Topic 6: Capital Budgeting Techniques (Copyright © 2022 Joseph W. Trefzger)
Capital budgeting is the planning for, and analysis of, investment projects in which a company buys assets that are expected to provide financial returns over multiple periods. An investment should be made if it is expected to provide high enough financial returns to give the investors the lenders and owners - a return of their money plus a fair rate of return on their money. (The cost to the firm of delivering a fair return on that money is the weighted average cost of capital.) An investment project whose expected periodic rate of return exceeds the periodic WACC is desirable because it creates wealth for the company's owners. Recall that as managers our goal is to maximize the firm owners' wealth, by maximizing the market value of the existing shares of common stock (a total known as the company's market capitalization.) The more investment projects the company's managers can identify that have expected average periodic returns greater than the periodic WACC, the more wealth they create for the owners they serve.

Capital budgeting is a multidisciplinary activity that requires information/input from managers in the company's marketing, production, human resources, engineering, and possibly other, areas in addition to finance and accounting. Capital budgeting analysis can be applied to projects ranging from replacing worn-out equipment or complying with environmental/other laws (for which the analysis might be somewhat informal, because the investment must be made to keep the business operating) to buying equipment for expansion into new products or overseas markets (for which the analysis would be quite detailed and involve managers at the highest levels of the firm). The most difficult task in a formal capital budgeting analysis is estimating the cash flows that would be generated by a project (addressed in Topic 7; here we treat the cash flows as "givens").

## I. An Intuitive Approach to Capital Budgeting

Our prior discussion was focused on the weighted average cost of capital (WACC): a firm's average periodic cost of obtaining money from its lenders and owners (we often assume annual time periods in capital budgeting analysis, since the cash flow figures must be broad estimates).

Question: if the annual WACC is $10 \%$, would we create wealth for a company's owners by committing to an investment opportunity - a purchase of new machinery, for example - with an expected average annual rate of return of $12 \%$ ? [Yes.] What if the expected annual rate of return were $8 \%$ ? [No.] The answers should be pretty obvious. What about $10 \%$ ? The answer may not seem so obvious; we will address that case (expected periodic rate of return equal to periodic WACC) later in connection with the Net Present Value discussion.

Is the analysis really that simple to complete? It almost is, though there are a few complications to deal with. The first is that an investment project's annual (or other periodic) inflows are likely to be predicted in terms of dollar (cash outflows and inflows) rather than percentage measures.

So consider instead this question: would we enhance our wealth by giving up $\$ 100$ today in return for someone's promise to give us $\$ 23$ each year for the next four years? What if the promise instead were to give us $\$ 25$ each year for four years? What about $\$ 26$, or $\$ 35$ ?

Let's begin our numerical analysis with an even simpler one-period example. We are told we will receive $\$ 100$ in one year if we will give up $\$ 90$ today. Is this investment a good deal for us; would committing to such an arrangement increase our wealth? The answer (you may have guessed) depends on our annual percentage cost of money, what we have to pay each year to have the ongoing use of the $\$ 90$ (maybe it is an opportunity rate: what we have to give up by taking the money out of savings). Let's say that this cost is $5 \%$ per year. We can analyze the situation in three general ways.
A) What we later will call Net Present Value (NPV)

Discount the $\$ 100$ back to a present value at the $5 \%$ annual WACC:

$$
\begin{array}{ll}
\$ 100\left(\frac{1}{1.05}\right)^{1}=\$ 100 \mathrm{x} .9524 & =\$ 95.24 \\
\text { Then subtract the initial investment } & -\underline{90.00} \\
\text { Gain in wealth (profit in present value terms) } & \$ 5.24
\end{array}
$$

Or just consider that we are giving up $\$ 90$ today and getting back something with a todayequivalent (i.e., present) value of $\$ 95.24$, thereby increasing our wealth by $\$ 5.24$. A couple of side notes to consider:

- What if the cost of getting money were only $2 \%$ per year? Then the deal would be even more attractive:

$$
\begin{array}{llr}
\$ 100\left(\frac{1}{1.02}\right)^{1}=\$ 100 \times .9804 & \$ 98.04 \\
\text { Then subtract the initial investment } & - & \underline{90.00} \\
\text { Gain in wealth (profit in present value terms) } & \$ 8.04
\end{array}
$$

- What if the annual cost of getting money were $13 \%$ ? Then the deal would be unattractive:

$$
\begin{array}{llr}
\$ 100\left(\frac{1}{1.13}\right)^{1}=\$ 100 \times .8850 & =\$ 88.50 \\
\text { Then subtract the initial investment } & - & \underline{90.00} \\
\text { Gain in wealth (a loss in present value terms) } & (\$ 1.50)
\end{array}
$$

Money, like any other input, has a cost. If the cost of any input declines (increases) while everything else - here, the cash flow projections - stays the same, the project creates more (less) wealth for the owners of the enterprise.
B) Second way to analyze: what we will later call Internal Rate of Return (IRR) Because the $\$ 100$ to be collected after a year is greater than the $\$ 90$ given up initially, we can tell that the periodic rate of return built into this investment is positive. But it may not be easy to see exactly what that rate of return is. We have to figure out the periodic rate of return represented by the indicated cash flows, and then compare that rate to the money's annual percentage cost (the WACC):

$$
\begin{gathered}
\text { BAMT } \mathrm{x}(1+r)^{n}=\mathrm{EAMT} \text { so } \\
\text { EAMT } \div(1+r)^{n}=\text { BAMT } \\
\$ 100 \div(1+r)^{1}=\$ 100\left(\frac{1}{1+r}\right)^{1}=\$ 90 \\
\left(\frac{1}{1+r}\right)^{1}=\$ 90 / \$ 100 \\
1+r=\$ 100 / \$ 90=1.111 \\
\text { So } r \text {, which is the IRR is } .111 \text {, or } 11.1 \%
\end{gathered}
$$

Since $11.1 \%$ is the annual rate of return created when a $\$ 100$ payout comes a year after a $\$ 90$ investment, the project creates wealth if the annual cost of getting the money is less than $11.1 \%$, and reduces our wealth if the annual cost of getting the money is higher than $11.1 \%$. If the annual cost of getting the money is exactly $11.1 \%$, we break even in time-value-adjusted terms:

$$
\begin{aligned}
& \$ 100\left(\frac{1}{1.111}\right)^{1}=\$ 100 \times .9000 \\
& \text { Then subtract the initial investment } \\
& \text { Gain in wealth (profit in present value terms) }
\end{aligned}
$$

C) Third way to analyze: what we will later call Modified Internal Rate of Return (MIRR) Here we incorporate a future value amount, and not just present value amounts, into the analysis. In our simple 1-period example, $I R R$ and $M I R R$ end up being the same value. (In an example with repeated cash flows occurring over multiple periods, however, they typically will not be.)

