 \% \text {, and a high } \\
& .095(1-0)=\underline{9.500 \%} \text { if the company does not pay income taxes (and thus gets no } \\
& \text { income tax benefit for the interest deduction). }
\end{aligned}
$$

(Changes to U.S. federal income tax law implemented in 2018 brought the highest federal income tax rate for corporations down to $21 \%$ from an earlier figure in the high $30 \%$ ranges, so today even with a state income tax added on top of the federal levy a company would not pay $45 \%$ of its income in tax. But if income that is recognized as earned were to be taxed at that high rate, the ability to avoid paying the tax through legal means would be especially valuable.)

A final note: we might reasonably assume that firms facing higher marginal income tax rates would attempt to shelter their income against the high tax by incorporating more debt into their capital structures than lower-taxed companies use. But why would they not simply borrow all of their operating money; why would Gallatin be planning to expand with a mix of debt and equity financing? Recall that all of the cost figures we work with in this chapter are based on the assumption of a given capital structure that is the best one for the company to follow. The yield to maturity on existing bonds (the interest rate the company would expect to pay on new debt) depends on the proportions of debt and equity financing that the borrower employs; lenders would insist on higher returns if a reasonable proportion of the company's money were not provided by owners (lenders would perceive higher risk if there were not a cushion of equity to absorb the first financial losses).
5. Holders of Grundy Gristmills \& Grain preferred stock receive $\$ 7.20$ in annual dividends (four quarterly payments of $\$ 1.80$ each). The par value per share is $\$ 100$, but the price at which the preferred shares are currently trading in the market is $\$ 110.77$ per share. Compute the before-tax and after-tax costs of preferred stock financing for Grundy.

Type: Cost of preferred stock. "Getting inside the heads" of potential preferred stockholders (or of lenders) is typically easier than is "getting inside the heads" of potential common stockholders, because preferred stockholders, in the typical case, expect to be paid an unchanging dollar amount in per-share dividends every year forever. (Thus with preferred stock, as with bonds, we can understand the money providers' expected rates of return by directly observing consistent behavior in the market.) The percentage cost of delivering a fair financial return to "middle of the line" investors thus is simply the unchanging expected annual dividend total (typically the sum of four expected quarterly preferred dividend payments), divided by the price $P_{0}$ that rational investors pay per share today. In an earlier problem we estimated $k_{p}$ as the cost of preferred stock that a similar company recently incurred. Here, we draw inferences on the annual dividend percentage Grundy would pay if it created new spots at the "middle of the line" (issued new preferred stock) by looking at the rate of return preferred stock buyers have insisted on receiving when they took existing spots at the "middle of the line" in recent transactions (bought existing Grundy preferred shares). With an unchanging expected annual dividend, $D_{0}=D_{1}=D_{100}$, etc., we can simply show the dividend as $D$, and compute

$$
k_{p}=\frac{D}{P_{0}}=\frac{\$ 7.20}{\$ 110.77}=.065, \text { or } \underline{\underline{6.5 \%}} .
$$

Here we assume that the preferred stock was initially sold, perhaps many years ago, for $\$ 100$ per share (a frequently encountered par value for preferred shares). At that time, people providing Trefzger/FIL 240 \& 404
money as preferred stockholders expected an annual return of $\$ 7.20 / \$ 100=7.2 \%$. But that was then; this is now. Today people who join the "middle of the line" are willing to accept a lower annual rate of return for bearing the risks of being preferred stockholders in Grundy. Maybe the company has overcome some financial problems and thus become a less risky company to provide money to; or maybe it is simply the case that across the economy investors are content with lower annual rates of return than they required when this preferred stock was first issued (high inflation might have been anticipated at that time). By looking at the annual rate of return those recently joining the "middle of the line" have built into the price they were willing to pay, we can easily infer the annual dividend percentage, or "yield," that new preferred stockholders would expect to receive.

Preferred stockholders are, in many ways, like lenders: the payments they are to receive (quarterly dividends) are specified in advance; preferred stockholders do not receive a residual claim the way common stockholders do. However, a preferred stockholder is viewed legally as a type of owner, not as a type of lender, so returns to preferred stockholders are, from a legal/taxation standpoint, paid out of net income (after the company has already paid its income taxes, such that it is too late to receive an income tax benefit). Therefore it would cost Grundy $6.5 \%$ to deliver a $6.5 \%$ annual rate of return to preferred stockholders, so the $6.5 \%$ computed above is both the before-tax and after-tax expected annual cost of preferred stock financing.
6. The total per-share dividend $D_{1}$ that Hancock Handcarts is expected to pay to its common stockholders over the coming year is $\$ 6.84$. Both the annual dividend total and the price of each share are expected to increase at a fairly constant $\mathrm{g}=3.5 \%$ annual growth rate for many years into the future. Using the constant dividend growth (sometimes called discounted cash flow) model, and assuming that Hancock common stock currently can be purchased in the market for $\$ 88.26$ per share, calculate the company's cost of common stock (or equity) financing, $\mathrm{k}_{\mathrm{e}}$. What if the market price per share were instead only $\$ 60.25$ ?

Type: Cost of common stock. We have seen that it is possible to "get inside the heads" of lenders and preferred stockholders simply by looking at the interest or dividend rates paid to new investors in the current market by similar companies, or by looking at the rates of return built into the prices investors pay when they buy bonds or preferred shares that were issued earlier by the company in question. The reason is that lenders and preferred stockholders receive none of the "leftovers;" they get only what the company agrees, in advance, to pay them. But common stockholders have the residual claim; they get the financial leftovers. Those leftovers can differ greatly from one company to another, and from one period to another. Therefore we need more systematic approaches to "getting inside the heads" of potential common stockholders, so we can estimate the annual financial returns that "back of the line" investors would expect and the cost to the company of delivering these returns. One such approach is the constant dividend growth model, based on the theorized relationship

$$
k_{e}=\frac{D_{1}}{P_{0}}+g .
$$

To compute $\mathrm{k}_{e}$ this way, we need an estimate of the coming year's total expected dividend payment (actually the sum of four expected quarterly dividends), $D_{1}$. Sometimes we have to compute a $D_{1}$ estimate by increasing the most recent year's dividend total, $D_{0}$, to reflect expected growth: $D_{1}=D_{0}(1+g)$. But here we are "spoon fed" the estimated $D_{1}$ figure. So we can directly compute

$$
\mathrm{k}_{e}=\frac{\$ 6.84}{\$ 88.26}+.035=.0775+.035=.1125 \text { or } \underline{\underline{11.25 \%}} \text {. }
$$

If, on the other hand, the shares could be purchased in the market for only $\$ 60.25$ each, then the indicated expected annual rate of return on equity (our measure of the company's cost of common stock financing $k_{e}$ ) would be

$$
\mathrm{k}_{\mathrm{e}}=\frac{\$ 6.84}{\$ 60.25}+.035=.1135+.035=.1485 \text { or } \underline{\underline{14.85 \%}}
$$

(paying less for the same stream of dividend and growth benefits represents a higher annual rate of return to the investor, or higher annual rate of cost to the company). But the constant dividend growth model is appropriate to use only if the expected growth rate $g$ is a consistent, market-wide estimate. Otherwise, we lack solid projections of expected future cash flows that we can relate to the current per-share price in estimating a percentage cost of delivering fair financial returns to a company's owners, and must use a different approach to estimating $k_{e}$.
7. The total per-share dividend $D_{0}$ that Iroquois Independent Irrigation's common stockholders received during the most recent year was $88 \phi$. Both the yearly dividend total and the price of each share are expected to increase at a fairly constant $\mathrm{g}=4.5 \%$ annual growth rate for many years into the future. Using the constant dividend growth (also called discounted cash flow) model, and assuming that Iroquois common stock currently can be purchased in the market for $\$ 14.58$ per share, calculate the company's cost of common stock (equity) financing, $\mathrm{k}_{\mathrm{e}}$. What if the longterm annual increase in dividends and the common stock's price were instead expected to be only $\mathrm{g}=2.5 \%$ ?

Type: Cost of common stock. The cost of common equity $\mathrm{k}_{e}$ is the percentage cost to the company, each year, of delivering fair financial returns to its true owners, the common stockholders. ( $k_{e}$ is also called the cost of retained earnings, because of our expectation that a company would try to get equity money for new investments by retaining earnings rather than selling new common stock since issuing new common stock requires paying considerable sums of money to investment banking firms.) As discussed in the previous question, we need more systematic approaches for "getting inside the heads" of common stockholders than for estimating the costs of delivering fair returns to lenders and preferred stockholders. One such approach is the constant dividend growth model, based on the theorized relationship

$$
k_{e}=\frac{D_{1}}{P_{0}}+g=\frac{D_{0}(1+g)}{P_{0}}+g .
$$

We need an estimate of the coming year's total expected per-share dividend payment (the sum of four expected quarterly dividends), $D_{1}$, but here we have instead been given the most recent year's dividend total, $D_{0}$. Thus we must convert the given $D_{0}$ (most recent actual) figure to our needed $D_{1}$ (coming year's estimated) figure by multiplying $D_{0}(1+g)=D_{1}$, here $\$ .88(1.045)=\$ .92$, rounded to whole cents. So we can compute

$$
\mathrm{k}_{e}=\frac{\$ .88(1.045)}{\$ 14.58}+.045=\frac{\$ .92}{\$ 14.58}+.045=.0631+.045=.1081 \text { or } \underline{\underline{10.81 \%}} \text {. }
$$

