ANALYZING BONDS: PROBLEMS *(copyright © 2021 Joseph W. Trefzger)*

This problem set covers all of our bond valuation situations, with a general increase in degree of difficulty as we progress (some of the later problems are for FIL 404 only). Be sure that you have mastered the easier problems before moving ahead, because the more difficult examples tend to expand on the ideas presented in the easier ones.

1. You are analyzing a bond that will mature in four years. Actually, ABC Company issued several hundred million dollars' worth of these bonds, but we conduct our analysis in terms of the individual units, described as follows. These bonds had 25-year original maturities, but they were issued twenty-one years ago; thus they will mature four years from today. Each individual bond has a $1,000 par value ($1,000 was the amount originally lent to the company and the amount the company will pay back at maturity; par value for bonds issued by U.S. corporations is typically $1,000) and a 7% annual coupon interest rate, with interest paid *annually*. Today, rational buyers of bonds with similar risk, 4-year remaining lives, and annual interest payments require a 10% effective annual rate of return (also called the bond's yield to maturity). What should a rational investor pay for one of these ABC bonds?

2. BCD Company issued several hundred million dollars' worth of bonds twenty-one years ago. These bonds had 25-year original maturities; therefore they will mature in four years. Each individual bond has a $1,000 par value and a 7% annual coupon interest rate, with interest paid *semiannually* (unlike what question 1 above might seem to suggest, bonds issued by U.S.-based corporations typically provide interest payments semiannually, not annually). Today, rational buyers of bonds with similar risk, 4-year remaining lives, and semiannual interest payments require a 10% *annual percentage rate* (APR) of return. What should a rational investor pay for one of these BCD bonds? If BCD were to issue 4-year bonds today, what annual coupon interest rate would we expect them to pay?

3. CDE Company issued several hundred million dollars' worth of bonds twenty-one years ago. These bonds had 25-year original maturities; therefore they will mature in four years. Each individual bond has a $1,000 par value and a 7% annual coupon interest rate, with interest paid *semiannually*. Today, rational buyers of bonds with similar risk, 4-year remaining lives, and semiannual interest payments require a 10% *effective annual rate* (EAR) of return (also called the bond's yield to maturity). What should a rational investor be willing to pay for one of these CDE bonds? If CDE were to issue 4-year bonds today, what coupon interest rate would we expect them to pay?

4. DEF Company issued several hundred million dollars' worth of bonds twenty-one years ago. These bonds had 25-year original maturities; therefore they will mature in four years. Each individual bond has a $1,000 par value. But there are no annual or semiannual coupon interest payments; these bonds are of the *zero-coupon* variety. Rational buyers of zero-coupon bonds with similar risk and 4-year remaining lives require a 10% effective annual rate (EAR) of return, or yield to maturity (YTM). What should they pay for each of these DEF bonds? If rational lenders had required a 7% EAR 21 years ago, what should they have paid for these bonds when they were issued? If rational investors required a 7% EAR (instead of 10%) today, what would each of these bonds be worth now?

5. EFG Company just issued two billion dollars' worth of zero-coupon bonds, each with a $1,000 par (or maturity) value and a 10-year maturity. Investors buying these bonds today require an 11.35% effective yield to maturity. You believe that investors will continue to expect an 11.35% effective yield to maturity as the bonds approach their maturity date. At what price should each of these bonds sell today, and at the start of each of the next 10 years? How much interest will the bond holder be earning each year, in the eyes of U.S. federal income tax authorities?

6. [Similar to problem 1, for extra practice.] FGH Company issued several hundred million dollars' worth of bonds seven years ago. These bonds had 20-year original maturities; therefore they will mature in thirteen years. Each individual bond has a $1,000 par value and an 8.5% annual coupon interest rate, with interest paid *annually*. If rational buyers of bonds with similar risk, 13-year remaining lives, and annual interest payments today require a 9.5% effective yield to maturity (YTM), what should they pay for each of these FGH bonds? What if such buyers require a 7.5% effective YTM? What if they require an 8.5% effective YTM?

