More About Structures: Scholarships

FIL 240 – Trefzger, Honors Section

ISU Foundation can earn a 5% annual rate of return on money donated to the university. How much must a grad contribute today to fund a scholarship that will provide the following payments to worthy students?

a-1. \$3,000/year, paid at beginning of each of years 1 – 10 (PV of immediate level 10-year annuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1}{1.05}\right)^{10}}{.05}\right)(1.05) = \$3{,}000\ (8.107822) = \$24{,}323.47$$

a-2. \$3,000/year, paid at beginning of each of years 7 – 16 (PV of deferred level 10-year annuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1}{1.05}\right)^{10}}{.05}\right)(1.05)\left(\frac{1}{1.05}\right)^{6} = \$3{,}000\ (6.050181) = \$18{,}150.54$$

b-1. \$3,000/year, paid indefinitely starting at beginning of year 1 (PV of immediate level perpetuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1}{1.05}\right)^{\infty}}{.05}\right)(1.05) = \$3{,}000\left(\frac{1}{.05}\right)(1.05) = \$63{,}000.00$$

b-2. \$3,000/year, paid indefinitely starting at beginning of year 7 (PV of deferred level perpetuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1}{1.05}\right)^{\infty}}{.05}\right)(1.05)\left(\frac{1}{1.05}\right)^{6} = \$3{,}000\left(\frac{1}{.05}\right)(1.05)\left(\frac{1}{1.05}\right)^{6} = \$47{,}011.57$$

c-1. Start at \$3,000/year and increase by 2%/year, to be paid at beginning of years 1 – 10 (PV of immediate changing 10-year annuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1.02}{1.05}\right)^{10}}{.05-.02}\right)(1.05) = \$3{,}000\ (8.807511) = \$26{,}422.53$$

c-2. Start at \$3,000/year and increase by 2%/year, to be paid at beginning of years 7 – 16 (PV of deferred changing 10-year annuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1.02}{1.05}\right)^{10}}{.05-.02}\right)(1.05)\left(\frac{1}{1.05}\right)^{6} = \$3{,}000\ (6.572300) = \$19{,}716.90$$

d-1. Start at \$3,000/year and increase by 2%/year, to be paid indefinitely starting at beginning of year 1 (PV of immediate changing perpetuity due) [g must be < r in long run, so $\left(\frac{1+g}{1+r}\right)^{n} \rightarrow 0$ as $n \rightarrow \infty$]

$$\$3{,}000\left(\frac{1-\left(\frac{1.02}{1.05}\right)^{\infty}}{.05-.02}\right)(1.05) = \$3{,}000\left(\frac{1}{.05-.02}\right)(1.05) = \$105{,}000.00$$

d-2. Start at \$3,000/year and increase by 2%/year, to be paid indefinitely starting at beginning of year 7 (PV of deferred changing perpetuity due)

$$\$3{,}000\left(\frac{1-\left(\frac{1.02}{1.05}\right)^{\infty}}{.05-.02}\right)(1.05)\left(\frac{1}{1.05}\right)^{6} = \$3{,}000\left(\frac{1}{.05-.02}\right)(1.05)\left(\frac{1}{1.05}\right)^{6} = \$78{,}352.62$$

▪