The constant dividend growth model is an attempt to observe consistent behavior at a market-wide level. It is our preferred method for estimating $k_{e}$ IF we do, indeed, observe consistent behavior. Note that $P_{0}$ and $D_{0}$ are current or historic values that can be verified, so the entire analysis rests on finding a consistent estimate of the annual growth rate $g$. For the technique to be valid to use, we must believe two things about $g$ : that it is a consensus estimate held by most company managers and investors and financial analysts, and that it is expected to remain fairly constant for many years into the future. If we feel that $g=4.5 \%$ is a consistent, market-wide view, then the only
inference we can draw is that people paying $\$ 14.58$ per share expect to earn a $10.81 \%$ annual rate of return on equity (and that it will cost Iroquois $10.81 \%$ annually to deliver that return if it can get needed new equity money by retaining earnings). If $g$ were not widely expected to consistently be $4.5 \%$ per year, however, then the $\$ 14.58$ current price would not relate to a $10.81 \%$ expected annual rate of return. For example, if the investing public felt that $g$ would be a constant $2.5 \%$ per year going far out into the future, then our inference would be that it would cost the company only

$$
k_{e}=\frac{\$ .88(1.025)}{\$ 14.58}+.025=\frac{\$ .90}{\$ 14.58}+.025=.0617+.025=.0867 \text { or } \underline{\underline{8.67 \%}}
$$

per year to deliver fair financial returns when it retains earnings that belong to its common stockholders. If there is not a consistent, market-wide estimate of $g$ (if company managers, investors, and analysts offer a range of different $g$ estimates), then the "constant" dividend growth model is not valid to use, and we must resort to a less preferred approach.
8. The dividends paid by Jasper Jade Jewelry Stores, Inc. to its common stockholders have followed no particular growth pattern over recent years, and are not expected to follow any particular growth pattern in the foreseeable future. Therefore, analysts can not estimate Jasper's cost of common equity financing $\mathrm{k}_{\mathrm{e}}$ with the constant dividend growth model. However, market observers are confident in their belief that the annual rate of return expected by investors who purchase "risk-free" U.S. government Treasury Bills is $4 \%$. The rate of return $\mathrm{k}_{\mathrm{m}}$ on the stock market as a whole is expected to average $10.5 \%$ per year going into the distant future. If the beta for Jasper common stock is 1.35 , what is your estimate of the firm's cost of common equity financing, $\mathrm{k}_{\mathrm{e}}$, based on the security market line approach?

Type: Cost of common stock. In the previous two questions we tried to compute the annual cost of common stock or equity financing, $\mathrm{k}_{e}$, based on our preferred technique, the constant dividend growth model. Here we lack the information needed in using that approach, but we do have the information needed in estimating $k_{e}$ using the security market line (SML) model, which we can represent as:

$$
k_{e}=k_{r f}+\left(k_{m}-k_{r f}\right) \beta .
$$

This approach is based on logic, rather than on the observation of consistent behavior in the market. (We would typically prefer to "get inside people's heads" by observing consistent behavior rather than by reasoning through what we think they should or would do, but in the absence of consistent behavior our next best alternative is to use logic.) The seemingly-solid logic is that the common stockholders (the "back of the line" investors, who are the true owners) of some corporation should expect to earn an annual rate of return that equals the annual return earned by those who buy risk-free securities, plus something extra as an added return for bearing additional risk:

$$
\mathrm{k}_{\mathrm{e}}=\mathrm{k}_{\mathrm{rf}}+\text { something extra. }
$$

The "something extra" should relate to two measures. The first is how much higher an annual return common stockholders across the economy expect to earn, above what the risk-free security buyers expect; this "market risk premium" is represented as ( $k_{m}-k_{r f}$ ). The second is a measure of the risk of investing in the particular common stock in question; this "beta" or $\beta$ measure tells us the degree to which annual returns earned on the stock in question have, historically, followed/no $\dagger$ followed returns on the stock market as a whole. Thus we can state
$k_{e}=k_{r f}+$ something extra relating to the stock market overall and the specific risk of this stock

$$
=k_{r f}+\left(k_{m}-k_{r f}\right) \beta .
$$

With the numbers given, we can compute

$$
k_{e}=.04+(.105-.04) \cdot 1.35=.04+.065 \cdot 1.35=.04+.08775=.12775 \text {, or } \underline{12.775 \%} \text {. }
$$

Here the market risk premium is $k_{m}-k_{r f}=.105-.04=.065$. Be careful to distinguish between the expected market return $\mathrm{k}_{\mathrm{m}}$ (here .105 , or $10.5 \%$ ) and the expected market risk premium $\mathrm{k}_{\mathrm{m}}-\mathrm{k}_{\mathrm{rf}}$. How good do we feel about this $12.775 \% k_{e}$ estimate? While the idea that $k_{e}=k_{r f}+$ something extra for risk is hard to dispute, the specific values of $\beta$, $k_{m}$, and even $k_{r f}$ are subject to question. For example, a beta value is based on observations of the annual returns earned by the specific company's common stockholders relative to the average annual returns on the stock market as a whole, over a long series of years. If the relationship has not been fairly steady from year to year (or if the company's business activities have changed such that past relationships would not likely continue to hold in the future), then we might not be overly confident in our beta measure, and thus in the cost of common stock (retained earnings) $k_{e}$ value estimated with the SML.
9. The dividends paid by Jersey Jogging Shoes to its common stockholders have followed no particular growth pattern over recent years, and are not expected to follow any particular growth pattern in the foreseeable future. Therefore, analysts can not estimate Jersey's cost of common equity financing $\mathrm{k}_{\mathrm{e}}$ with the constant dividend growth model. However, market observers are confident in their view that the annual rate of return expected by investors who buy "risk-free" U.S. government Treasury Bills is $3.5 \%$. The annual market risk premium earned by stock market investors is expected to average $7.25 \%$ per year going into the distant future. If the beta for Jersey Company is .85 , what is your estimate of the firm's cost of common equity financing, $\mathrm{k}_{\mathrm{e}}$ ? What if Jersey's beta were instead 1.85 , or 1.00 ?

Type: Cost of common stock. As in the previous question, we are trying to compute the cost of common stock or equity (more technically, cost of retained earnings) financing, $k_{e}$, based on the security market line (SML) model:

$$
k_{e}=k_{r f}+\left(k_{m}-k_{r f}\right) \beta .
$$

But whereas in the previous problem we were told the expected return on the stock market, $k_{m}$, here we are told the expected market risk premium, $k_{m}-k_{r f}$. With the values provided, we can compute

$$
\mathrm{k}_{e}=.035+(.0725) \cdot .85=.035+.061625=.0966625, \text { or } \underline{\underline{9.6625 \%}} \text {. }
$$

Again, be sure to carefully identify what you have been given, such as distinguishing between the expected market return $k_{m}$ and the expected market risk premium $k_{m}-k_{r f}$. Here the expected average annual market risk premium is $k_{m}-k_{r f}=.0725$. So what is the expected average annual rate of return on the overall stock market, $k_{m}$ ? For $k_{m}-k_{r f}=k_{m}-.035$ to be $.0725, k_{m}$ has to be $.0725+$ $.035=.1075$, or $10.75 \%$ (thus $k_{m}-k_{r f}=.1075-.035=.0725$ ).

Note that Jersey's beta is less than 1, suggesting that Jersey's common stockholders have tended, historically, to earn returns that are less high in good years, but less low in bad years, than do stock market investors on average. Because they face less than average risk, their $9.6625 \%$ expected average annual return is less than the $10.75 \%$ expected average annual return on the stock market overall. But a beta of 1.85 would indicate that annual returns to Jersey's common stockholders have been, historically, $85 \%$ more volatile than the stock market overall (a riskier investment, hitting higher highs and lower lows), and with greater than average risk they would expect an annual return exceeding the market's $10.75 \%$ expected average, specifically:

$$
\mathrm{k}_{e}=.035+(.0725) \cdot 1.85=.035+.134125=.169125 \text {, or } \underline{\underline{16.9125 \%}} \text {. }
$$

Finally, if Jersey's common stock had a beta of 1.00, we would find that

$$
k_{e}=.035+(.0725) \cdot 1.00=.035+.0725=.1075 \text {, or } \underline{\underline{10.75 \%}} \text {. }
$$

A beta of 1 indicates that Jersey's common stockholders have tended, historically, to earn annual returns equal to what stock market investors earned on average. It should not be surprising that a stock with average risk would have an expected annual return equal to the expected average annual return on the stock market overall (an average stock should be expected to earn the stock market's average annual rate of return).
10. Kane Kazoo \& Karaoke Corporation paid a cash dividend (total of four quarterly payments) of $\$ 4.20$ per share to its common stockholders in the most recent year. The stock currently can be purchased for $\$ 47.80$ per share. A consensus view among investors and professional analysts is that Kane's dividend stream will increase at a fairly constant annual rate of $1.5 \%$ for many years into the future. Based on observations over many years, analysts feel that the beta for Kane's common stock is 1.65 . Stock market observers expect the average annual return on the market as a whole to be $9.75 \%$ into the distant future, while they expect the "risk-free" rate earned by holders of short-term U.S. government bonds to be $3.75 \%$ per year. Estimate Kane's cost of common stock (equity) financing $\mathrm{k}_{\mathrm{e}}$ based on both the constant dividend growth (also called discounted cash flow) approach and the security market line approach. Why do these two approaches not lead to the same $\mathrm{k}_{\mathrm{e}}$ estimate?

Type: Cost of common stock. Here we have the information to estimate the cost of common equity (or retained earnings), $\mathrm{k}_{\mathrm{e}}$, using both the constant dividend growth model and the security market line model. We can compute:

$$
\begin{aligned}
k_{e}= & \frac{D_{1}}{P_{0}}+g=\frac{D_{0}(1+g)}{P_{0}}+g=\frac{\$ 4.20(1.015)}{\$ 47.80}+.015=\frac{\$ 4.263}{\$ 47.80}+.015=.089184+.015=\underline{\underline{10.42 \%}}, \\
& \text { and } k_{e}=k_{r f}+\left(k_{m}-k_{r f}\right) \beta=.0375+(.0975-.0375) \cdot 1.65=.0375+.099=13.65 \% .
\end{aligned}
$$

For the constant dividend growth approach, we are given the most recent year's dividend total $D_{0}$, so if we expect growth to occur at a $1.5 \%$ constant annual rate in the future, including the coming year, we compute the coming year's expected total dividend $D_{1}$ as $D_{0}(1+g)=\$ 4.20(1.015)=$ $\$ 4.263$.