7. [Similar to problem 2, for extra practice.] GHI Company issued several hundred million dollars' worth of bonds seven years ago. These bonds had 20-year original maturities; therefore they will mature in thirteen years. Each individual bond has a $1,000 par value and an 8.5% annual coupon interest rate, with interest paid *semiannually*. If rational buyers of bonds with similar risk, 13-year remaining lives, and semiannual interest payments today require a 9.5% stated *annual percentage rate* (APR) of return, what should they pay for each of these GHI bonds? What if such buyers require a 7.5% APR? What if they require an 8.5% APR?

8. [Similar to problem 3, for extra practice.] HIJ Company issued three billion dollars' worth of bonds seven years ago. These bonds had 20-year original maturities; therefore they will mature in thirteen years. Each individual bond has a $1,000 par value and an 8.5% annual coupon interest rate, with interest paid *semiannually*. If rational buyers of bonds with similar risk, 13-year remaining lives, and semiannual interest payments today require a 9.5% *effective annual rate* (EAR) of return, or yield to maturity (sometimes specified as the effective yield to maturity), what should they pay for each of these HIJ bonds? What if such buyers require a 7.5% EAR? What if they require an 8.6806% EAR?

9. IJK Company has issued more than one billion dollars' worth of bonds. Each individual bond has a $1,000 par value and a 10.6% annual coupon interest rate, with interest paid semiannually. Today rational buyers of bonds with similar features require an 8.785% yield to maturity (which we interpret as an effective annual rate of return). What price should each of these bonds sell for today if 20 years remain until the maturity date? 10 years? 2 years? What if rational investors instead required a 13% yield to maturity? A 10.88% yield to maturity?

10. Eleven years ago, JKL Company issued several hundred million dollars' worth of bonds with 30-year maturities. Each individual bond has a $1,000 par value and a 7.75% annual coupon interest rate, with interest payments made *annually*. What yield to maturity does an investor receive if she buys one of these bonds today for $892.23?

11. Eleven years ago, KLM Company issued several hundred million dollars' worth of bonds with 30-year maturities. Each individual bond has a $1,000 par value and a 7.75% annual coupon interest rate, with interest payments made *semiannually*. What yield to maturity does an investor receive if he buys one of these bonds today for $1,035.41?

12. Eleven years ago, LMN Company issued several hundred million dollars' worth of bonds with 30-year maturities. Each individual bond will mature at a price of $1,000, but no periodic interest payments are to be made (thus we are looking at zero-coupon bonds). What yield to maturity does an investor receive if she buys one of these bonds today for $111.29?

13. Two years ago, MNO Company issued several hundred million dollars' worth of bonds with 30-year original maturities and an 8.25% annual coupon interest rate, with interest payments to be made semiannually. Today Ms. Investor, a wealthy individual, buys some of these $1,000 par value bonds at the market price of $1,068.33 each. What is her yield to maturity on this investment?

14. Now Ms. Investor (see the previous question) realizes that the MNO bonds she bought contain a *call provision*. Specifically, the company can *call* the bonds (buy them back from Ms. Investor and other bond holders, even if those holders do not wish to sell) as soon as ten years have passed from the date of issue, which was two years ago. If the bonds are called, MNO will pay a price of $1,082.50 (the par value plus an extra year's worth of interest) for each. If Ms. Investor buys each bond today for $1,068.33, and then MNO calls the entire bond issue at the earliest possible date – which Ms. Investor fears they may do, since the roughly 7.8% APR required by the market today is lower than the 8.25% annual coupon rate on the callable bonds she holds – what will be her yield to first call?

15. Now let's move forward in time eleven years with Ms. Investor (see the previous two questions). It turns out that the MNO bonds she bought were not called 8 years after she bought them; in fact, market interest rates began rising such that MNO ultimately had no incentive to call the bonds. However, Ms. Investor also did not hold the bonds until they matured. Instead, eleven years after she bought her bonds for $1,068.33 each, she sold them to Mr. Financial for $952.84 each. Looking back, what has been her average annual holding period yield for the 11 years she held her MNO bonds?