The ending (future) value is $\$ 100$, and the initial investment is $\$ 90$. What compounding rate causes $\$ 90$ to grow to $\$ 100$ in one year?

$$
\begin{gathered}
\text { BaMt } \mathrm{x}(1+r)^{n}=\text { EAMT } \\
\$ 90(1+r)^{1}=\$ 100 \\
(1+r)=\$ 100 / \$ 90=1.111 \\
\text { So } r=M I R R=11.1 \%
\end{gathered}
$$

As in our $I R R$ analysis, since $11.1 \%$ is the measured annual rate of return, the project creates wealth if the annual cost of obtaining the money is less than $11.1 \%$, reduces wealth if the annual cost of getting the money is higher than $11.1 \%$ per year, and breaks even in time value-adjusted terms (neither creates nor reduces wealth) if the yearly cost of getting the money is $11.1 \%$.

## D) A More Complex Example

Now let's do a slightly more involved example. A new machine can be purchased for $\$ 244$. (This figure can represent thousands of dollars, millions, or any other units we might specify.) The expectation is that the machine will produce goods that will generate cash flows of $\$ 100$ per Trefzger/FIL 240 \& 404
year for 3 years. (Cash flows are dollar amounts expected to remain each period to compensate the lenders and owners who contributed the $\$ 244$ after the firm's labor, material, administrative, utility, income tax, and other cash expenses to compensate parties other than lenders or owners but not depreciation, which is not a cash outlay in the specified period - have been met.) So it is clear that the total cash the investors expect to get back ( $\$ 100+\$ 100+\$ 100)$ exceeds the cash they initially give up (\$244), such that the periodic rate of return embedded in the investment is positive. But is that periodic return positive enough; is it greater than the periodic weighted average cost of capital? Only if it is can the investment be expected to create wealth for the company's owners, indicating that it is profitable in time value-adjusted terms.

The quick eyeballing $(\$ 100+\$ 100+\$ 100>\$ 244)$ we completed above shows that the periodic rate of return is positive, so the project certainly increases the owners' wealth if money is available at a zero annual cost:

$$
\begin{array}{ll}
\text { Year 1: } \$ 100\left(\frac{1}{1.00}\right)^{1}=\$ 100 \times 1.00= & \$ 100.00 \\
\text { Year 2: } \$ 100\left(\frac{1}{1.00}\right)^{2}=\$ 100 \times 1.00= & \$ 100.00 \\
\text { Year 3: } \$ 100\left(\frac{1}{1.00}\right)^{3}=\$ 100 \times 1.00= & \underline{\$ 100.00} \\
\text { Total PV of Expected Cash Inflows } & \$ 300.00 \\
\text { Minus PV of Expected Cash Outflows (Initial Outlay) } & -\underline{244.00} \\
\text { Gain in wealth (profit in present value terms, or NPV) } & \$ 56.00
\end{array}
$$

If the cost of capital is $15 \%$ per year, then the investment is undesirable, reducing the wealth of the firm's owners:

$$
\begin{array}{lr}
\text { Year 1: } \$ 100\left(\frac{1}{1.15}\right)^{1}=\$ 100 \times .8696= & \$ 86.96 \\
\text { Year 2: } \$ 100\left(\frac{1}{1.15}\right)^{2}=\$ 100 \times .7561= & \$ 75.61 \\
\text { Year 3: } \$ 100\left(\frac{1}{1.15}\right)^{3}=\$ 100 \times .6575= & \underline{\$ 65.75} \\
\text { Total PV of Expected Cash Inflows } & \$ 228.32 \\
\text { Minus PV of Expected Cash Outflows (Initial Outlay) } & -\underline{244.00} \\
\text { Gain in wealth (loss in present value terms, or NPV) } & (\$ 15.68)
\end{array}
$$

Note that because the expected period $1-3$ cash flows are all equal, we could compute their present value with the annuity shortcut:
$\$ 100\left(\frac{1-\left(\frac{1}{1.15}\right)^{3}}{.15}\right)=\$ 100 \times 2.2832=$
\$228.32
Minus PV of Expected Cash Outflows (Initial Outlay) - $\underline{244.00}$
Gain in wealth (loss in present value terms, or NPV)
[If money had an infinite periodic cost, the NPV would be simply - $\$ 244$, signifying a very high reduction in the owners' wealth, since the PV of any amount received after today would be zero - it would be infinitely discounted - while the PV of today's outlay would be the full $-\$ 244$.]

Let's proceed with the analysis assuming an $8 \%$ annual WACC, and also include the Payback, Discounted Payback, and Profitability Index (not always presented in textbooks) methods.

## Method 1: Payback

The firm's managers need all of year 1's $\$ 100$, all of year 2's $\$ 100$, and $\$ 44$ of year 3's $\$ 100$ to pay the lenders and owners back the $\$ 244$ they provide to purchase the machinery. The payback period therefore is $2+44 / 100$ or 2.44 years. Payback analysis is based on the idea that a shorter payback period is better, in that it suggests a faster recouping of the investment. A company's managers might reject any proposed project with an expected payback period longer than four years, for example.

Weaknesses of the Payback method: No accounting for time value of money (the $\$ 100$ cash flow expected in year 3 is treated as being just as desirable as the $\$ 100$ expected in year 1), and no accounting for cash flows expected after the payback date. Consider this example:

| $\frac{\text { Year }}{0}$ | $\frac{\text { Project A CF's }}{(\$ 100)}$ | $\frac{\text { Project B CF's }}{(\$ 100)}$ |
| :---: | :---: | :---: |
| 1 | $\$ 50$ | $\$ 25$ |
| 2 | $\$ 50$ | $\$ 25$ |
| 3 | $\$ 25$ | $\$ 50$ |
| 4 | -- | $\$ 100$ |
| 5 | --- | $\$ 600$ |
| 6 | -- | $\$ 900$ |
| Payback | 2 Years $\checkmark$ | 3 Years |

Project A is ranked to be the better investment under the payback criterion, even though the high cash flows expected in years $4-6$ (which never make their way into the payback computation) would seem to add substantial value to Project $B$ that is ignored in the payback discussion.

In fact, an even more severe weakness of Payback (or of Discounted Payback, discussed below) is that there is no theoretical basis for choosing any acceptable payback or discounted payback cutoff. If managers decide that a payback longer than four years is unacceptable, how do they come up with the four-year figure? It might be company tradition or current executives' "gut" preferences, but it is not based on any systematic logic or established theory.

Strengths of the Payback method: The primary strength of payback analysis is that it stresses near-term expected cash flows (which are more predictable and thus entail less risk than more distant expected cash flows). How confident can we be that, for Project B above, the high cash flows predicted in years $4-6$ will actually materialize, not be reduced or eliminated by changes in technology, unexpected competitors, new government regulations, or a host of other dangers?