For the security market line approach, we are given the expected annual return on the stock market, $k_{m}=.0975$. So we have to compute the market risk premium $k_{m}-k_{r f}=.0975-.0375=.06$. Once again, watch your terminology; be careful not to mistake the prior year's total per-share dividend payment $D_{0}$ for the coming year's expected payment $D_{1}$, or to mistake the expected market return $k_{m}$ for the expected market risk premium $k_{m}-k_{r f}$.

We get very different estimates of $k_{e}$ with the two different methods because they are based on different assumptions and input values. We prefer the constant dividend growth model if we are confident that the market expects a constant annual growth rate $g$ and that we have identified it. We would choose the security market line method if we lacked confidence in our $g$ estimate but were confident in our estimates of $k_{r f}, k_{m}$, and $\beta$. Lacking confidence in either method, we might have to resort to a not-very-systematic $k_{e}$ estimate by simply adding a few percentage points to our estimate of the company's before-tax cost of debt $\mathrm{k}_{\mathrm{d}}$.
11. The managers of Kendall Kennels \& Kibble, Inc. have determined that their company’s optimal debt (debt to assets) ratio is $38 \%$. If the appropriate capital structure is maintained, the annual cost of common equity financing $\mathrm{k}_{\mathrm{e}}$ should be $9.8 \%$, and the before-tax cost of debt financing $\mathrm{k}_{\mathrm{d}}$ (the interest rate Kendall would pay to new lenders) should be $5.9 \%$ annually. The firm does not use preferred stock financing. If Kendall pays income tax at a $24 \%$ combined state-plus-federal marginal rate, what is its weighted average cost of capital, $\mathrm{k}_{\mathrm{A}}$ or WACC? With debt costing so much less than equity, why should Kendall not finance more heavily with borrowed money? And why do we compute a weighted average cost of money (capital)?

Type: WACC. This example illustrates the most straightforward type of WACC problem. With no preferred stock financing, the preferred stock term $w_{p} \cdot k_{p}$ in the more general form of the WACC equation becomes zero, and our working equation becomes simply

$$
W A C C=w_{d} \cdot k_{d}(1-t)+w_{e} \cdot k_{e} .
$$

The capital structure is also presented in a straightforward way here; with debt ratio $w_{d}$ identified as $38 \%$, and with no preferred stock financing, we know that the proportion of assets financed with common equity has to be $w_{e}=62 \%$ (the various w's must sum to $100 \%$, since we want to account for all of Kendall's financing). So we can compute

$$
\left.\begin{array}{rl}
\text { WACC } & =.38 \cdot .059(1-.24)+.62 \cdot .098 \\
& =.38 \cdot .04484+.62 \cdot .098 \\
= & .017039+.06076=.077799
\end{array}\right) \underline{\underline{7.78 \%}} .
$$

Note that the after-tax cost of debt financing is $k_{d}(1-t)=.059(1-.24)=.04484$, or $4.484 \%$. Kendall expects to pay new lenders a $5.9 \%$ annual interest rate, but the cost to the company of providing that $5.9 \%$ return should be only $4.484 \%$ because of the expected income tax savings (paying interest reduces taxable income, whereas financial returns to preferred and common stockholders are deemed to come from the firm's after-tax net income, so there is no difference between the before-tax and after-tax costs of preferred or common stock financing). And with part ( $38 \%$ ) of the company's money costing $4.484 \%$ and part ( $62 \%$ ) costing $9.8 \%$, the weighted average has to be something in between, as we see with our 7.78\% answer (a bit closer to the 9.8\% than to the $4.484 \%$, because more of Kendall's financing comes from the costlier equity source).

We would not want to "pile on" debt financing because the company's adoption of its $38 \%$ debt/62\% common stock capital structure reflected management's belief that this mix has been best for keeping the company's overall cost of money as low as possible. Upsetting that balance would cause the weighted average cost of money (capital) to rise. With more debt financing, each lender would share the "front of the line" claim with more fellow lenders (and have a smaller equity cushion) - and thus would rightly perceive more risk, and as a result would expect an annual return more than $5.9 \%$.

Finally, we compute a weighted average cost of money, or capital, so that we can understand what it would cost to raise money for a new investment project. We try to "get inside the heads" of potential money providers so we can determine whether a particular investment could be expected to "pay for itself" by giving money providers the financial returns they would require. Let's say that Kendall could expect to earn a $10.5 \%$ annual rate of return by buying production equipment for its factory. This investment would be favorable, because that $10.5 \%$ return exceeds the company's $7.78 \%$ annual cost of compensating the investors who would provide the money to pay for the equipment; it makes sense to obtain money at a cost of $x \%$ if it can be invested to earn a higher $y \%$.

Without computing a weighted average of the costs of the various money components, we would have trouble seeing exactly what rate of return has to be earned for the investment to pay for itself. (Is it the $9.8 \%$ cost of compensating the owners, or the $4.484 \%$ cost of compensating the lenders? Neither; it is the $7.78 \%$ weighted average of those two component costs.) It should be intuitively clear that it makes sense to obtain money at a $7.78 \%$ annual cost if that money can be invested to earn a $10.5 \%$ annual rate of return.
12. The managers of LaSalle Laproscopic Laboratories, Ltd. have determined that their company's optimal debt to equity ratio is $72.41 \%$. If the appropriate capital structure is maintained, the cost of common equity financing $\mathrm{k}_{\mathrm{e}}$ should be $10.55 \%$, and the before-tax cost of debt financing $\mathrm{k}_{\mathrm{d}}$ (the interest rate LaSalle would pay new lenders) should be $6.65 \%$. The firm does not use preferred stock financing. If LaSalle pays income tax at a $24 \%$ combined state-plus-federal marginal rate, what is its weighted average cost of capital, $\mathrm{k}_{\mathrm{A}}$ or WACC?

Type: WACC. Here we have another straightforward example, with no preferred stock financing, so again the preferred stock term $w_{p} \cdot k_{p}$ becomes zero and we work with the simpler

$$
\text { WACC }=w_{d} \cdot k_{d}(1-t)+w_{e} \cdot k_{e}
$$

form of the WACC equation. The only slight complication here is that the capital structure is given in terms of the debt/equity ratio, rather than the debt ratio $w_{d}$ (or $D / V$ ) and the equity ratio $w_{e}$ (or $E / V$ ). So we simply note that if debt/equity is .7241 , then the lenders invest $72.41 \varnothing$ for every $\$ 1.00$ the owners invest, for $a w_{d}$ of $\$ .7241 / \$ 1.7241=42 \%$ and $a w_{e}$ of $\$ 1.00 / \$ 1.7241=58 \%$. Then we compute

$$
\begin{aligned}
\text { WACC } & =.42 \cdot .0665(1-.24)+.58 \cdot .1055 \\
& .42 \cdot .0665(.76)+.58 \cdot .1055 \\
& =.42 \cdot .05054+.58 \cdot .1055 \\
= & .021227+.061190=.082417 \text { or } \underline{\underline{8.24 \%}} .
\end{aligned}
$$

(We would likely round to a couple of decimal places because we know that "getting inside the heads" of potential money providers involves approximating.) Again we have an answer that is above the .05054 lower cost and below the .1055 higher cost, so the $8.24 \%$ passes an initial reasonableness test. (It would cost the company $5.054 \%$ each year to deliver a $6.65 \%$ return to lenders, and would cost $10.55 \%$ each year to deliver a $10.55 \%$ return to owners when earnings are retained. So an answer below $5.054 \%$ or above $10.55 \%$ could not possibly be a correct weighted average of the component costs of money.) Since its annual weighted average cost of money is estimated to be a little more than $8 \%$, we can see that LaSalle should reject any proposed investment project that is not expected to produce an annual rate of return at least that high.
13. The managers of Livingston Loading \& Lading feel that their company's optimal capital structure consists of $33 \%$ debt, $13 \%$ preferred stock, and $54 \%$ common stock. Livingston pays income tax at a marginal rate (combined state-plus-federal) of $26 \%$. If its before-tax cost of debt $\mathrm{k}_{\mathrm{d}}$ is $9.375 \%$, its cost of preferred stock $\mathrm{k}_{\mathrm{p}}$ is $8.5 \%$, and its cost of common equity $\mathrm{k}_{\mathrm{e}}$ is $13.25 \%$, what is its weighted average cost of capital ( $\mathrm{k}_{\mathrm{A}}$, or WACC)? With $\mathrm{k}_{\mathrm{p}}$ less than $\mathrm{k}_{\mathrm{d}}$, why would Livingston not issue less debt (and common stock) and make greater use of preferred stock financing?

Type: WACC. Now we have a case in which all three general forms of financing are present: debt, preferred stock, and common stock or equity. Therefore we must use the more comprehensive form of the WACC equation:

$$
W A C C=w_{d} \cdot k_{d}(1-t)+w_{p} \cdot k_{p}+w_{e} \cdot k_{e} .
$$

Based on the values provided, we compute:

$$
\begin{aligned}
& \text { WACC }=.33 \cdot .09375(1-.26)+.13 \cdot .085+.54 \cdot .1325 \\
&=.33 \cdot .09375(.74)+.13 \cdot .085+.54 \cdot .1325 \\
&=.33 \cdot .069375+.13 \cdot .085+.54 \cdot .1325 \\
&=.022894+.011050+.07155=.105494 \text { or } \underline{\underline{10.5494 \%}} .
\end{aligned}
$$

So it would cost Livingston $10.5494 \%$ annually, on average, to deliver fair returns to new money providers. Thus it should take on an investment project only if that project is expected to provide an annual rate of return of at least about 10.5494\%.

