TIME VALUE: PROBLEM SET B *(copyright © 2022 Joseph W. Trefzger)*

This problem set covers all of our basic time value of money applications, with a general progression in degree of difficulty as we proceed from problem 1 to problem 21. This set is designed to provide added practice for FIL 240 students. A full understanding of all steps in these problems should indicate solid knowledge of our basic time value of money ideas. Detailed solutions (with the problems repeated) are available in an accompanying file. Be sure you have mastered the easier problems before moving ahead, because the more difficult examples tend to expand on the ideas from the easier ones. Opportunities for added practice are provided in the main problem set, which is organized with the same ordering as Set B, and in largely open-ended Problem Set C.

1-B. Seventeen years ago Leesa bought the antique bell of an old Rock Island locomotive for $375. If antiques of this type have increased in value by an average of 3.5% per year, what should the bell be worth today?

2-B. Tim expects to earn a 3.25% average annual interest rate on any money in his bank account. How much must he have on deposit today if he wants his balance to grow with interest to $150,000 by the end of year 29?

3-B. Lilly bought a very rare book for $4,550, and then sold it eight years later for $9,895. What was her average annual compounded rate of return? What if she sold after holding the book for eight years and received only $3,000? At what price would she have had to sell it to earn a 12% average annual compounded rate of return?

4-B. If saver Chizuko in Japan puts 9,000 yen (¥) today into an investment expected to earn a 7.75% compounded average annual rate of return, how many years will it take for the account's value to double? Triple? Quadruple?

5-B. Matt earns a 4.6% annual interest rate on any money that remains in his savings account from year to year. a.) What will his balance be at the end of year 3 if he deposits $730 at the end of year 1, $830 at the end of year 2, and $990 at the end of year 3? What if instead he deposits $850 at the end of each of the three years? b.) What if instead he makes deposits for the two cases above at the beginning of each year? c.) What if instead the deposits begin at $850 and then increase by 1.4% per year for the subsequent two years?

6-B. a.) How much money must David have on hand today in an account that earns a 4.6% annual interest rate if he wants to take out $1,250 at the start of year 1, $1,500 at the start of year 2, and $2,350 at the start of year 3? What if instead he wants to withdraw a level $1,700 at the start of each of the three years? (We could alternatively ask how much he can borrow today if he is willing to make the indicated payments, although a loan with beginning-of-period payments would be very unlikely.) b.) What if instead he wants to take the withdrawals (or make loan payments) in the two cases above at the end of each year? c.) What must he have (can he borrow) today if instead he wants to take three withdrawals (make three repayments) that start at $1,700 and then increase by 2.1% per year?

7-B. Forty-two-year-old Steve plans to contribute a total of $6,000 at the end of each year into his retirement savings plan in the LeMay High Yield Mutual Fund. If he can earn a 9% average compounded annual rate of return on this investment, and continues making the $6,000 yearly contributions until he is 65, how much money will he have when he retires in 23 years? What if instead he makes his $6,000 deposits into the fund at the start of each year? (Note we could as easily analyze from the perspective of the mutual fund manager, asking how much the fund will owe Steve in 23 years if it collects $6,000 from him yearly and credits him with a 9% average annual return.)

8-B. Logan Security Systems needs to purchase new telecommunications equipment. It can obtain a business loan that carries an 8.5% annual interest rate and calls for equal payments to be made at the end of each year for four years. How much can the firm borrow if it can afford to make annual payments of $100,745? Develop a year-by-year amortization schedule. How much can be borrowed if payments are made at the start of each year?

9-B. After some initial analysis, Steve in problem 7-B above decides that his good pension plan through work means he needs to accumulate a total of only $300,000 in his personal retirement savings plan by the time he retires in 23 years. But now he decides to reach that goal through the more conservative LeMay Safe Yield Mutual Fund, and therefore expects to earn a lower 5.85% average compounded annual rate of return. If he continues making contributions until he is 65, how much money must he deposit into his mutual fund account at the end of each year? What if instead he makes the deposits at the beginning of each year? (Note that we could as easily analyze from the mutual fund manager's perspective, asking how much the fund must collect each year if it is to owe Steve $300,000 in 23 years while applying a 5.85% average annual return to the growing amount under its care.)

10-B. Tucker just retired today with $480,000 saved at the Smithville Bank. If he expects his remaining account balance to earn a 6.25% average annual rate of return, how much should he be able to withdraw in each of 31 years of expected retirement if he takes the money out at the end of each year? What if he takes withdrawals at the start of each year? (Note we could as easily analyze from the bank's perspective, asking how much it should plan to pay out every year for the next 31 years if it credits the account's remaining balance with returns averaging 6.25% yearly.)

11-B. Blaine puts $2,500 every year into a stock market index mutual fund. After 19 years, his account balance has grown to $83,341.43. What has his average annual compounded rate of return been if he has made his deposits at the end of each year? What if instead he has made beginning-of-year deposits? What if instead the balance is only $40,000 after he has made end-of-year or beginning-of-year deposits for 19 years?