16. NOP Company issued several hundred million dollars' worth of bonds eighteen years ago. These bonds had 35-year original maturities; therefore they will mature in 17 years. Each individual bond has a $1,000 par value, and interest is paid semiannually. Today, rational buyers of bonds with similar risk, 4-year remaining lives, and semiannual interest payments require an 8.6% effective annual rate (EAR) of return, or yield to maturity, and they are paying a price of $1,074.07 per bond. What annual coupon interest rate is NOP paying on this bond issue?

17. OPQ Company issued several hundred million dollars' worth of bonds 7½ years ago. Each individual bond has a $1,000 par value, and the annual coupon interest rate is 11.25%, with interest paid semiannually. Today, rational buyers of bonds with similar risk and semiannual interest payments, and with remaining lives equal to the remaining life of this OPQ bond issue, require a 12.75% effective annual rate (EAR) of return, or yield to maturity. If rational

investors are currently paying $911.46 for each of these OPQ bonds, how many years will pass before they mature? What was the original life of each of these bonds?

18. PQR Company issued one billion dollars' worth of fifty-year bonds twenty-three years ago. Each bond has a $1,000 par value and a 6.75% annual coupon interest rate, with interest paid semiannually. Mr. and Mrs. Front O'Line just purchased some of these bonds at the market price of $1,091.39 each. They plan to hold each bond until their oldest child begins college in 12½ years, and then sell for the prevailing market price. If they expect their holding period yield to be 6.25%, at what price do they expect to sell each bond? What yield to maturity would that price represent for whoever buys the bonds from them?

19. QRS Company issued $300 million worth of 50-year bonds 20 years ago; thus these bonds will mature in 30 years. They carry a 13.5% annual coupon interest rate, with coupon payments made semiannually. Today people who might consider lending money to QRS for 30 years would expect to earn a 5.25% semiannual rate of return, so the company's financial managers feel that they could issue $300 million of 30-year bonds with an annual coupon interest rate (an APR measure) of 5.25% x 2 = 10.5%. QRS's marginal federal + state income tax rate is 37%, and the administrative cost of replacing the old bonds with new bonds (call premiums on the old bonds, investment banking fees on the new bonds) would total $28 million. Should QRS refund the old bonds by issuing new ones?

20. [FIL 404 only] RST Company issued hundreds of millions of dollars' worth of 10-year bonds several months ago. Specifically, these bonds will mature in 8 years and 137 days. Each individual bond has a $1,000 par value and an 11.5% annual coupon interest rate, with interest paid semiannually. If a rational investor buying a bond with the same life and similar risks requires a yield to maturity of 12.5%, what should a rational investor willingly pay for each of these RST bonds?

21. A bond's yield to maturity (or yield to first call or holding period yield) is an internal rate of return measure, which does not explicitly take into account the bond holder's reinvestment of interest payments. Some analysts therefore prefer to compute a bond's *realized compound yield* (RCY, sometimes called total return; we called it modified internal rate of return in capital budgeting analysis) by looking explicitly at reinvesting the coupon payments received before the bond matures (or is called or sold). [With no coupon payments to reinvest, a zero-coupon bond has a RCY equal to its yield to maturity (YTM).] Assume that STU Company issued several hundred million dollars' worth of 35-year bonds 24 years ago. Each $1,000 par value bond has a 7% annual coupon interest rate, with semiannual interest payments, and is priced to provide a 7.1225% YTM. What will Mr. Lender's RCY be if he buys an STU bond today and then reinvests over the bond's remaining 11-year life at a 9.2025% effective annual rate (EAR) of return? What if his reinvestments instead earn a 5.0625% EAR? A 7.1225 EAR?

22. TUV Corp. issued a large dollar volume of bonds with 50-year original maturities nineteen years ago. Each bond has a $1,000 par value and a 7.2% annual coupon interest rate, with interest paid semiannually. Today you buy some of those bonds for the current market price of $930.25 each. What will your realized compound yield on this investment be if you hold each bond until it matures and you can reinvest each semiannual coupon payment, from the day you get it until the maturity date, for a return expressed as an 8.3681% effective annual rate (EAR)?