Let's make sure the answer makes sense. Look at the second line in the computations above; we can see that $33 \%$ of the company's money would cost it $6.9375 \%$ annually (it would cost $6.9375 \%$ to deliver new lenders a $9.375 \%$ return after the tax savings), $13 \%$ of the money would cost it $8.5 \%$ annually, and $54 \%$ of the money would cost it $13.25 \%$ annually. In averaging these costs (recall that we are computing a weighted average cost of capital), we should get an answer higher than the lowest cost component ( $6.9375 \%$ ) and lower than the highest cost component ( $13.25 \%$ ); 10.5494\% meets that initial "reasonableness" test. Recall that we do not adjust the cost of preferred or common stock for income tax savings, because the financial returns to both common and preferred stockholders are seen legally as coming from the company's net income, which is left after income tax already has been paid (so at that point it is too late to realize an income tax savings).

Finally, we would not want to "pile on" the preferred stock financing for two reasons. First, while preferred stock would cost less than debt on a pre-tax basis, debt would cost less after the income tax savings is factored in (6.9375\% is less than $8.5 \%$ ). Second, Livingston adopted its $33 \%$ debt/ $13 \%$ preferred $/ 54 \%$ common capital structure because that mix seemed best for keeping the company's overall cost of money as low as possible. Upsetting that balance would cause the weighted average cost of money (capital) to rise. For example, if more preferred stock were issued, each preferred stockholder would have to share the "middle of the line" claim with more fellow preferred stockholders - and thus would rightly perceive more risk, and as a result would expect to earn an annual rate of return higher than that nice, low $8.5 \%$.
14. The managers of Macoupin Magnet Manufacturing, Inc. feel that their company's optimal capital structure contains $28 \%$ debt and $17 \%$ preferred stock financing. Macoupin pays income tax at a $25 \%$ marginal rate (combined state-plus-federal); its before-tax cost of debt $\mathrm{k}_{\mathrm{d}}$ is $7.85 \%$ annually and its cost of preferred stock $\mathrm{k}_{\mathrm{p}}$ is $6.75 \%$ annually. If Macoupin's annual weighted average cost of capital ( $\mathrm{k}_{\mathrm{A}}$, or WACC) is $8.32 \%$, what is the company's annual cost of common stock, or equity, financing, $\mathrm{k}_{\mathrm{e}}$ ?

Type: WACC. Here we have another example with all three general forms of financing (we are not directly told that Macoupin has common equity financing, but we know that it does, because every company has to have owners). But now the unknown to solve for is one of the component costs, rather than the WACC. First we must identify the three weights. We know that $w_{d}$ is $28 \%$ and $w_{p}$ is $17 \%$; since the proportions must sum to 1.00 the remaining weight $w_{e}$ has to be $1.00-.28-.17=$ .55 , or $55 \%$. Then we simply solve for the unknown $k_{e}$ in our WACC equation:

$$
W A C C=w_{d} \cdot k_{d}(1-t)+w_{p} \cdot k_{p}+w_{e} \cdot k_{e} .
$$

Based on the values provided, we compute:

$$
\begin{gathered}
.0832=.28 \cdot .0785(1-.25)+.17 \cdot .0675+.55 \cdot \mathrm{k}_{e} \\
.0832=.28 \cdot .0785(.75)+.17 \cdot .0675+.55 \cdot \mathrm{k}_{e} \\
.0832=.28 \cdot .058875+.17 \cdot .0675+.55 \cdot \mathrm{k}_{e} \\
.0832=.016485+.011475+.55 \cdot \mathrm{k}_{e} \\
.0832=.027960+.55 \cdot \mathrm{k}_{e} \\
.05524=.55 \cdot \mathrm{k}_{e} \\
.100436 \text { or } \underline{\underline{010.04 \%}}=\mathrm{k}_{e}
\end{gathered}
$$

(approximately/just above 10\%). After performing so many algebraic steps, it can be helpful to double-check our work:

$$
\begin{aligned}
\text { WACC } & =.28 \cdot .0785(1-.25)+.17 \cdot .0675+.55 \cdot .100436 \\
& =.28 \cdot .058875+.17 \cdot .0675+.55 \cdot .100436 \\
& =.016485+.011475+.055240=.0832 \text { or } 8.32 \% .
\end{aligned}
$$

## Work These for Extra Practice

15. Massac Masking Tape wants to borrow money by issuing bonds. Investment bankers have advised Massac that it should expect to pay a $5.95 \%$ annual interest rate to make the bonds attractive to lenders. If the company pays income tax at a $27 \%$ combined state-plus-federal marginal rate, what is its after-tax annual cost of debt financing?

Type: Cost of debt. Massac's before-tax cost of debt (the interest rate $\mathrm{k}_{d}$ that new lenders would expect to receive) is $5.95 \%$ annually. But because the payment of interest reduces a company's income taxes, the after-tax cost $k_{d}(1-t)$ generally is less than the before-tax cost. Here we compute

$$
k_{d}(1-t)=.0595(1-.27)=.0595 \cdot .73=.043435 \text {, or } 4.3435 \% \text {. }
$$

So if Massac borrows money under a bond issue with the terms that have been examined by the firm's investment banking advisors, lenders would expect to be paid $5.95 \%$ interest each year. But since interest is deducted from operating income in the computation of taxable income, the company saves $27 \$$ in income tax every time it pays $\$ 1.00$ in interest. So after factoring in the income tax savings, it should cost Massac only $4.3435 \%$ each year to deliver a $5.95 \%$ annual return to new lenders from whom it borrows money.
16. McLean Manila Envelope Enterprises, Inc. issued preferred stock many years ago. The per-share par value is $\$ 50$, and the per-share annual dividend (total of four quarterly payments) is $\$ 3.32$. If the shares are currently selling in the market for $\$ 39.50$ each, what is our estimate of the company's annual cost of preferred stock financing, $\mathrm{k}_{\mathrm{p}}$ ?

Type: Cost of preferred stock. Here we want to estimate what it would cost McLean to raise new money from preferred stockholders. We can "get inside the heads" of potential preferred stockholders fairly easily, since preferred stockholders typically expect to be paid an unchanging dollar amount in per-share dividends every year forever. By looking at the expected yearly dividend as a percentage of the price paid, we can see what annual rate of return investors have been creating for themselves when they have taken the places of existing preferred stockholders in recent transactions. The inference we typically draw is that, unless we have strong reasons to believe otherwise, those who would take new spots at the "middle of the line" would expect the same annual rate of return as those who have recently taken existing spots at the "middle of the line." With an unchanging expected annual dividend, $D_{0}=D_{1}=D_{100}$, etc., we can simply show the dividend as $D$, and can compute

$$
k_{p}=\frac{D}{P_{0}}=\frac{\$ 3.32}{\$ 39.50}=.084, \text { or } 8.4 \% .
$$

Because preferred stockholders' returns are viewed legally as coming from after-tax net income, this $8.4 \%$ is both the before-tax and after-tax cost of preferred stock financing. In this situation, the shares were sold many years ago for $\$ 50$ each (the par value). At that time, investors paid $\$ 50$ to get $\$ 3.32$ in yearly dividends, for a $\$ 3.32 \div \$ 50=6.64 \%$ annual rate of return. But now transaction prices in the market tell us that a fair financial return for those who provide money to McLean as preferred stockholders is higher; either the required rate of return for investors is higher in general than it was when this preferred stock was issued, or else McLean has come to be seen as a riskier company to invest in, such that $8.4 \%$ is now a fair annual rate of return.
17. The common stock of Menard Menswear, Inc. can currently be purchased in the market for $\$ 75$ per share. The firm's earnings per share were $\$ 10.91$ in the most recent year, of which $55 \%$ was paid to the common stockholders as dividends. Dividends per share are expected to grow by a fairly steady rate approximating 3\% per year into the distant future. What was $\mathrm{D}_{0}$ ? What is our estimate of $\mathrm{D}_{1}$ ? What is Menard's annual cost of common stock financing $\mathrm{k}_{\mathrm{e}}$, as estimated with the constant dividend growth model?

Type: Cost of common stock. The constant dividend growth approach for estimating a company's annual cost of delivering fair financial returns to its owners is based on the equation

$$
k_{e}=\frac{D_{1}}{P_{0}}+g=\frac{D_{0}(1+g)}{P_{0}}+g .
$$

Here we must convert an earnings figure to a dividend figure: $55 \%$ of the most recent year's $\$ 10.91$ in earnings was paid as dividends, for $.55 \times \$ 10.91=\$ 6.00$. Because that value represents the total per-share dividend (sum of four quarterly payments) paid over the year that just ended, we know that $D_{0}=\$ \underline{\underline{6.00}}$ per share. But we ultimately need an estimate of $D_{1}$, the expected total per-share dividend for the coming year, to compute $k_{e}$. So we must estimate $D_{1}$ as $D_{0}(1+g)=\$ 6.00(1.03)=$ $\$ 6.18$. And then we can compute

$$
\mathrm{k}_{e}=\frac{\$ 6.00(1.03)}{\$ 75.00}+.03=\frac{\$ 6.18}{\$ 75.00}+.03=.0824+.03=.1124 \text { or } \underline{\underline{11.24 \%}} \text {. }
$$

If Menard were to create a larger total of claims at the "back of the line" by retaining earnings or issuing new common stock, we infer that the investors who hold these new claims would expect an annual rate of return equal to the $11.24 \%$ rate that investors have been creating for themselves when they have paid $\$ 75$ for existing shares, if the buyers of existing shares expect dividends to begin at $\$ 6.18$ and then grow by $3 \%$ per year. Thus it is important that the $3 \%$ growth rate represent a widely held consensus expectation; otherwise we do not have a consistent $g$ estimate and can not meaningfully use the "constant" dividend growth model to "get inside the heads" of potential "back of the line" investors.
18. Professional investment analysts expect that the average annual rate of return on the overall stock market will be $9.25 \%$ in the future. These same analysts feel that the beta for Moultrie Mouse \& Modem's common stock is 1.25. The "risk-free" rate of return earned by purchasers of short-term U.S. government bonds is expected to be $3.15 \%$ annually. Using the security market line model, estimate Moultrie's cost of common equity financing, $\mathrm{k}_{\mathrm{e}}$.