## Method 2: Discounted Payback

If its stressing of the nearer-term expected cash flows makes the payback technique worth including in a capital budgeting analysis, is there a way to address at least one of the standard payback approach's weaknesses? Consider the discounted payback approach, in which we compute the present values of the expected cash inflows before computing a payback period.

$$
\begin{array}{cc}
\begin{array}{c}
\text { Year 1-3 } \\
\text { Cash Flows } \\
\text { "Column 1" }
\end{array} & \begin{array}{c}
\text { Year 1-3 } \\
\text { PV of Cash Flows } \\
\text { "Column 2" }
\end{array} \\
\text { Year 1: } \$ 100\left(\frac{1}{1.08}\right)^{1} & =\$ 100 \times .9259=\$ 92.59
\end{array} \begin{aligned}
& \text { Year 2: } \$ 100\left(\frac{1}{1.08}\right)^{2}=\$ 100 \times .8573=\$ 85.73 \\
& \text { Year 3: } \$ 100\left(\frac{1}{1.08}\right)^{3}=\$ 100 \times .7938=\$ 79.38
\end{aligned}
$$

Think of the unadjusted cash flow projections (\$100 per year for three years) as "Column 1," and the present values of those cash flow estimates, discounted at the $8 \%$ annual cost of capital, as "Column 2." With ordinary payback we ask how many of the unadjusted "Column 1" cash flow values are needed to repay the lenders and owners the $\$ 244$ they provided to buy the equipment. With discounted payback we are merely asking how many of the present value "Column 2" cash flow values are needed to repay the lenders and owners the $\$ 244$ they provided to buy the assets, plus a fair annual rate of return on their invested dollars (which costs the managers $8 \%$ annually to deliver).

What can make discounted payback seem complex is that present values often turn out to be messy numbers that we can not easily keep mental track of, so we must keep running totals on paper. In this example the sum of the year 1 and 2 cash flows' present values is $\$ 92.59+\$ 85.73$ $=\$ 178.32$, which is short of the $\$ 244$ target by $\$ 244-\$ 178.32=\$ 65.68$. So we need all of the first two year cash flows' present values, plus $\$ 65.68$ of the third year's $\$ 79.38$ present value, or $2+65.68 / 79.38=2.83$ years .

Weaknesses of the Discounted Payback method: Discounted payback analysis still provides no accounting for cash flows expected to occur after the discounted payback date, and there is no theoretical basis for identifying an allowable discounted payback cutoff. Sometimes it is also suggested that payback (the earliest capital budgeting technique that was developed) and discounted payback are relatively easy tools to work with and thus might be better than doing no analysis at all - but if they can lead to incorrect investment decisions (which they certainly could do if used in isolation), then their use might indeed be worse than doing no analysis at all.

Strengths of the Discounted Payback method: It stresses near-term cash flows, and does take into account the time value of money. So its use might be worth considering as part of a package of techniques that would be likely to include Net Present Value analysis.

## Method 3: Net Present Value (NPV)

Conceptually, NPV is the present value of all expected cash inflows minus the present value of all expected cash outflows. We introduced the Net Present Value idea above, with the $0 \%$ and $15 \%$ cost of capital examples. Now let's look at things more systematically. NPV presented in its most general form is:

$$
\mathrm{NPV}=\mathrm{CF}_{0}\left(\frac{1}{1+r}\right)^{0}+\mathrm{CF}_{1}\left(\frac{1}{1+r}\right)^{1}+\mathrm{CF}_{2}\left(\frac{1}{1+r}\right)^{2}+\mathrm{CF}_{3}\left(\frac{1}{1+r}\right)^{3}+\cdots+\mathrm{CF}_{\mathrm{n}}\left(\frac{1}{1+r}\right)^{n}
$$

an equation in which we are merely adding the PVs of all expected cash flows (so we are, in effect, netting the negatives against the positives). We define a "normal" business investment project as one with an initial negative cash flow, followed by multiple periods when all cash flows are positive, or at least zero (the company makes an investment today and then expects to get benefits each period for the rest of the project's life). So if a project is "normal," as are most projects used as examples in this introductory coverage, then $\mathrm{CF}_{0}$ should be a negative value and all of $\mathrm{CF}_{1}-\mathrm{CF}_{\mathrm{n}}$ should be non-negative. [It would be possible for periods other than year 0 to have negative net cash flow projections. One "non-normal" example would be a huge project requiring multiple years of expenditures before positive cash flows could be generated; another would involve the expected need for a major retooling midway through a long project's life.]

The NPV equation shown above is actually an application of a more general asset valuation formula that we will see again in later Topics 10 and 13, which deal with the traditional valuation models for bonds, preferred stock, and common stock. That general formula, in which we think of value as the price $\left(-\mathrm{CF}_{0}\right)$ that a rational buyer would pay for an asset today, is:
$-\mathrm{CF}_{0}\left(\frac{1}{1+r}\right)^{0}\left(\right.$ or just $\left.-\mathrm{CF}_{0}\right)=\mathrm{CF}_{1}\left(\frac{1}{1+r}\right)^{1}+\mathrm{CF}_{2}\left(\frac{1}{1+r}\right)^{2}+\mathrm{CF}_{3}\left(\frac{1}{1+r}\right)^{3}+\cdots+\mathrm{CF}_{\mathrm{n}}\left(\frac{1}{1+r}\right)^{n}$.
Buying a security can be thought of as a "project" just like a firm's purchase of new equipment; again, keep it simple by thinking of a "normal" project. The $\mathrm{CF}_{0}$ value we find through this process is the initial outlay (a negative cash flow) that would cause NPV to be $\$ 0$. Why $\$ 0$ ? Because in a very competitive market, like the one in which actively traded securities are bought and sold, we expect to see a transaction price that gives the buyer exactly the expected annual rate of return justified by the risk of holding that security. When the expected cash flows cover only the required rate of return, and nothing more (or less), the project's NPV is $\$ 0$. (Note that if $\mathrm{CF}_{0}$ is equal to the sum of the present values of $\mathrm{CF}_{1}-\mathrm{CF}_{\mathrm{n}}$, then the difference between them is $\$ 0$. The negative sign might seem confusing; when we move $\mathrm{CF}_{0}$ to the left-hand side of the equals sign we must apply a negative sign to it, but if the project is "normal" then $\mathrm{CF}_{0}$ is itself a negative value, such that $-\mathrm{CF}_{0}$ is a positive value equal to the sum of the present values of all subsequent expected positive cash flows.)

In the NPV version of the asset value equation we typically "know" (or at least have a working estimate of) the negative $\mathrm{CF}_{0}$ value (what the company must pay initially for the equipment), and we determine whether expected $\mathrm{CF}_{1}$ through $\mathrm{CF}_{n}$ generate a periodic rate of return higher than
the periodic cost of money (the weighted average cost of capital, explained in Topic 5) and thereby create wealth for the enterprise owners.

Now we can apply the NPV idea using the given cash flow estimates and the $8 \%$ cost of capital assumption. Find the present value of each expected cash flow, and then total these individual amounts:

$$
\begin{gathered}
\text { NPV }=-\$ 244\left(\frac{1}{1.08}\right)^{0}+\$ 100\left(\frac{1}{1.08}\right)^{1}+\$ 100\left(\frac{1}{1.08}\right)^{2}+\$ 100\left(\frac{1}{1.08}\right)^{3} \\
=-\$ 244(1.000)+\$ 100(.9259)+\$ 100(.8573)+\$ 100(.7938) \\
=-\$ 244+\$ 92.59+\$ 85.73+\$ 79.38
\end{gathered}
$$

$=-\$ 244+\$ 257.70=\$ 13.70$ Gain in wealth (profit in present value terms, or NPV)
or, because $\mathrm{CF}_{1}$ through $\mathrm{CF}_{\mathrm{n}}$ are projected to be equal, we can treat them as a team and compute more easily as an annuity:

$$
-\$ 244\left(\frac{1}{1.08}\right)^{0}+\$ 100\left(\frac{1-\left(\frac{1}{1.08}\right)^{3}}{.08}\right)=-\$ 244+\$ 257.70=\$ 13.70
$$

