

TIME VALUE: PROBLEM SET C (copyright © 2022 Joseph W. Trefzger)

This problem set covers all of our basic time value of money applications. It is designed to provide yet additional practice for FIL 240 students beyond the main problem set and Set B. But whereas problems in the earlier sets are organized in the order of our coverage, generally progress in degree of difficulty, and are accompanied by complete explanations, here problems are in a random order, and long narrative explanations generally are not offered with the numerical answers. Set C therefore offers the chance to practice under something more like exam conditions.

1-C. Illinois State graduate Brad sets the long-term goal of buying a campus area condominium for his newborn daughter to live in while she attends ISU. He plans to save equal amounts of money every year until she graduates from high school in 18 years to amass the \$218,000 that he believes a condo unit will cost at that time. If he expects to earn a 4.3% average annual rate of return on any money in his savings plan, how much should he deposit if he saves at the end of each year? At the start of each year?

2-C. Amy wants to save \$26,000 to buy a bass fishing boat. If she can save \$1,675 at the end of each year, and if she can earn a 3.92% average annual rate of return on her account's balance, how many years will it take for her to amass the \$26,000? What if instead she makes the \$1,675 deposit at the beginning of each year?

3-C. Wealthy graduate Aaron contributes \$1,350,000 to the Illinois State University Foundation to establish a scholarship fund. He directs that the fund should provide scholarships for the next 14 years, in honor of legendary ISU quarterback Red G. Redbird, who wore Number 14. If the foundation's investment director can earn a 5.31% average annual rate of return on any money that she manages, how much can be paid out annually in scholarships if the awards are made at the end of each year? What if the money is paid out at the start of each of the 14 years?

4-C. Tom just netted \$33,625 as a winner on the TV game show *Wheel of Jeopardy* (he actually won \$50,000, but after paying applicable income taxes has \$33,625 left). He wants to use this single amount of money to help build a retirement nest egg. What average annual rate of return will he need to earn for the \$33,625 to grow to \$100,000 by the time he retires in 21 years? What average annual rate of return would grow his balance to \$150,000?

5-C. Betty just opened an account at BloNoBank. The bank manager expects to collect savings deposits from Betty of \$530 in each of years 1 through 11 and \$660 in each of years 12 through 17, and feels the bank will pay savers a 3.22% average annual interest rate in the coming years. How much does the banker expect to owe Betty at the end of year 17 if she makes the deposits at the end of each year? What if instead she saves at the start of each year?

6-C. Billie is retiring today with \$1,265,000. She expects to survive for 31 years, and to earn a 4.26% average annual return on any money in her accounts. If her hope is to withdraw \$53,000 to live on at the end of each of the 31 years and then leave \$1,250,000 to Redbird Charities in her will, does she have enough money saved? What if instead she more realistically expects to withdraw the \$53,000 to pay living expenses at the beginning of each year?

7-C. Tim has some very old carpentry tools that he got from his late grandfather. An antiques appraiser feels that they are worth \$9,400 today. If collectible items of this type have been increasing in value by an average of 4.65% per year, what would we estimate the tools to have been worth when Tim received them seven years ago?

8-C. Marilyn is retiring today with \$804,000 saved. What average annual rate of return will she have to earn on any balance remaining in her account from year to year if she wants to take out \$73,000 at the end of each year in the 32 years she expects to remain living? What if instead she wants to take the \$73,000 out at the start of each year?

9-C. All-Season Sports is planning to borrow \$214,000 to replace sold inventory. The firm's managers want to repay the loan over 15 years, and the interest rate the bank charges can be represented as a 9.36% annual percentage rate (APR). What must each equal regular payment be if payments are to be made at the end of each year? At the beginning of each year? What if instead payments are to be made semi-annually, quarterly, or monthly, and interest accrues on the loan with the same frequency as the payments occur? With the 9.36% APR, what effective annual rate (EAR) of cost would the borrower incur under each of the four payment and compounding frequencies?

10-C. Nancy just graduated from ISU at age 23 with a BS/MPA degree in accounting and is starting work as an auditor with Jesse, Fell & Associates. She expects to work until age 66, and plans to save \$13,800 every year in the accounting firm's 401-K plan. What average annual rate of return will she have to earn on her account's balance if she is to retire in 43 years with \$1,850,000 saved? What if instead she deposits the \$13,800 at the beginning of each year?

11-C. Clearing Publishers House, headquartered near Midway Airport, sponsors the \$10 Million Sweepstakes, whose winner will receive \$500,000 per year for 20 years. How much money must Clearing's managers have on deposit today to fund the promised stream of payments if they expect to earn a 4.17% average annual rate of return on any money they have invested and the winner is to get the \$500,000 at the end of each year? What if instead the winner is to collect at the beginning of each year?

12-C. French wine dealer John just paid €2,322 (2,322 euros) for a bottle of a rare Chenin blanc at an auction in Angers. If it increases in value, as expected, by an average rate of 11.4% annually, how many years will it take for the bottle to be worth €10,000? To be worth €20,000?