Type: Cost of common stock. Here we have the information to estimate the cost of equity, $\mathrm{k}_{\mathrm{e}}$, using the security market line model, which is based on the logic that the annual rate of return expected by a common stockholder in Moultrie should be the annual return expected by buyers of risk-free securities, plus something extra to reflect the risk of investing in the stock market in general and the risk of being a Moultrie common stockholder in particular. Therefore we can compute

$$
\begin{gathered}
\mathrm{k}_{e}=\mathrm{k}_{\mathrm{rf}}+\left(\mathrm{k}_{\mathrm{m}}-\mathrm{k}_{\mathrm{rf}}\right) \beta \\
=.0315+(.0925-.0315) \cdot 1.25 \\
=.0315+(.061) \cdot 1.25=.0315+.07625=\underline{\underline{10.775 \%}} .
\end{gathered}
$$

As always, use caution in interpreting and working with the information given. Here we are told the expected annual rate of return on the stock market overall, $\mathrm{k}_{\mathrm{m}}=.0925$. Note that we could as easily have been given the expected market risk premium, $k_{m}-k_{r f}=.061$. (If that difference had been given, then we could have computed the expected market-wide return as $k_{m}-.0315=.061 \Rightarrow k_{m}=$ $.061+.0315=.0925$ or $9.25 \%$ annually.) Here we assume that there is not a consistent estimate of growth in dividends going into the future $g$, so we can not use the constant dividend growth model and must rely on this logic-based approach to "getting inside the heads" of potential common equity investors.
19. The optimal debt to assets ratio for Ogle Overland Express is believed to be $37 \%$. If this optimal capital structure is maintained, the before-tax cost of debt financing $\mathrm{k}_{\mathrm{d}}$ (the interest rate Ogle would expect to pay new lenders) should be $6.625 \%$ per year and the cost of common equity financing $\mathrm{k}_{\mathrm{e}}$ should be $11.875 \%$ annually (Ogle does not use preferred stock financing). If the company's marginal combined state-plus-federal income tax rate is $23 \%$, what is its weighted average cost of capital, $\mathrm{k}_{\mathrm{A}}$ or WACC? What if the marginal income tax rate were $39 \%$ ?

Type: WACC. With no preferred stock financing, the preferred stock term $w_{p} \cdot k_{p}$ in the more general form of the WACC equation becomes zero, and our working equation becomes simply

$$
\text { WACC }=w_{d} \cdot k_{d}(1-t)+w_{e} \cdot k_{e} .
$$

With a $w_{d}=37 \%$ debt ratio and no preferred stock financing, the equity ratio must be $w_{e}=63 \%$ (so the various w's account for 100\% of Ogle's financing). Thus we compute

$$
\begin{aligned}
& \text { WACC }=.37 \cdot .06625(1-.23)+.63 \cdot .11875 \\
&=.37 \cdot .051013+.63 \cdot .11875 \\
&=.018875+.074813=.093687 \text { or } \underline{\underline{9.3687 \%}} .
\end{aligned}
$$

Lenders to Ogle would expect to earn an interest rate of $6.625 \%$ annually, but with the $23 \%$ income tax savings (every time it pays a dollar in interest, Ogle saves $23 \$$ in income tax) it would cost the company only $5.1013 \%$ each year to deliver this $6.625 \%$ return. Owners would expect an $11.875 \%$ annual return on their equity investment, and because returns are provided to owners after income taxes have been paid (when there is no longer a basis for realizing an income tax savings) it would cost Ogle $11.875 \%$ to deliver this $11.875 \%$ return. Because part of the company's money would cost $5.1013 \%$ and part would cost $11.875 \%$ each year, it should not be surprising that the weighted average of these costs would fall somewhere in between (like 9.3687\%).

If it paid income taxes at a 39\% marginal rate, as could have been true before 2018 when corporate income tax rates were much higher (and thus the tax sheltering impact of the interest deduction was much greater), Ogle's WACC would be a lower

```
WACC =. 37 - .06625 (1-.39) +.63 ·. .11875
    =. .37 -. 040413 +. . 3 -. 11875
=.014953 + . 074813 = . 089765 or 8.9765%.
```

Note that the after-tax cost of debt financing, and thus the weighted average cost of capital (if there is any debt financing, which for most companies there generally is), is lower when the marginal income tax rate is higher, holding everything else equal. The reason is that the debt financing leads to greater income tax savings. But it is not the case that paying more income tax is good for a company's financial picture. Paying income tax at a higher rate makes it cheaper for a firm to borrow money, holding everything else the same - but it also means that operating cash flows (money left for lenders and owners after all costs, including income tax on the operations, have been paid) will be lower.
20. The managers of Piatt Pie \& Pastry feel that the optimal capital structure for their company is $23 \%$ debt, $16 \%$ preferred stock, and $61 \%$ common stock. They feel that if they were to borrow money for a new capital project their lenders would expect to receive an $8.2 \%$ annual interest rate, while preferred stockholders would expect a $7.8 \%$ annual return and common stockholders would expect a $10.9 \%$ annual return. If Piatt pays income tax at a $24 \%$ combined state-plus-federal marginal rate, what is its weighted average cost of capital (WACC, or $\mathrm{k}_{\mathrm{A}}$ )?

Type: WACC. Piat† makes use of all three general forms of financing, so we estimate the WACC with the equation

$$
\begin{gathered}
\text { WACC }=w_{d} \cdot k_{d}(1-t)+w_{p} \cdot k_{p}+w_{e} \cdot k_{e} . \\
=.23 \cdot .082(1-.24)+.16 \cdot .078+.61 \cdot .109 \\
=.23 \cdot .06232+.16 \cdot .078+.61 \cdot .109 \\
=.014334+.01248+.06649=.093304 \text { or } 9.3304 \% .
\end{gathered}
$$

For an investment project of average risk, it would cost Piatt $9.3304 \%$ annually to deliver fair financial returns to new money providers. $23 \%$ of its invested money comes from lenders who expect an $8.2 \%$ annual interest rate (but with the tax savings it would cost Piatt only $6.232 \%$ to deliver that $8.2 \%$ return), $16 \%$ would cost $7.8 \%$ annually, and the remaining $61 \%$ would cost $10.9 \%$ annually. A weighted average of these costs should be higher than the lowest cost component ( $6.232 \%$ ) and lower than the highest cost component (10.9\%), as here with the $9.3304 \%$ WACC estimate.
21. The directors of Pulaski Pulp \& Pulverizing Company feel that their company's optimal capital structure contains $48 \%$ debt and $52 \%$ common equity (they do not make use of preferred stock financing). Pulaski pays income tax at a combined state-plus-federal marginal rate of $24 \%$. The company's annual weighted average cost of capital (WACC, or $\mathrm{k}_{\mathrm{A}}$ ) has been estimated to be $10.875 \%$. If Pulaski's annual after-tax cost of debt financing is $6.435 \%$, what would we estimate its annual before-tax cost of debt and its annual cost of common equity financing to be? If the annual WACC is $10.875 \%$ and Pulaski's annual cost of common equity financing is $14.15 \%$, what would we estimate its before-tax and after-tax annual costs of debt financing to be?

Type: Computing WACC components. Here we have an algebraic exercise involving the simpler form of the WACC equation (there is no preferred stock financing), with $k_{e}$ or $k_{d}$ as the unknown to solve for. We are given the after-tax cost of debt $k_{d}(1-t)=6.435 \%$, so with a $24 \%$ marginal income tax rate we compute the before-tax cost $k_{d}$ as

$$
\begin{gathered}
k_{d}(1-.24)=.06435 \\
k_{d}(.76)=.06435 \Rightarrow .084671 \text { or } \underline{\underline{8.4671 \%}}=k_{d}
\end{gathered}
$$

(new lenders would expect to get an $8.4671 \%$ annual interest rate, but with the income tax savings it would cost Pulaski only $6.435 \%$ annually to deliver that $8.4671 \%$ return). Then we can compute

$$
\begin{gathered}
\text { WACC }=w_{d} \cdot\left[k_{d}(1-t)\right]+w_{e} \cdot k_{e} \\
.10875=.48 \cdot .084671(1-.24)+.52 \cdot k_{e} \\
.10875=.48 \cdot .06435+.52 \cdot k_{e} \\
.10875=.030888+.52 \cdot k_{e} \\
.077862=.52 \cdot k_{e} \Rightarrow .149735 \text { or } 14.9735 \%=k_{e} .
\end{gathered}
$$

After doing all those computations it can be helpful to double-check:

$$
.48 \cdot .084671(1-.24)+.52 \cdot .149735=.030888+.077862=.10875 . \checkmark
$$

On the other hand, if we are given the WACC and the cost of common equity $k_{e}$ we can compute

$$
\begin{gathered}
\text { WACC }=w_{d} \cdot k_{d}(1-t)+w_{e} \cdot k_{e} \\
.10875=.48 \cdot k_{d}(1-.24)+.52 \cdot .1415 \\
.10875=.48 \cdot k_{d} \cdot .76+.52 \cdot .1415 \\
.10875=.3648 \cdot k_{d}+.07358 \\
.03517=.3648 \cdot k_{d} \\
.096409 \text { or } \underline{\underline{9.6409 \%}}=k_{d} .
\end{gathered}
$$

Again, let's double-check: $\quad .48 \cdot .096409(1-.24)+.52 \cdot .1415=.035170+.037580=.10875 . \checkmark$
The before-tax cost of debt $k_{d}$ is $9.6409 \%$, so the after-tax cost $k_{d}(1-t)$ is $096409(1-.24)=$ 7.3271\%. Based on these figures, we see that new lenders would expect to earn a $9.6409 \%$ annual interest rate, but with the income tax savings it would cost Pulaski only $7.3271 \%$ each year to deliver that $9.6409 \%$ interest return.
22. Financial analysts expect a per-share dividend (total of four quarterly payments) of $\$ 3.80$ to be paid to Randolph Ranch \& Range Equipment Company's common stockholders in the coming year, and expect the dividend stream to increase by approximately $5.5 \%$ steadily per year thereafter into the distant future. The company's common stock can currently be purchased for $\$ 93.75$ per share. Randolph obtains all of its money for capital investment projects from lenders ( $32 \%$ ) and common stockholders ( $68 \%$ ); it has never issued preferred stock and is not expected to in the future. If they do maintain this capital structure, Randolph's managers expect to be able to borrow money at a $7.75 \%$ annual interest rate. Randolph pays income tax each year at a $24 \%$ marginal (combined state-plus-federal) rate. Using the constant dividend growth model, estimate Randolph's cost of common equity financing, and then compute the company's weighted average cost of capital (WACC, or $\mathrm{k}_{\mathrm{A}}$ ).