The method above offers the advantage of showing cash flows in the order of when they are expected to occur. A slight variation that achieves the same end result is to compute the total of the present values of the expected cash inflows and then subtract the total of the present values of the presumed cash outflows; in a "normal" project the only cash outflow occurs at the outset and thus already is in PV terms:

$$
\begin{aligned}
& \text { PV of Expected Cash Inflows }=\$ 100\left(\frac{1}{1.08}\right)^{1}+\$ 100\left(\frac{1}{1.08}\right)^{2}+\$ 100\left(\frac{1}{1.08}\right)^{3} \\
& =\$ 100(.9259)+\$ 100(.9573)+\$ 100(.7938) \\
& =\$ 92.59+\$ 85.73+\$ 79.38
\end{aligned}=\$ 257.70
$$

or, because the year 1-3 cash flows are expected to be equal, such that we can treat them as a team and compute their combined present values with the present value of an annuity factor:

PV of Expected Cash Inflows $\$ 100\left(\frac{1-\left(\frac{1}{1.08}\right)^{3}}{.08}\right)=\$ 100 \times 2.577=\$ 257.70$
Minus PV of Expected Cash Outflows (Initial Outlay)

- 244.00

Gain in wealth (profit in present value terms, or NPV)
\$ 13.70

The latter method has the advantage of always providing distinct present value totals for the expected cash outflows and inflows, even when $\mathrm{CF}_{1}-\mathrm{CF}_{\mathrm{n}}$ are epected to be unequal and can not be treated as annuities. But the initial outlay for a "normal" project can be subtracted out as the first or last computational step, so all the approaches shown above are mathematically equivalent and generate the exact same NPV estimates.

NPV is our preferred method of analysis in most capital budgeting situations, for two reasons.

- First, if the computed NPV is positive we know that wealth is being created for the firm's owners (and recall that financial managers' goal is to maximize the firm owners' wealth), such that the project's predicted cash flows represent a periodic rate of return that exceeds the periodic percentage cost of capital. NPV analysis stresses wealth creation rather than focusing on time, or even percentage, measures that may not always be relevant.
- But just as importantly, the computed net present value is the amount of wealth that should be created for the owners if the firm's managers commit to doing the proposed project: $\$ 13.7$ thousand, million, or whatever the units are in the example above.

Indeed, the current wealth of the owners, meaning the total value of the existing ownership claims, known as the "market capitalization" of the existing shares of common stock, should be the total of the NPVs of the company's existing projects.

This preference for NPV is not to say that the other methods are without merit; firms sometimes base "real world" decisions on criteria other than, or in addition to, NPV. But the other methods all are subject to interpretation or other problems that tend to render them theoretically less useful than NPV.

A note on terminology. In basic time value analysis (Topic 4) we saw cases such as: if deposits into a $4 \%$ annual interest rate account are $\$ 600$ at the end of year $1, \$ 750$ at the end of year 2, and $\$ 400$ at the end of year 3 , then the balance at the end of year 3 will be $\$ 600(1.04)^{2}+\$ 750$ $(1.04)^{1}+\$ 400(1.04)^{0}=\$ 648.96+\$ 780+\$ 400=\$ 1,828.96$. The $\$ 1,828.96$ is sometimes called a net future value (NFV), the sum of the future values of several individual cash flows. We also saw essentially the opposite case: for an account that earns $5 \%$ annually to allow withdrawals of $\$ 200, \$ 900$, and $\$ 300$ at the ends of each of years $1-3$, the balance today must be $\$ 200 /(1.05)^{1}$ $+\$ 900 /(1.05)^{2}+\$ 300 /(1.05)^{3}=\$ 190.48+\$ 816.33+\$ 259.15=\$ 1,265.96$. The $\$ 1,265.96$ is, in fact, a net present value, the sum of the present values of several individual cash flows. By this reasoning, the large amount we call Total or Tot in a future value of an annuity problem is an NFV, and the large amount we call TOT in a present value of an annuity problem is an NPV.

But in our discussions we have reserved the NPV label for cases, like those in our Topic 6 capital budgeting coverage, in which we add the PV's of an initial investment and then one or more subsequent expected cash inflows, such that we are netting out negative present values against positive present values in the same example. Your instructor has not found the use of the NFV and NPV labels in basic time value problems to be helpful in enhancing students' understanding.

## Method 4: Profitability Index (PI)

Profitability Index is a relative NPV measure. Whereas NPV is the total present value of the expected cash inflows minus the total present value of the expected cash outflows, PI is the total present value of the expected cash inflows divided by the total present value of the expected cash outflows. (We have seen this kind of dollar difference $v s$. proportion measure before; recall current assets - current liabilities $=$ net working capital while $\mathrm{CA} \div \mathrm{CL}=$ the current ratio.) In a "normal" project like our current example, the only expected net cash outflow occurs in period zero (today) and thus already is in present value terms. So once we have computed NPV, we have the two final values needed in computing the profitability index. Here

$$
\begin{aligned}
\mathrm{NPV} & =\$ 257.70-\$ 244=\$ 13.70 \\
\mathrm{PI} & =\$ 257.70 \div \$ 244
\end{aligned}=1.05
$$

Note that a project with a positive NPV will have a PI greater than 1.0 (a negative NPV project has a PI less than 1.0, and a zero NPV project's PI is equal to 1.0 ). This technique (which should give the same "acceptance" decision as NPV but not necessarily the same "ranking" decision among multiple proposed projects) measures the time value-adjusted return per dollar invested. Our sample project provides $\$ 1.05$ in total time value-adjusted returns, or $\$ .05$ in NPV, for every dollar invested. Some analysts argue that this relative measure of wealth creation lets us make meaningful comparisons among different sized proposed investment projects; how much NPV does a project create for its size? But others view such a relative comparison as irrelevant; they ask: if the goal is to maximize owners' wealth, is a lumbering giant that produces more wealth in total not better than a "mighty mite" that produces more wealth per unit invested but less wealth in total?

## Method 5: Internal Rate of Return (IRR)

Recall that an investment project can enhance a company owners' wealth (i.e., generate a positive NPV) if, and only if, the predicted cash flows create a periodic rate of return that exceeds the periodic percentage cost of invested money (WACC). NPV analysis tells us how much wealth a proposed project creates, and if that wealth is positive in amount we know the periodic return is a percentage greater than the periodic cost of capital. Because our sample project's NPV is positive (\$13.70), we know that money providers expecting three $\$ 100$ annual cash flows after investing $\$ 244$ expect to earn an annual rate of return greater than an $8 \%$ annual cost of capital. Thus we know from NPV analysis that the internal rate of return (IRR) is greater than $8 \%$ per year. But what, exactly, is it? NPV analysis does not tell us. Some analysts argue that the rate of return inherent in the expected cash flows is not essential to know, since generally it is the increase in owners' wealth, and not some percentage measure, that matters. But others believe that the annual percentage return generated by the estimated cash flows is important. (Later we will show why, in some instances, identifying a project's IRR can be of critical importance.)