12-B. Jenny buys an annuity product from a large financial services company. She pays $315,000 today, and in return the company promises to pay her $19,150 at the end of each year for 27 years. What average annual rate of return does Jenny expect to receive on the $315,000 investment (which equals the rate of return the financial firm is expected to provide)? What would the average annual rate of return be if instead she were scheduled to receive the $19,150 payment at the beginning of each year?

13-B. Recent ISU graduate Scott plans to save $11,500 every year in a retirement savings account. He expects to earn a 5.8% average annual rate of return on the account's growing balance. How many years of working and saving will be required before he can retire with $1,500,000 if he makes his deposits at the end of each year? What if he saves the $11,500 at the beginning of each year?

14-B. Carissa has saved $13,195. The money currently is in an account that earns a 4.375% annual interest rate. She wants to withdraw $1,885 from the account each year to buy software upgrades. For how many years can she continue to take out $1,885? Compute for both end-of-year and beginning-of-year withdrawals.

15-B. Emily plans to purchase a perpetual annuity (perpetuity) that provides its holder with a payment of $372.50 at the end of each year forever. If 7.45% is the average annual rate of return that the investment is expected to deliver based on the accompanying risk, how much should a sensible buyer be willing to pay? What if the $372.50 were to be received at the beginning of each year? Show what the cash flows are expected to look like during the first four years of this plan for end-of-year payments.

16-B. Second-generation billionaire I.M. Rich wants to create a $1,750,000 annual scientific research prize, in honor of his late father I.B. Rich, to reward developments in technology. He wants the prize to be given for 25 years, but the first of the 25 awards will not be given until four years have passed so researchers can plan long-term projects. If investment managers can expect to earn a 6.125% average annual rate of return, how much should Rich donate today to create the endowment fund? Compute both for having the money awarded at the end of each year and at the start of each year. What if instead he wants to give small $625,000 awards rather than $0 in the first four years before starting the big 25-year $1,750,000 series in year 5? What if instead he elects to award $625,000 in each of years 1 through 4, then $1,500,000 in each of years 5 through 13, and then $1,750,000 for the subsequent 25 years?

17-B. Arts afficionados and members of the British aristocracy, young Lord and Lady Gahgagh, want to establish a classical music summer camp for pre-teens. There will be no sessions for six years, but then the camp is expected to open in year 7 (to commemorate the couple's 10th wedding anniversary) and remain in operation perpetually. If expected costs of running the camp each year are £28,500 and a 3.875% average annual rate of return can be earned on money invested to support the camp, how large an endowment should the Gahgaghs create today?

18-B. Jacob plans to save for retirement in two stages. During the first 22 years of his working and savings period he will save $7,000 annually and then in the subsequent 18 years he will save $8,000 annually. If any account balance he holds earns a 6.83% average annual rate of return, how much will he have by the end of year 40 if he makes the deposits at the end of each year? At the beginning of each year?

19-B. Sandy will provide for income over 36 expected years of retirement by saving money every year during 41 expected working years. Long-term investments' performance over recent decades suggests that she can expect to earn a 5.36% average annual rate of return on any money that is in her account from year to year.

a. If she deposits $15,750 at the end of each of the 41 savings years, how much will she be able to withdraw at the end of each of the 36 retirement years? What if instead she wants to take the money out at the start of each retirement year? What if instead she makes the savings contributions at the start of each year?
b. After thinking about potential inflation in future years Sandy fears that the figures shown in part a above will not provide for a comfortable retirement, so she will have to save more than $15,750 annually. How much must she deposit in each of the 41 saving years if she wants to be able to withdraw $180,000 in each of the 36 retirement years? Compute for all possible end-of-year and beginning-of-year cash flow combinations.

20-B. Rita plans to save for retirement by depositing $1,125 every month. If the average rate of return she can earn on her accumulating balance is presented as a 3.48% annual percentage rate (APR), how much money will she have after 33 years of saving if she makes deposits at the end of each month? At the beginning of each month? How much would she have to deposit each month to reach a $1 million balance? How much will she have at retirement if instead she deposits $1,025 each month for 4 years, $1,125 monthly for 21 years, and $1,225 monthly for 8 years?

21-B. Henry wants to pursue his dream of searching for Sasquatch. His eccentric millionaire Aunt Mary has just endowed a $555,000 fund to help pay for the venture. An investment manager feels that Henry can expect to earn a return on any balance in the endowment fund that can be represented as a 7.48% annual percentage rate (APR). How much can he plan to take out at the end of each quarter for the next 18 years to finance his search? What if he takes the money out at the beginning of each quarter? How much less could Aunt Mary contribute today if Henry would be happy to get just $12,865 per quarter over the 18 years? If Aunt Mary endows the fund with $555,000 and Henry takes out only $12,865 at the end of each quarter, how long could he keep that withdrawal stream going? What rate of return, in annual percentage rate (APR) and effective annual rate (EAR) terms, would the fund have to earn if the endowment were $555,000 and Henry's plan required withdrawing $16,000 at the end of each quarter for 18 years? (Just set up the equation for this last part, since solving for the rate requires trial and error iterations.)