Type: Cost of common stock, WACC. Here we compute the cost of common equity financing and then use that figure in computing the company's WACC. One point of caution is to recognize that here we are given $D_{1}$, not the most recent year's $D_{0}$, so we do not have to multiply $D_{0}(1+g)$ to get our needed $D_{1}$ value. (Be careful about what you are given!!) Therefore we can simply compute

$$
k_{e}=\frac{D_{1}}{P_{0}}+g=\frac{\$ 3.80}{\$ 93.75}+.055=.040533+.055=\underline{\underline{9.5533 \%}} .
$$

We can see (though we are not asked to compute this figure) that, if dividends have been rising consistently by about $g=5.5 \%$ per year, then the most recent year's total dividend $D_{0}$ should have
been about $\$ 3.80 \div 1.055=\$ 3.60$. (The steady expected growth rate of $5.5 \%$ per year that we use in this example would be awfully high in a real world example. We could also debate whether it really makes sense to carry our component costs out as many decimal places as we are doing here since "getting inside the heads" of potential money providers involves some degree of estimating.) Now, with a ke estimate in hand, we can compute

$$
\begin{gathered}
W A C C=w_{d} \cdot k_{d}(1-t)+w_{e} \cdot k_{e} \\
=.32 \cdot .0775(1-.24)+.68 \cdot .095533 \\
.32 \cdot .0589+.68 \cdot .095533 \\
=.018848+.064962=.08381 \text { or } 8.381 \% .
\end{gathered}
$$

Randolph would expect to pay new lenders an $8.15 \%$ annual interest rate, but with the $30 \%$ income tax savings it would cost only $.0815(1-.30)=5.705 \%$ for the company to deliver that $8.15 \%$ return. Then it would cost $9.5533 \%$ annually to deliver a fair financial return to Randolph's owners if the firm got new equity money by retaining earnings. On average, it would cost $8.3218 \%$ each year to provide the investors with appropriate financial returns. Randolph therefore should not undertake a capital investment project unless the expected annual rate of return would be at least as high as the $8.3218 \%$ needed to fairly compensate those who would put up the money to make the investment.
23. The managers of Saline Sales \& Salvage Company believe that, if they were to keep their optimal capital structure ( $36 \%$ debt, $8 \%$ preferred stock, $56 \%$ common equity) intact, they would be able to borrow money at a $6.95 \%$ before-tax annual interest rate. Saline's existing preferred stock, which pays a $\$ 7.04$ total annual per-share dividend, has been selling in recent transactions for $\$ 114.94$ per share; and its existing common stock, on which the company paid a $\$ 1.16$ total per-share dividend over the most recent year, has been selling in recent transactions for $\$ 12.95$ per share. Investment analysts feel that the company's stream of dividends to its common stockholders will increase by a fairly constant rate approximating $2 \%$ annually going into the distant future. Saline pays income tax each year at a $25 \%$ marginal (combined state-plus-federal) rate. Compute the cost of each of the company's capital components and the weighted average cost of capital ( $\mathrm{k}_{\mathrm{A}}$, or WACC). Should Saline invest in a capital project that is expected to provide a $10 \%$ annual rate of return? What about a $7 \%$ expected annual rate of return?

Type: Comprehensive. In this example we compute various component costs in the company's financing mix, and then tie them together - within a given capital structure - into a WACC estimate. Because Saline uses all three general forms of financing (debt, preferred stock, and common equity), we use the more comprehensive form of the WACC equation:

$$
\text { WACC }=w_{d} \cdot k_{d}(1-t)+w_{p} \cdot k_{p}+w_{e} \cdot k_{e} .
$$

Here, the before-tax cost of debt $k_{d}$ (the interest rate the firm would expect to pay if it borrowed money) is given as $6.95 \%$. But with the $25 \%$ expected income tax savings from paying interest, the after-tax cost of debt financing should be only $k_{d}(1-t)=.0695(1-.25)=.052125$ or $5.2125 \%$. If new investors were to join the "front of the line," they would expect to receive a $6.95 \%$ annual interest return, but it would cost Saline only $5.2125 \%$ (with the tax savings) each year to deliver that $6.95 \%$ return.

We infer the annual dividend percentage Saline would have to pay to attract new investors to the "middle of the line" by looking at the rate of return preferred stock buyers have been creating for themselves when they have purchased existing preferred shares (when they took existing spots at the "middle of the line" in recent transactions). The per-share annual (generally the total of four quarterly installments) dividend paid to a company's preferred stockholders typically remains at a
constant level indefinitely, so we can represent the annual dividend as $D_{0}=D_{1}=D_{100}=D$, and thus can compute the expected cost of preferred stock financing $k_{p}$ as

$$
k_{p}=\frac{D}{P_{0}}=\frac{\$ 7.04}{\$ 114.94}=.06125, \text { or } \underline{\underline{6.125 \%}} \text {. }
$$

We might assume that the existing preferred shares were created many years ago, and were sold initially for a par value of $\$ 100$ per share. At that time, $\$ 7.04 / \$ 100=7.04 \%$ per year was an appropriate risk-adjusted rate of return for Saline's preferred stockholders. But now, if Saline were to create new preferred shares, we assume that new investors would join the "middle of the line" for a $6.125 \%$ expected yearly return (either required returns across the investing marketplace have fallen, or else the risk of providing money to Saline is perceived to be lower than it was when the existing preferred stock was issued). In inferring that buyers of new preferred shares would expect the same annual return as is being earned by recent buyers of existing preferred shares, we assume that investors would expect Saline to be just as skilled in managing new money under its control as it has been in managing existing investors' funds.

To compute the cost of common stock financing $k_{e}$ (the annual cost of delivering a fair financial return to the company's current stockholders when earnings that belong to them are retained), if we feel the market expects dividends to rise at a fairly steady rate year by year in the future (such that we can estimate $k_{e}$ with the constant dividend growth model), we need an estimate of $D_{1}$, the coming year's total expected per-share dividend payment (the sum of four expected quarterly payments). If we are given a $D_{1}$ figure then we are ready to compute; if instead (as here) we are told only the most recent year's dividend total $D_{0}$, then we have to translate the given $D_{0}$ into $D_{1}$ (the expected dividend for the coming year) as $D_{0}(1+g)=D_{1}$, here $\$ 1.16(1.02)=\$ 1.18$, rounded to whole cents. Therefore:

$$
k_{e}=\frac{\$ 1.16(1.02)}{\$ 12.95}+.02=\frac{\$ 1.18}{\$ 12.95}+.02=.0911+.02=.1111 \text { or } \underline{\underline{11.11 \%}} .
$$

Then, with the $36 \% / 8 \% / 56 \%$ capital structure specified, we find:

$$
\begin{aligned}
\text { WACC } & =.36 \cdot .0695(1-.25)+.08 \cdot .06125+.56 \cdot .1111 \\
& =.36 \cdot .052125+.08 \cdot .06125+.56 \cdot .1111 \\
& =.018765+.0049+.062216=.085881 \text { or } 8.5881 \% .
\end{aligned}
$$

So if Saline were to raise new money for capital investment projects, it would expect to incur an annual cost of $8.5109 \%$, on average, in delivering fair returns to its money providers. It would cost less than $8.5109 \%$ annually to provide fair risk-adjusted returns to lenders and preferred stockholders (who are themselves very much like lenders), and more than $8.5109 \%$ to provide fair risk-adjusted returns to owners, with the average (a weighted average, since the three types of investors provide different proportions of Saline's financing) falling somewhere between. Because a company should not take on an investment project unless it expects to earn a return higher than the cost of compensating the parties who put up the money for the investment to be made, Saline should accept the project with a $10 \%$ expected annual rate of return but reject the project with the $7 \%$ expected annual return.

## Work These for Extra Insights

24. The annual weighted average cost of capital (WACC, or $\mathrm{k}_{\mathrm{A}}$ ) for Schuyler Skydiving Schools has been estimated to be $8.5291 \%$. The company's cost of common equity financing $k_{e}$ is estimated to be $10.45 \%$ annually, and its estimated annual before-tax cost of debt $\mathrm{k}_{\mathrm{d}}$ is $6.25 \%$ (the firm has no preferred stock financing). If Schuyler pays income tax at a $24 \%$ marginal (combined state-plus-federal) rate, what proportions of its financing would it appear to obtain, respectively, from lenders and common stockholders?