IRR analysis is based on the NPV equation, which, as explained in the NPV discussion above, is just a restated version of the general asset valuation equation. But whereas in NPV analysis we know the discount rate r (the WACC) and solve for the NPV dollar value, when applying the

NPV version of the general valuation equation to solve for IRR we set NPV equal to $\$ 0$ and solve for $r$ :
$\$ 0=\mathrm{CF}_{0}\left(\frac{1}{1+I R R}\right)^{0}+\mathrm{CF}_{1}\left(\frac{1}{1+I R R}\right)^{1}+\mathrm{CF}_{2}\left(\frac{1}{1+I R R}\right)^{2}+\mathrm{CF}_{3}\left(\frac{1}{1+I R R}\right)^{3}+\cdots+\mathrm{CF}_{\mathrm{n}}\left(\frac{1}{1+I R R}\right)^{n}$,
such that $-\mathrm{CF}_{0}=\mathrm{CF}_{1}\left(\frac{1}{1+I R R}\right)^{1}+\mathrm{CF}_{2}\left(\frac{1}{1+I R R}\right)^{2}+\mathrm{CF}_{3}\left(\frac{1}{1+I R R}\right)^{3}+\cdots+\mathrm{CF}_{\mathrm{n}}\left(\frac{1}{1+I R R}\right)^{n}$.

In the case of a "normal" project we find the periodic rate of return that is just enough to cause the present values of the period 1 through $n$ cash flows to exactly equal the initial outlay, in time value-adjusted terms (non-normal projects can be more complicated). The second representation above is essentially just the general asset valuation equation as used with estimating bond or stock values. However, in a bond or stock valuation problem we know the required periodic rate of return $r$ and solve for $\mathrm{CF}_{0}$; in an IRR analysis we know $\mathrm{CF}_{0}$ and solve for the rate of return inherent in the expected cash flows, the IRR.

Computationally, IRR is the discount rate that equates the sum of the PV's of the expected net cash inflows with the sum of the PV's of the expected net cash outflows; it is, therefore, the discount rate that causes NPV to be $\$ 0$. We set up an IRR analysis the same way we set up an NPV analysis; both are based on the NPV equation. The only difference is that in NPV analysis we know the discount rate (the WACC) and have to solve for the time value-adjusted gain in wealth (the NPV dollar value), whereas in IRR analysis we know the NPV (we set it at $\$ 0$ ) and have to figure out the discount rate that causes the $\$ 0$ NPV. Our goal is not to achieve an NPV of $\$ 0$ (we want NPV to be positive). We are simply using an algebraic tool to find the desired quantity $r=$ IRR:

$$
-\$ 244\left(\frac{1}{1+r}\right)^{0}+\$ 100\left(\frac{1-\left(\frac{1}{1+r}\right)^{3}}{r}\right)=\$ 0
$$

If the project is expected to generate cash flows in more than one time period after the initial investment is made, we must use trial and error to find the IRR; we will be trying to solve for $r$ when we have both $r$ and $r$-to-a-power in the same equation. Recall that an $8 \%$ discount rate caused NPV to be greater than $\$ 0$ (wealth would be created), so the IRR has to be greater than $8 \%$. Let's see what would happen if money instead carried a $10 \%$ annual cost:

$$
-\$ 244\left(\frac{1}{1.10}\right)^{0}+\$ 100\left(\frac{1-\left(\frac{1}{1.10}\right)^{3}}{.10}\right)=-\$ 244+\$ 248.69=\$ 4.69
$$

Our target is $\$ 0$, so $10 \%$ is still too low a discount rate (if money actually did cost $10 \%$ per year the investment would still be wealth-enhancing, so the cash flows must represent an annual rate of return - an IRR - greater than $10 \%$ ). We need to discount more heavily to reduce the sum of the present values of the expected cash flows down toward zero; let's try $12 \%$ :

$$
-\$ 244\left(\frac{1}{1.12}\right)^{0}+\$ 100\left(\frac{1-\left(\frac{1}{1.12}\right)^{3}}{.12}\right)=-\$ 244+\$ 240.18=(\$ 3.82)
$$

Now we have gone below our target of $\$ 0$, so $12 \%$ is too high a discount rate (if money actually did cost $12 \%$ per year the investment would destroy wealth for the firm's owners, so the cash flows must represent an annual rate of return - an IRR - less than $12 \%$ ). Thus we need to discount less severely to increase the sum of the PV's of the expected cash flows up toward zero. Since $12 \%$ is too high while $10 \%$ is too low, let's try $11.1 \%$ :

$$
-\$ 244\left(\frac{1}{1.111}\right)^{0}+\$ 100\left(\frac{1-\left(\frac{1}{1.111}\right)^{3}}{.111}\right)=-\$ 244+\$ 244=\$ 0
$$

So $11.1 \%$ is the IRR!! Is it clear why we set the NPV $=\$ 0$ to find the IRR? It is an algebraic trick; we do not really want NPV to be zero, but rather want to find the rate of return that must exist if the present value of the expected inflows just covers the present value of the expected outflows. We achieve a $\$ 0 \mathrm{NPV}$ only if the annual rate of return equals the annual cost of the money, so conversely if we compute a $\$ 0$ NPV after pretending we have a particular cost of money then the IRR has to be whatever we pretended the cost of money was. Another way to state the point is that we identify an unknown periodic (annual or other) rate of return by finding a zero dollar difference with respect to a known, or at least hypothesized, periodic cost of money.

Recall that our IRR criterion is to accept a project if its periodic IRR is greater than the periodic weighted average cost of capital (thus we sometimes call the WACC the company's hurdle rate for new investment projects); in other words, we accept a project if it earns a periodic return for the money providers that exceeds the managers' periodic cost of providing fair returns to those investors. Because the $11.1 \%$ IRR is greater than the $8 \%$ WACC, this project is acceptable per IRR analysis. Earning an annual return higher than the annual cost of money generates added wealth for the company's owners, although IRR analysis does not tell us how much wealth.

NPV and IRR analyses together "cover all the bases." A positive NPV tells us that the proposed project creates wealth for the company's owners and exactly how much wealth is created, and concurrently tells us that the indicated periodic rate of return exceeds the periodic cost of capital - but not what that periodic rate of return is. IRR analysis identifies that periodic rate of return.

## Method 6: Modified Internal Rate of Return (MIRR)

Is the IRR, as computed above, the relevant rate of return measure? Some analysts argue that it is, based on the logic that a project should be judged on its own internal merits. But other analysts contend that when analyzing a proposed capital project we should view the project's expected life as a unified time period, incorporating both the rate of return represented by the internal cash flows and the rate earned when cash flows are reinvested until the project ends.