13-C. a. Richard wants Mid-Illinois Food Bank to have \$130,000 available at the end of each year so it can continue operations indefinitely into the future. If the food bank's trustees can earn a 4.34% average annual rate of return on money under their care, how much should Richard give to the organization today? What if instead the \$130,000 yearly budget should be provided at the beginning of each year forever? If he is able to give only \$2,000,000 how much can the food bank budget to spend at the end vs. beginning of each year forever?

b. Now assume that the food bank trustees can invest donations in a plan that provides investment returns more frequently than once per year. How much must Richard contribute today to generate \$65,000 in receipts every six months (\$130,000 annual total) if earnings compound semiannually and the rate of return is represented as a 5.4% annual percentage rate (APR)? What if the plan is to provide \$32,500 every three months (again \$130,000 annually) if earnings compound quarterly and the rate of return is represented as a 4.88% APR?

14-C. James wants to save \$172,000 over the next 11 years to buy a lakeside cabin. He expects to earn a rate of return on his savings plan that can be represented as a 4.86% annual percentage rate (APR). How big must his regular equal deposits be if he wants to reach the \$172,000 goal by putting money in at the end of each year? At the beginning of each year? What if instead he makes deposits semi-annually, quarterly, or monthly, and interest compounds on the account with the same frequency as the deposits? With the 4.86% APR, what effective annual rate (EAR) of return would he earn under each of the four deposit and compounding frequencies?

15-C. Dorothy hopes to buy her neighbor's motor home when the neighbor retires to Florida in nine years. If Dorothy can save \$2,700 every year in a savings plan that earns a 3.6% average compounded annual rate of return, how much will she be able to offer for the vehicle? What if instead she makes the \$2,700 deposits at the beginning of each year?

16-C. Robert just won \$54,000 (net of income taxes) playing the Illinois State Lottery. With those winnings he creates an account to pay for annual year-end trips to Monte Carlo that are expected to cost \$6,250 each. If he can earn a 3.14% average annual rate of return on any money in his account, for how many years will he be able to fund his trips? What if instead he withdraws the \$6,250 to pay for his Monte Carlo visits at the beginning of each year?

17-C. Susan wants to have \$515,000 when she retires in 12 years. The average rate of return she expects to earn on any money she holds can be represented as a 5.36% annual percentage rate (APR), with compounding to take place quarterly. If she already has \$29,000, how much must she save at the end of each quarter for the next 12 years to reach her goal? What if instead she makes savings deposits at the beginning of each quarter? What effective annual rate (EAR) of return would she be earning?

18-C. Redbird fan Allison plans to donate \$243,000 to provide for a scholarship to be given yearly to Illinois State University's top ornithology student. She wants the first scholarship to be awarded in year 11, and for the program then to continue indefinitely. The ISU Foundation expects to earn a 4.12% average annual rate of return on any money in dedicated accounts of this type. How big an annual scholarship can be given if the money is paid at the end of each year? What if instead scholarships are to be awarded at the beginning of each year? How much would Allison have to donate today to fund an \$18,000 annual scholarship starting in year 11 and continuing indefinitely?

19-C. How much money can a bank justify lending Alex today if it charges a 7.3% annual interest rate and Alex agrees to make payments of \$3,100 at the end of year 1; \$4,200 at the end of year 2; \$5,100 at the end of year 3; and \$4,800 at the end of year 4? What if instead he agrees to pay the bank \$4,600 at the end of each year? What if instead he is willing to make payments of \$4,600 at the start of each of the four years?

20-C. The Income Boost Annuity plan offered by Redbird Investments provides the investor with payments of \$12,365 at the end of each of years 1 through 15 and then \$14,365 at the end of each of years 16 through 24. If Emily believes that the risk of being in this plan calls for an 8.17% average annual rate of return, what should she be willing to pay for this contract? What if instead Redbird pays the plan holder at the beginning of each year?

21-C. Paul expects money in his savings account to earn interest at a 5.9% compounded average annual rate. If he has \$10,250 today what does he expect the account balance to grow to be in 24 years?

22-C. Sarah's four-year old son is so argumentative that she feels he should be a lawyer. She plans to save systematically over the coming 14 years to accumulate the money to pay for his schooling over the subsequent 7 years (BS and JD degrees). She expects to earn a 3.64% average annual rate of return on any money in her account.

a. If she wants to be able to give him \$42,500 at the start of each of the 7 school years, how much must she save at the end of each of the 14 deposit years? What if instead she saves at the beginning of each of the 14 deposit years?

b. Sarah comes to realize that she may have been overly optimistic in projecting her ability to save. If she can put aside only \$10,980 at the end of each of the 14 savings years how much will she be able to give her son at the beginning of each of his 7 years of school? What if instead she saves at the beginning of each year?

23-C. The Virginia State Bank expects to receive a \$6,200 deposit from saver Roger at the beginning of year 1; \$7,300 at the beginning of year 2; \$8,500 at the beginning of year 3; and \$9,900 at the beginning of year 4. If the bank managers predict that they will pay interest on deposit accounts at a 2.8% average annual rate, how much should they expect to owe Roger at the end of year 4? What will they owe him if instead he deposits \$7,900 at the start of each of the four years? What if instead he deposits the \$7,900 at the end of each year?