Type: Computing WACC components. Now we use the WACC equation to solve for $w_{d}$ or $w_{e}$. (We only have to solve for one of the two, because there are only two types of financing here - no preferred stock - and thus $w_{d}+w_{e}=100 \%$, so if we know one the other is just $100 \%$ minus the one we know.) So let's focus on $w_{d}$, and represent $w_{e}$ as $\left(1-w_{d}\right)$. Plugging what we know into the equation, we have:

$$
\begin{gathered}
\text { WACC }=w_{d} \cdot k_{d}(1-t)+w_{e} \cdot k_{e} \\
.085291=w_{d} \cdot .0625(1-.24)+\left(1-w_{d}\right) \cdot .1045 \\
.085291=w_{d} \cdot .0475+\left(1-w_{d}\right) \cdot .1045 \\
.085291=.0475 w_{d}+1 \cdot .1045-w_{d} \cdot .1045 \\
.085291=.0475 w_{d}+.1045-.1045 w_{d} \\
.085291=.1045-.057 w_{d} \\
-.019209=-.057 w_{d} \\
.337=w_{d} .
\end{gathered}
$$

If lenders provide $w_{d}=\underline{\underline{33.7 \%}}$ of the company's financing, then common stockholders (since there are no preferred stockholders) have to be providing the other $w_{e}=\left(1-w_{d}\right)=100 \%-33.7 \%=66.3 \%$. Thus the debt ratio is $33.7 \%$, or the debt/equity ratio is $33.7 \% / 66.3 \%=.508296$, or $50.8296 \%$. Let's double-check our values:

$$
.337 \cdot .0625(1-.24)+.663 \cdot .1045=.016008+.069284=.085291 . \checkmark
$$

25. The total per-share dividend that Tazewell Tambourine \& Timpani paid its common stockholders in the firm's first year of operations was $\$ 2.20$. Nine years later, in the most recent year, the total per-share dividend was $\$ 3.34$. What has been the approximate compounded annual growth rate in dividends to common stockholders, g?

Type: Computing growth rate. There are different methods we can use in computing a compounded annual growth rate. One is to view the situation as a simple non-annuity rate of growth or return problem, with the initial dividend level growing to the final dividend level over the indicated number of periods:

$$
\begin{gathered}
\text { BAMT }(1+g)^{n}=\text { EAMT } \\
\$ 2.20(1+g)^{9}=\$ 3.34 \\
(1+g)^{9}=1.51818 \\
\sqrt[9]{(1+g)^{9}}=\sqrt[9]{1.51818} \\
1+g=1.51818^{1 / 9}=1.51818 .11111=1.04748 \\
g=.04748 \text { or about } \underline{4.75 \%} .
\end{gathered}
$$

We would use this growth rate in estimating the cost of common stock $k_{e}$ with the constant dividend growth model, if this rate represented a fairly steady rate of growth over the period in question, and if we felt that it would continue fairly steadily into the distant future. If the dividend did not grow fairly steadily over recent years - for example, if it remained at $\$ 2.20$ per share yearly for 7 years, and then was abruptly raised to $\$ 3.34$ for each of the past two years -
then growth has not been steady, and the "constant" dividend growth model would not be an approprioate tool to use in estimating $\mathrm{k}_{\mathrm{e}}$.
26. The common stock of Union Unified Uniform Wholesalers currently sells in the market for $\$ 26.25$ per share. The total annual per-share dividends paid by Union to its common stockholders over the most recent six years have been:

| Year 1 | $\$ 1.56$ | Year 4 | $\$ 1.73$ |
| :--- | :--- | :--- | :--- |
| Year 2 | $\$ 1.60$ | Year 5 | $\$ 1.77$ |
| Year 3 | $\$ 1.66$ | Year 6 | $\$ 1.83$ |

Compute Union's growth rate in dividends $g$, and the company's cost of common equity financing $k_{e}$ based on the constant dividend growth model.

Type: Computing growth rate, cost of common stock. Here we want to compute the dividend growth rate $g$, and then compute the cost of equity $k_{e}$, with the constant dividend growth model. Since a quick "eyeballing" of the dividends shows that there has been fairly steady growth over the recent period shown, we might quickly estimate a compounded rate of growth by finding the rate at which the first-year dividend (\$1.56) grew to the most recent sixth-year dividend (\$1.83) over the five years that separated them:

$$
\begin{gathered}
\text { BAMT }(1+g)^{n}=\text { EAMT } \\
\$ 1.56(1+g)^{5}=\$ 1.83 \\
(1+g)^{5}=1.173077 \\
\sqrt[5]{(1+g)^{5}}=\sqrt[5]{1.173077} \\
1+g=1.173077^{1 / 5}=1.173077^{2}=1.03244 \\
g=.03244 \text { or about } \underline{\underline{3.244 \%}} .
\end{gathered}
$$

Another way to estimate annual growth is to average the individual year-to-year growth rates:
Year 2 growth over year 1 is ( $\$ 1.60 / \$ 1.56$ ) $-1=2.56 \%$
Year 3 growth over year 2 is ( $\$ 1.66 / \$ 1.60$ ) $-1=3.75 \%$
Year 4 growth over year 3 is ( $\$ 1.73 / \$ 1.66$ ) $-1=4.22 \%$
Year 5 growth over year 4 is ( $\$ 1.77 / \$ 1.73$ ) $-1=2.31 \%$
Year 6 growth over year 5 is ( $\$ 1.83 / \$ 1.77$ ) - $1=3.39 \%$;
the average of these individual rates is $(2.56 \%+3.75 \%+4.22 \%+2.31 \%+3.39 \%) / 5=\underline{\underline{3.246 \%}}$.
If growth is reasonably steady from year to year, then we should tend to get about the same estimate of growth from either of the two methods shown. A problem with the earlier method is that we would compute the same answer no matter what the dividends in years 2-5 happened to be. Of course, if the dividend growth were not both fairly steady in the recent past and expected to remain fairly steady going into the future, then we would not want to use the constant dividend growth model to estimate $\mathrm{k}_{\mathrm{e}}$. Here we will treat the series of individual growth rates shown above (and our expectation for future dividends) as being sufficiently "constant" that we feel confident using the constant dividend growth model, with $g=3.245 \%$. Note that the most recent year's dividend (which is for year 6 in the progression shown above) is $D_{0}=\$ 1.83$, such that

$$
k_{e}=\frac{D_{0}(1+g)}{P_{0}}+g=\frac{D_{1}}{P_{0}}+g
$$

$$
=\frac{\$ 1.83(1.03245)}{\$ 26.25}+.03245=\frac{\$ 1.89}{\$ 26.25}+.03245=.072+.03245=.10445 \text { or } \underline{\underline{10.445 \%}} \text {. }
$$

27. Most investors and professional analysts believe the dividend per share (total of four quarterly payments) that Vermilion Violin \& Viola Varnish, Inc. will pay its common stockholders in the coming year will be $\$ 1.92$, and that dividends will increase by a fairly constant $2.25 \%$ annually going into the future. Vermilion's existing common shares sell in the market for $\$ 26.70$ each, and it is expected that investors would pay that same price for newly created common shares, but the company's investment bankers would charge a fee of $\$ 1.10$ per share for their underwriting services. Compute Vermilion's cost of common stock (more technically viewed as cost of retained earnings) $\mathrm{k}_{\mathrm{e}}$, and its cost of new common stock financing $\mathrm{k}_{\mathrm{ncs}}$.

Type: Cost of common stock with flotation costs. Under the constant dividend growth model, we compute a company's cost of common stock financing $k_{e}$ (which we might more technically call cost of retained earnings) as

$$
k_{e}=\frac{D_{0}(1+g)}{P_{0}}+g=\frac{D_{1}}{P_{0}}+g .
$$

Here we are given the expected first-year dividend per share total $D_{1}=\$ 1.92$, rather than a prior year's actual dividend per share figure $D_{0}$, so we can directly compute

$$
\mathrm{k}_{e}=\frac{\$ 1.92}{\$ 26.70}+.0225=.0719+.0225=.0944 \text { or } \underline{\underline{9.44 \%}} \text {. }
$$

[If dividends have been growing at a fairly constant $2.25 \%$ per year, then $D_{0}$ should have been about $\$ 1.92 \div 1.0225=\$ 1.88$, such that $D_{0}(1+g)=\$ 1.88(1.0225)=\$ 1.92$ for estimated $D_{1 .}$ ] The risk of standing at the "back of the line" among Vermilion's money providers merits earning a $9.44 \%$ annual rate of return on equity. So we can think of $9.44 \%$ as the "cost of retained earnings," the annual cost to Vermilion of delivering a fair rate of return to common equity investors if the company can get its common equity money for new capital investment projects simply by retaining earnings that belong to the current common stockholders.

But what if the net income expected over the near term is too small an amount to provide the common stockholders' share of money for Vermilion's planned capital spending? Then the company will have to pay an investment banking firm to locate new common equity money, and will have to pay both a fair rate of return to the new investors and a "finder's fee" (here, $\$ 1.10$ per share) to the investment bankers. So we would expect the buyers of new common stock to pay $\$ 26.70$ per share (if the investing public believed that Vermilion's managers would be no more or less efficient in managing new money for new projects than they have been in managing investors' money currently under their control). However, Vermilion would net only $\$ 26.70-\$ 1.10=\$ 25.60$ per share after incurring a per-share "flotation" cost of $\$ f=\$ 1.10$, which the investment banking firm would keep for "underwriting" each share of the new common stock issue. Under such conditions, the annual cost to Vermilion of providing fair financial returns to new common stockholders would be

$$
\begin{gathered}
k_{n c s}=\frac{D_{0}(1+g)}{P_{0}-\$ f}+g=\frac{D_{1}}{P_{0}-\$ f}+g \\
=\frac{\$ 1.92}{\$ 26.70-\$ 1.10}+.0225=\frac{\$ 1.92}{\$ 25.60}+.0225=.075+.0225=.0975 \text { or } \underline{\underline{9.75 \%}} .
\end{gathered}
$$

So here, based on the constant dividend growth analysis, we find that a fair return for being a "back of the line" money provider to Vermilion is $9.44 \%$ annually. If the company could obtain the common equity portion of money for new investments simply by retaining earnings from the current common stockholders (an activity that has essentially no administrative costs), then it would cost $9.44 \%$ to deliver this $9.44 \%$ annual return to the group (the current common stockholders) who would put up the equity money for new capital investment projects, and the relevant cost of common stock financing would be the cost of retained earnings, $\mathrm{k}_{e}$. However, if Vermilion wanted to engage in so much new capital spending that it would run out of earnings to retain, then it would have to pay its investment bankers a $\$ 1.10$ per share fee for underwriting (locating new common equity money), along with providing a $9.44 \%$ annual return on equity to the new common stockholders.