Modified Internal Rate of Return (MIRR) is the compounding rate that allows the present value of the expected net cash outflows (the initial investment in a simple "normal" project) to grow, by the end of the project's expected life, to a total value that reflects both the sum of the expected net cash inflows and the impact of reinvesting those cash inflows (at an explicit periodic rate of return, usually identified as the project's WACC). So we are incorporating two ideas not used in the NPV/IRR analysis: explicit treatment of reinvestment, and a comparison with a future rather than just present values. Consider how the expected cash inflows would compound to a terminal value at an $8 \%$ annual reinvestment rate by the end of the project's expected life:

First compute the terminal value

| End of <br> Period | Expected <br> Cash Inflow | \# of Reinvestment <br> 1 | $\$ 100$ | $\frac{\text { Periods until end yr 3 }}{}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | | $8 \%$ FV Int |
| :---: | :---: | :---: | :---: |
| Factor |$~$| Compounded |
| :---: |
| 2 |

So a $\$ 244$ investment is expected to grow, with reinvestment, to $\$ 324.64$ in 3 years. Note that we could compute the terminal value as the future value of an annuity in this case, since all cash flows after period zero are expected to be equal and thus can be treated as a team:

$$
\$ 100\left(\frac{(1.08)^{3}-1}{.08}\right)=\$ 100 \times 3.2464=\$ 324.64
$$

Then find the periodic rate of return that causes the initial investment to grow to the terminal value. A $\$ 244$ investment is expected to grow over 3 years to $\$ 324.64$; we can solve with our basic rate of return technique:

$$
\begin{gathered}
\text { BAMT } \times(1+r)^{n}=\text { EAMT } \\
\$ 244 \times(1+r)^{3}=\$ 324.64 \\
(1+r)^{3}=\$ 324.64 / \$ 244=1.3305 \\
1+r=\sqrt[3]{1.3305}=1.3305^{1 / 3}=1.3305^{.3333}=1.0999
\end{gathered}
$$

$$
\text { So } r=\text { MIRR }=.0999 \text { or } 9.99 \%
$$

[In a simple example with the cash flows after period 0 expected to be equal, we can think of MIRR computation as the two-step process:

MIRR Step 1: $\quad \mathrm{CF}_{1-\mathrm{n}}\left(\frac{(1+\mathrm{WACC})^{n}-1}{\mathrm{WACC}}\right)=$ Terminal Value
MIRR Step 2:
$\mathrm{CF}_{0}(1+r)^{n}=$ Terminal Value
[But since each step equals terminal value, the two steps can be further simplified into one computational exercise as

$$
\text { Combined Single Step: } \mathrm{CF}_{0}(1+r)^{n}=\mathrm{CF}_{1-\mathrm{n}}\left(\frac{(1+\mathrm{WACC})^{n}-1}{\mathrm{WACC}}\right)
$$

Based on the values in the example above, we would compute

$$
\$ 244(1+r)^{3}=\$ 100\left(\frac{(1.08)^{3}-1}{.08}\right)
$$

$\$ 244(1+r)^{3}=\$ 324.64$ (and then solve as shown above)]

The Modified IRR technique is favored by some analysts because it explicitly accounts for reinvestment of the intermediate-period cash flows, which IRR analysis does not. But recall that some theorists see the reinvestment idea as irrelevant to the capital budgeting decision process (in their view an investment project should be judged based on the cash flows it is expected to generate, not based on reinvestment possibilities that can reflect the company's other activities).

Our assumed reinvestment rate in MIRR analysis generally is the WACC, based on the idea that a wealth-maximizing firm would already have exploited investment opportunities with expected periodic returns greater than the periodic WACC. Note that the MIRR blends the internal rate of return (here the $\operatorname{IRR}=11.1 \%$ ) with an "external" rate of return (here the presumed reinvestment rate is the WACC $=8 \%$ ). MIRR should fall somewhere between the WACC and the IRR; the $9.99 \%$ we compute here obviously passes that test.
(If we assumed that the reinvestment rate were $0 \%$, then the terminal value would be simply the unadjusted sum of the expected period $1-3$ net cash inflows, $\$ 300$ in our example above. The corresponding MIRR would be $\sqrt[3]{\$ 300 / \$ 244}-1=7.13 \%$. If we assumed that the annual reinvestment rate would be the $11.1 \%$ IRR, the terminal value would be $\$ 334.53$, and the corresponding MIRR would be $\sqrt[3]{\$ 334.53 / \$ 244}-1=11.1 \%$. In a situation with multiple expected cash inflows following an initial investment, the computed MIRR equals the IRR only if we assume that the annual reinvestment rate will be the IRR. This brief discussion is a stab at explaining the rather cryptic statement sometimes seen in textbooks that "NPV analysis is based on an assumption of reinvesting cash flows at the cost of capital, while IRR analysis is based on an assumption of reinvestment at the IRR.")

## II. Capital Budgeting: A Quick Review of the Process

- Estimate the Cash Flows:

For a simple "normal" project of the type analyzed above - with an initial investment followed by multiple periods of projected net cash inflows - the period 0 cash flow $\mathrm{CF}_{0}$ is the expected upfront investment (a negative value), consisting of the amounts spent on new long-lived assets, getting them installed and ready to use, and any added inventory or receivables the company must hold in connection with the project. The periods $1-n$ projected cash flows $\mathrm{CF}_{1}-\mathrm{CF}_{\mathrm{n}}$ are the expected revenues minus expected cash expenses involving payment to parties other than money providers (so we don't subtract depreciation since it is not a cash outlay, and we don't subtract interest since interest is part of what is available for the money providers, but we do subtract income taxes since taxes affect what is expected to be left as profit for the owners).

- Determine the appropriate discount rate (WACC), based on the riskiness of the expected cash flows (how likely it is that the predicted cash outflows and inflows will actually be realized, as perceived by potential money providers).
- Compute the NPV: sum the present values of all the expected cash flows (including the negative cash flow represented by the cost of the project at time 0 , which already is stated in present value terms). Apply other capital budgeting techniques if they are deemed to provide valuable information in the decision-making process.
- Accept the project if the NPV is positive (i.e., if the sum of the present values of the expected cash inflows exceeds the sum of the PV's of the expected cash outflows - which is just the initial cash outlay representing the project's cost in a simple "normal" project), and if any secondary criteria are not violated (for example, a firm's managers might impose a maximum payback or discounted payback period restriction because of the added risk of projects with cash flows expected to be realized far in the future).


## III. Capital Budgeting: Some Points to Note

- Capital budgeting analysts can not simply make projections and then walk away from their work; each project has a post-audit phase in which actual results are compared to what had been projected. Goals of the post-audit include holding managers accountable for their projections and helping the company to improve its forecasting techniques.
- When considering a single investment project in isolation, the decision criterion is to accept the project if it has a positive NPV (or an IRR or MIRR > WACC, or a Profitability Index > 1, or a Payback or Discounted Payback period < some stated limit - if these latter criteria, which all generally are inferior to NPV, are to be used).
- When considering a series of independent investment projects (i.e., we could conceivably do them all), the decision criterion is to select all of the projects that have positive NPVs (or IRRs > WACC, etc.).
- When considering a series of mutually exclusive investment projects (i.e., we would choose only one, such as when considering two different machines to do the same task), the decision criterion is to select the one that has the highest NPV (frequently it will also have the highest IRR, MIRR, and Profitability Index, but that result does not always happen).
- If we compare two proposed projects and $A$ has the higher NPV while $B$ has the higher IRR (or MIRR or Profitability Index) - a result that can occur with projects that are different in size, or have different timing on the expected cash flows - our decision criterion is to choose the project that has the higher NPV and thus contributes more wealth to the owners of the firm. (IRR is a percentage measure; would we rather earn a high percentage return on a small base, or generate more wealth by earning a smaller percentage on a bigger base?)
- Another weakness of the IRR criterion is that there actually can be more than one solution to the IRR equation. This situation can occur when 1) there is more than one directional change in expected cash flows, such as negative in period 0 , then positive for one or more periods, then negative again (a non-normal project) and 2) the unadjusted total of the net cash outflows exceeds the unadjusted net cash inflow total. If two or more rates are both solutions to the IRR equation, what is the correct IRR answer? The standard answer of "both" may be technically correct, but has little practical value as a decision tool.
- A strength of the IRR criterion, however, is that comparing the IRR to the WACC gives a quick insight into how much of a "safety margin" exists in case the project's costs or inflows end up being less favorable than the analyst had projected (whereas a large project with a slightly positive measured NPV could turn out to be undesirable if any of the cash flows turn out to be less favorable than had been projected).
- Another strength of the IRR criterion is that for a "normal" project, with one net cash outlay followed by a series of expected net cash inflows, there is only one IRR. But NPV depends on the assumed WACC - and if we do not know the WACC, we can not compute an NPV, or would have to compute a series of NPVs based on different WACC assumptions (such a series is called an NPV Profile). But a "normal" project has a single, unique IRR.