23. [FIL 404 only] UVW Company issued several hundred million dollars' worth of 30-year bonds 26 years ago. Each $1,000 par value bond has an 8% annual coupon interest rate, with semiannual interest payments, and is priced today to provide a 9.2025% yield to maturity (YTM). What will Ms. Holder's realized compound yield (RCY) be if she buys one of these bonds today and holds it for only 3½ years of its 4-year remaining life, while reinvesting the coupon payments to earn an average 11.3025% effective annual rate (EAR) of return? What if her reinvestments instead earn, on average, a 7.1225 or 9.2025% EAR?

24. a. Mr. VWX wants his money to be invested in U.S. government bonds for the next three years. Broker Reginald von Redbird at Normal Securities notes that there is an active market for U.S. government bonds with a 3-year maturity. If Mr. VWX buys one of these 3-year bonds, he will be assured of earning a 4.25% annual interest rate in each of the three years. But von Redbird also sees that there is an active market for U.S. government bonds with a 2-year maturity. If Mr. VWX buys one of these 2-year bonds, he will be assured of earning a 4.75% annual interest rate, but only for two years. Then, at the end of year 2/start of year 3, he will have to buy a new U.S. government bond with a 1-year maturity to complete the third year of his planned investment period. Based on the Expectations Theory, what annual interest rate should he expect to earn on a 1-year U.S. government bond during year 3?

b. Before Mr. VWX can make his decision, von Redbird suggests yet another possibility. The U.S. government also issues bonds ("T-bills") with just a 1-year maturity. If Mr. VWX buys a 1-year bond, he will be assured of earning a 4.95% annual interest rate for one year. Then, at the end of year 1/start of year 2, he could buy a new U.S. government bond with a 1-year maturity to complete the second year of his planned investment period, following at the end of year 2/start of year 3 by buying another new bond with a 1-year maturity to complete the third year. Or, after earning 4.95% on the 1-year bond he could, at the end of year 1/start of year 2, buy a new U.S. government bond with a 2-year maturity to complete the second and third years of his planned investment period. According to the Expectations Theory, what annual interest rate should he expect to earn on a 1-year bond during year 3? On a 1-year bond during year 2? On a 2-year bond during years 2 and 3? (In other words, compute the applicable implied forward rates.)

25. WXY Securities has announced that the following interest rates are being paid on U.S. government bonds purchased today:

One-year maturity: pays 6.15% annual interest for one year
Two-year maturity: pays 6.45% annual interest each year for two years
Three-year maturity: pays 6.85% annual interest each year for three years

Compute and identify the applicable implied forward rates, based on our additive approximation approach.

26. XYZ Investments reports that the following interest rates are being paid on U.S. government bonds purchased today:

One-year maturity: pays 7.45% annual interest for one year
Two-year maturity: pays 7.55% annual interest each year for two years
Three-year maturity: pays 7.70% annual interest each year for three years
Four-year maturity: pays 7.90% annual interest each year for four years

Compute and identify the applicable implied forward rates, based on our additive approximation approach.

27. [Similar to problems 3 and 8, for extra practice.] YZA Corporation issued several hundred million dollars' worth of bonds thirty-three years ago. These bonds had 50-year original maturities. Each individual bond has a $1,000 par value and a 5.8% annual coupon interest rate, with interest paid semiannually. Today, rational buyers of bonds with similar risk, similar remaining lives, and semiannual interest payments require a 5.3% *effective annual rate* (EAR) of return (what we call the bond's yield to maturity, although some sources call an EAR measure a bond's *effective* yield to maturity). What should a rational investor be willing to pay for one of these YZA bonds?

28. [Similar to problems 3, 8, and 27, for extra practice.] ZAB Corporation issued a large dollar amount of bonds sixteen years ago. These bonds had 55-year original maturities. Each individual bond has a $1,000 par value and an 8.2% annual coupon interest rate, with interest payments made semiannually. Today, rational buyers of bonds with similar risk (including the default risk that the borrower will not pay interest and repay the lender's principal on the agreed schedule), similar remaining lives, and semiannual interest payments require a 9.83% *effective annual rate* (EAR) of return (what we call the bond's yield to maturity; sometimes that EAR measure is more explicitly called a bond's *effective* yield to maturity). What should a rational investor be willing to pay for one of these ZAB bonds?