Investors would pay $\$ 26.70$ per share, but Vermilion would get the use of only $\$ 26.70-\$ 1.10=$ $\$ 25.60$ per share, and then the new common stockholders would participate in the $2.25 \%$ expected annual growth in dividends (and in the stock's value). So if Vermilion had to obtain its common equity money for new capital spending projects by selling new common stock, the relevant cost of common stock financing would be the $9.75 \%$ cost of new common stock, $k_{n c s}$. While the difference between $k_{e}$ and $k_{n c s}$ may be relatively small, the higher $k_{n c s}$ component cost of equity that would prevail if new common stock had to be sold might increase the company's measured WACC just enough to make an otherwise-acceptable capital investment project unacceptable.

Finally, we should note that there are flotation costs on new issues of debt and preferred stock, as well. But in our cost of capital computations we often ignore these costs because they tend to be relatively small in percentage terms (bonds and preferred stock tend to be sold in big transactions to a small number of large investors, so the flotation cost per dollar raised tends to be quite low, whereas common stock is often sold in smaller transactions to a large number of small investors, so the flotation cost per dollar raised can be more substantial and worth taking note of).
28. The average annual rate of return on the stock market is expected to be $8.75 \%$ in the future, while the annual "risk-free" rate of return earned by holders of short-term U.S. government bonds is expected to be $3.35 \%$. Financial analysts have measured the beta for Wabash Wheel \& Washer's common stock to be 1.4. Using the security market line (SML) model, estimate Wabash's annual cost of common equity (viewed more precisely as cost of retained earnings) financing, $\mathrm{k}_{\mathrm{e}}$. If the company would incur flotation costs of $7.5 \%$ in hiring an investment banking firm to help it issue new common stock, what is our estimate of the cost of new common stock financing, $\mathrm{k}_{\mathrm{ncs}}$ ?

Type: Cost of common stock with flotation costs. Based on the values given, we would estimate $\mathrm{k}_{\mathrm{e}}$ with the security market line approach as

$$
\begin{gathered}
k_{e}=k_{r f}+\left(k_{m}-k_{r f}\right) \beta \\
=.0335+(.0875-.0335) \cdot 1.4 \\
=.0335+.054 \cdot 1.4=.0335+.0756=.1091 \text { or } \underline{\underline{10.91 \%}} .
\end{gathered}
$$

A fair financial return for bearing the risks of being a "back of the line" money provider to Wabash would be $10.91 \%$ annually, under the SML's logic. And if the company could obtain all needed equity money for new capital spending simply by retaining earnings that belong to the current common stockholders, then it would incur no attendant administrative costs, and the relevant cost of common stock financing would be the "cost of retained earnings" $k_{e}$. It would cost just $10.91 \%$ to give the current common stockholders a $10.91 \%$ annual return on the greater investment they would make by giving up dividends if management decided to retain earnings.

If, however, new common stock had to be sold (presumably because net income was not expected to be high enough that the company could simply retain earnings to get all the money needed from "back of the line" investors for new investment projects), then it would cost more than $10.91 \%$ to give the money providers a $10.91 \%$ annual rate of return. With a flotation cost of $\% f=7.5 \%$, the cost of delivering a $10.91 \%$ annual rate of return to new common stockholders would be

$$
\mathrm{k}_{\mathrm{ncs}}=\frac{\mathrm{k}_{e}}{1-\% \mathrm{f}}=\frac{.1091}{1-.075}=\frac{.1091}{.925}=.1179 \text { or } \underline{\underline{11.79 \%}} \text {. }
$$

So if Wabash's managers felt they would be able to get the common equity portion of all needed new investment funds by retaining earnings, then the relevant cost of common stock financing to use in computing a WACC for new capital projects, under the SML approach, would be the cost of retained earnings, $\mathrm{k}_{e}=10.91 \%$. But if they expected to create and sell new common stock, then the cost of delivering a $10.91 \%$ return on equity to new common stockholders would be a higher 11.79\%, because Wabash would have to compensate the investors for their risks of providing money and compensate the investment bankers for providing underwriting services. Thus the relevant cost of common stock financing to use in computing a WACC for new capital projects would be the cost of new common stock, $\mathrm{k}_{\text {ncs }}=11.79 \%$. This small increase in the component cost of common stock financing might cause the estimated annual WACC, in a case in which new common stock would have to be sold, to rise above the expected annual rate of return on the investment, thus causing the investment proposal to be rejected.
29. Managers of Whiteside Window \& Siding Corporation have computed Whiteside's weighted average cost of capital (WACC, or $\mathrm{k}_{\mathrm{A}}$ ), based on recent transactions in which informed investors purchased the company's existing securities, to be $9.15 \%$ annually. Is that the rate the company's managers should compare to the expected annual rate of return in determining whether to accept any proposed investment project? How might a company adjust its cost of capital measure for risk? Could a company incorrectly accept some unfavorable investment projects and incorrectly reject some favorable investment projects by ignoring the need to adjust for risk?

Type: Risk adjustment. The weighted average cost of capital is the estimated average cost to a firm of delivering fair financial returns to the investors who would provide new money for new capital spending projects. Our starting point for trying to "get inside the heads" of potential new investors is to draw inferences about the returns expected by current investors. We do that by looking at the prices paid recently by buyers of the company's existing bonds, preferred stock, and common stock, and relate these prices to the cash flows (interest, princinpal, dividends) we think the buyers expect to get in subsequent years. Thus when we compute WACC based on the returns expected by investors who have recently taken existing spots at the front, middle, or back of the line of money providers, we implicitly assume that future investment projects would be no more or less risky, in the eyes of potential money providers, than the firm's existing projects (which is the risk on which the current investors based their decisions to provide money). So the Whiteside managers would accept a proposed project of average risk only if it had an expected rate of return high enough to cover the $9.15 \%$ cost of getting money for an average risk investment project.

However, if Whiteside were to invest in a project of greater than average risk (relative to its prior projects), then in "getting inside the heads" of potential investors it would be reasonable to think that new investors would expect higher returns than did those who financed the company's earlier, average (and thus lower) risk endaevors. If Whiteside were to invest in a project of lower risk than it has sought out historically, then in "getting inside the heads" of potential investors it would be
reasonable to think that new investors would be content to earn lower returns than did those who financed the company's earlier average (and thus higher) risk endaevors.

There is no generally applicable method for adjusting the cost of capital to reflect the higher (or lower) risk that investors would be expected to perceive. One possibility is to add a few percentage points to (subtract a few percentage points from) the WACC computed for average-risk projects. For example, Whiteside's managers might accept a high-risk project only if its expected annual return were at least $9.15 \%+3 \%=12.15 \%$, and accept a low-risk project if its expected annual return were at least $9.15 \%-3 \%=6.15 \%$.

Here the company might incorrectly accept a high-risk project with a $10.5 \%$ expected rate of return (more than the $9.15 \%$ WACC for average-risk projects), or incorrectly reject a low-risk project with an $8.5 \%$ expected rate of return (less than the $9.15 \%$ WACC for average-risk projects), if it blindly applied a $9.15 \%$ standard to all projects. An investment project should be accepted only if its expected return exceeds what it would cost the company to get the money for the project if the investing public adjusted their expected returns based on the risks perceived.
(And note that the risk would best be looked at in a "portfolio" context. People who recently purchased Whiteside securities were becoming providers of money to the firm as it existed on the day the purchases were made; they got small financial claims on all of the company's existing projects. People providing money as lenders or owners for a new investment project would receive securities that give them claims on all of the company's existing projects and the new one as well. So we need to consider how risky the new money providers would view the company to be with the addition of the new project to the existing mix.)
30. Woodford Woodcrafting's total debt consists of 200,000 bonds that were issued a few years ago; each bond currently sells for $93 \%$ of its $\$ 1,000$ par (book) value. There are $15,000,000$ outstanding shares of the company's common stock; each share has a book value of $\$ 16.25$ and a market value of $\$ 18.28$. There is no preferred stock financing. Compute the weights $\mathrm{w}_{\mathrm{d}}$ and $\mathrm{w}_{\mathrm{e}}$ that we would use in computing Woodford's weighted average cost of capital, on both a book value basis and a market value basis. Which is the more correct way to compute the weights?

Type: Capital structure weights. The debt's total book value is 200,000 bonds $\times \$ 1,000$ per bond $=$ $\$ 200,000,000$. The common stock's total book value is $15,000,000$ shares $\times \$ 16.25$ per share $=$ $\$ 243,750,000$. The total book value of the debt and equity claims therefore is $\$ 200,000,000+$ $\$ 243,750,000=\$ 443,750,000$. So on a book value basis, $w_{d}=\$ 200,000,000 / \$ 443,750,000=$ $\underline{\underline{45.07 \%}}$, and $w_{e}=\$ 243,750,000 / \$ 443,750,000=\underline{\underline{54.93 \%}}$.

The debt's total market value is 200,000 bonds $\times 93 \% \times \$ 1,000$ per bond $=\$ 186,000,000$. The common stock's total market value is $15,000,000$ shares $\times \$ 18.28$ per share $=\$ 274,200,000$. The total market value of the debt and equity claims therefore is $\$ 186,000,000+\$ 274,200,000=$ $\$ 460,200,000$. So on a market value basis, $w_{d}=\$ 186,000,000 / \$ 460,200,000=40.42 \%$, and $w_{e}=$ $\$ 274,200,000 / \$ 460,200,000=59.58 \%$.

The market value-based weights are more correct than book value-based weights, because WACC is a measure of what it would cost under current market and economic conditions (not the historical conditions reflected in book values) for the company to raise money from new investors. Consider, for example, that the current common stockholders expect a return on equity based on what they have at stake in the company - which is the current market value of their stock (what they could
sell it for), not based on what the shares sold for way back when the company was created. If we assume that those who provide new common equity money (by having their earnings retained or by purchasing newly created common shares) would expect a similar percentage return, then we would have to base our estimate of their expected return on that same market value standard.

In earlier examples we did not explicitly distinguish between market and book values in our capital structure percentages, but if those percentages were to be seen as the appropriate ones to use in computing WACCs then they had to be market value-based weightings.