In fact, the ranking of two investment projects (i.e., which creates more wealth for the company's owners) can depend on our estimate of the WACC: one project might appear better if the WACC is low, while the other might appear better if the WACC is high.

Consider two proposed projects, which we will call $L$ (because we expect to have to wait a long time to realize a high net cash inflow) and $S$ (expected to generate the highest net cash inflow in a shorter time).

| $\frac{\text { Year }}{0}$ | $\frac{\text { Project L CF's }}{}$ | $(\$ 100)$ |
| :---: | :---: | :---: |$\quad \frac{\text { Project S CF's }}{}$| $(\$ 100)$ |
| :---: |
| 1 |

At an assumed WACC of 5\%, the respective NPVs are (note that $L$ 's is higher):

$$
\mathrm{NPV}_{\mathrm{L}}=-\$ 100\left(\frac{1}{1.05}\right)^{0}+\$ 10\left(\frac{1}{1.05}\right)^{1}+\$ 60\left(\frac{1}{1.05}\right)^{2}+\$ 80\left(\frac{1}{1.05}\right)^{3}=\$ 33.05
$$

or, with an equivalent but more streamlined algebraic representation:

$$
\begin{aligned}
& \mathrm{NPV}_{\mathrm{L}}=\frac{-\$ 100}{(1.05)^{0}}+\frac{\$ 10}{(1.05)^{1}}+\frac{\$ 60}{(1.05)^{2}}+\frac{\$ 80}{(1.05)^{3}}=\$ 33.05 \\
& \mathrm{NPV}_{\mathrm{S}}=\frac{-\$ 100}{(1.05)^{0}}+\frac{\$ 70}{(1.05)^{1}}+\frac{\$ 50}{(1.05)^{2}}+\frac{\$ 20}{(1.05)^{3}}=\$ 29.30
\end{aligned}
$$

But at an assumed WACC of $15 \%$, the NPVs are (note that $S$ 's NPV is higher):

$$
\begin{gathered}
\mathrm{NPV}_{L}=\frac{-\$ 100}{(1.15)^{0}}+\frac{\$ 10}{(1.15)^{1}}+\frac{\$ 60}{(1.15)^{2}}+\frac{\$ 80}{(1.15)^{3}}=\$ 6.67 \\
\mathrm{NPV}
\end{gathered}=\frac{-\$ 100}{(1.15)^{0}}+\frac{\$ 70}{(1.15)^{1}}+\frac{\$ 50}{(1.15)^{2}}+\frac{\$ 20}{(1.15)^{3}}=\$ 11.82 \checkmark ~ 又
$$

(Note also that if money costs more we create less wealth for the company's owners with either project, with NPV declining from $\$ 29$ or $\$ 33$ if money costs $5 \%$ per year to $\$ 7$ or $\$ 12$ if money costs $15 \%$ per year.) It turns out that $8.7 \%$ is the crossover point in the NPV profiles for Projects L \& S. For any WACC less than $8.7 \%$, project L creates more wealth (we do not mind waiting longer to get the big inflows if the cost of waiting - the discount rate - is low). For any WACC greater than $8.7 \%$ project $S$ is better (we would rather get the big dollars sooner if there is a big penalty for waiting; recall that discounting is a form of penalizing). If we discount at the $8.7 \%$ annual crossover rate we get the same mid-level NPV measures for both proposed projects:

$$
\begin{aligned}
& \mathrm{NPV}_{\mathrm{L}}=\frac{-\$ 100}{(1.087)^{0}}+\frac{\$ 10}{(1.087)^{1}}+\frac{\$ 60}{(1.087)^{2}}+\frac{\$ 80}{(1.087)^{3}}=\$ 22.28 \\
& \mathrm{NPV}_{\mathrm{S}}=\frac{-\$ 100}{(1.087)^{0}}+\frac{\$ 70}{(1.087)^{1}}+\frac{\$ 50}{(1.087)^{2}}+\frac{\$ 20}{(1.087)^{3}}=\$ 22.28
\end{aligned}
$$

But note that there is only one IRR for each project: $18.1 \%$ for $L$ and $23.6 \%$ for $S$ :

$$
\begin{aligned}
& \frac{-\$ 100}{(1.181)^{0}}+\frac{\$ 10}{(1.181)^{1}}+\frac{\$ 60}{(1.181)^{2}}+\frac{\$ 80}{(1.181)^{3}}=\$ 0 \\
& \frac{-\$ 100}{(1.236)^{0}}+\frac{\$ 70}{(1.236)^{1}}+\frac{\$ 50}{(1.236)^{2}}+\frac{\$ 20}{(1.236)^{3}}=\$ 0
\end{aligned}
$$

Thus while we can determine how much wealth each project generates only if we first know the WACC, we know that the expected cash flows of proposed project $S$ represent a higher annual rate of return than do those of proposed project L . So even if we lack a clear handle on what the WACC's for the projects would be, if we are confident that these WACC's would be equal then we know that $S$ creates more wealth for the firm's owners than $L$ does. (And if the unknown WACC would be expected at least to be something less than $23.6 \%$, we would know that the wealth produced by $S$ would be positive in amount, while if we could be confident that L's cost of capital at least would be something less than $18.1 \%$ we would know that the wealth produced by $L$ would be positive.) It is interesting that business executives sometimes seem to favor the IRR technique over NPV for evaluating capital investment projects. Reasons may include the desire to evaluate in terms of percentage returns, which relates to the "safety margin" idea discussed above. Smaller firms may rely to some extent on the payback approach, because of that technique's strength in stressing liquidity issues and the low cost of conducting the analysis.

## IV. Evaluating Projects with Unequal Lives

Consider two possible investment projects, S (with a shorter life) and L (with a longer life):

| $\frac{\text { Year }}{0}$ | $\frac{\text { Project S CF's }}{}$ |  |
| :---: | :---: | :---: |
|  | $\$ 100,000)$ |  |
| 1 | $\$ 60,000$ | $\$ 100,000)$ |
| 2 | $\$ 60,000$ | $\$ 33,500$ |
| 3 | --- | $\$ 33,500$ |
| 4 | -- | $\$ 33,500$ |
| 4 |  | $\$ 33,500$ |

If the annual weighted average cost of capital is $10 \%$, we compute the NPV's as

$$
\begin{aligned}
& N P V_{S}=-\$ 100,000+\$ 60,000\left(\frac{1-\left(\frac{1}{1.10}\right)^{2}}{.10}\right)=\$ 4,132 \\
& \mathrm{NPV}_{\mathrm{L}}=-\$ 100,000+\$ 33,500\left(\frac{1-\left(\frac{1}{1.10}\right)^{4}}{.10}\right)=\$ 6,190
\end{aligned}
$$

Based on our discussion of capital budgeting techniques up to this point, it would appear, at first glance, that L is the project that creates more wealth for the company's owners. But notice that the projects cover differing time horizons. In fact, over a 4 -year period we could conceivably complete project $S$ twice (depending on the nature of that project). Project $S+S$ ' would be analyzed on a year-by-year basis as follows:

| $\frac{\text { Year }}{0}$ | $\frac{\text { Project S CF's }}{}$ |  | $10 \%$ PV Factor |  |
| :---: | :---: | :---: | :---: | :---: | | PV of CF |
| :---: |
| 1 |

Or we could just compute

$$
\begin{array}{r}
\mathrm{NPV}_{\mathrm{S}}=-\$ 100,000+\$ 60,000\left(\frac{1-\left(\frac{1}{1.10}\right)^{2}}{.10}\right)=\$ 4,132 \\
\mathrm{NPV}_{\mathrm{S}^{\prime}}=\$ 4,132\left(\frac{1}{1.10}\right)^{2}=\underline{\$ 3,415} \\
\text { NPV Total over 4 Years } \$ \underline{\underline{7,547}}
\end{array}
$$

Because we anticipate doing a project like $S$ twice, we have to take S's \$4,132 NPV into account twice. But the total wealth created by our plan to do Project $S$ and then do another one just like it over the next four years is not $2 \times \$ 4,132=\$ 8,264$. Project $S$ ' is expected to generate $\$ 4,132$ in wealth two years from now, but the present value of that expected gain in wealth is only $\$ 3,415$. (How much wealthier do we feel today from our expectation of feeling wealthier by $\$ 4,132$ in two years?) So if we commit today to doing Project S starting now, and then to doing similar Project S' starting in two years, our gain in wealth today is $\$ 4,132$ plus the present value of another $\$ 4,132$ : $\$ 4,132+\$ 3,415=\$ 7.547$.

This method of analysis is called the replacement chain (or common life) approach. It tells us that if we could repeat Project $S$ during years $3-4$, we would prefer Project $S+S$ ' to project $L$ as a means of keeping our facilities occupied for a four-year period. With this technique, we compare two (or more) investment projects by pretending we can repeat them any number of times needed (a chain of projects, with a new one replacing the one that just ended) so that their extended lives are projected to end on the same date. For example, in comparing a two-year project with a four-year project, we pretend that we will undertake the two-year project 2 times and the four-year project 1 time, such that either choice would terminate at the end of year 4. In comparing a two-year project with a three-year project, we pretend that we will undertake the two-year project 3 times and the three-year project 2 times, such that either choice would end its extended life at the end of year 6 .

But this tool does not work well for comparing, say, a 13-year project with a 19-year project. (Because 13 and 19 are both prime numbers, we would have to assume that we could do the 13year project 19 times and the 19-year project 13 times over 247 years; the far distant cash flow projections would be all but meaningless.) Indeed, it is illogical to assume, as we implicitly do when using a replacement chain, that there will be no inflation, and no improvement in efficiency of the equipment (and no other changes in annual revenues or costs) as new incarnations of the project occur over an extended future time period.

A second approach to dealing with different-lived projects, in which these problems arguably are not so pronounced, is the equivalent annual annuity method. Here we simply look at the NPVs, $\$ 4,132$ for Project $S$ and $\$ 6,190$ for Project $L$, and determine the amount of wealth that each project would create for the company's owners on an annualized or other appropriate periodic basis (the implicit assumption is that a project of the specified type could continue generating that level of wealth each year or other period for any desired interval, within reason):

$$
\operatorname{EAAs}\left(\frac{1-\left(\frac{1}{1.10}\right)^{2}}{.10}\right)=\$ 4,132
$$

EAAs x $1.7355372=\$ 4,132$; therefore EAAs $=\$ 2,381$.

$$
\operatorname{EAA}_{\mathrm{L}}\left(\frac{1-\left(\frac{1}{1.10}\right)^{4}}{.10}\right)=\$ 6,190
$$

Because $S$ has the higher EAA, it is selected as the superior project, the one that creates more wealth each period for the company's owners (just as S is shown to generate more wealth than L under replacement chain analysis; the two approaches will always rank two projects the same way). Another way to think about EAA analysis is that the EAA represents a perpetual stream of dollars the company owners could receive from a project if the managers kept repeating it over the years, and that we can capitalize a perpetual stream (perpetuity) of that type into an overall gain in wealth by discounting it to a present value at the WACC. Since S's EAA of $\$ 2,381$ every year forever has a present value of $\$ 2,381 / .10=\$ 23,810$, while L's EAA of $\$ 1,953$ every year forever has a present value of only $\$ 1,953 / .10=\$ 19,530, \mathrm{~S}$ is creates more wealth than L .

## V. Real Options

Late in the last century some financial theorists determined that standard NPV analysis can lead to incorrect capital budgeting decisions. The reason is that any cash flow projection used in computing NPV is the midpoint of a range of possible outcomes in an always-uncertain world; an analyst who predicts a 15 -year project's annual cash inflow at $\$ 75$ if customer demand turns out to be weak, $\$ 100$ if average, and $\$ 125$ if strong (and the three cases all seem equally likely) will use $\$ 100$ as the working value. But then the NPV estimate reflects the logic that the project would proceed without changes for the entire 15 years, even if the yearly figure turned out to be $\$ 75$, so the impact the prospect of 15 years of $\$ 75$ has on the working value keeps the computed NPV lower, possibly even negative such that the project would be rejected. Yet if demand did turn out to be low and it became clear after a few years that cash flows would stay at the $\$ 75$ yearly figure going forward, the managers would actually add value by shutting the project down to cut their losses. In a similar manner, if product demand turned out to be at the high end of the range, with $\$ 125$ as the yearly cash flow realized once the product got to the marketplace and was not being judged only on paper, the managers would add value by expanding the project.

The ability to abandon a project that turns out not to meet initial expectations and to expand one that ultimately exceeds projections initially made on paper are two forms of "real option" that can make an investment worth more than its traditionally-computed NPV. By this logic the NPV as we have computed it in Topic 6 is the low-end measure of wealth a project creates for the firm's owners; adding the value of any real options (the "put option" to abandon, or the "call option" to expand or perhaps just learn how to do business involving unfamiliar technologies or geographic regions) to the standard NPV yields a figure sometimes called the "strategic NPV." A real option is the ability to take tangible action in the real economy, the sector where actual goods and services are created. A financial option, on the other hand, is the right in the financial sector to buy or sell a financial instrument, often shares of a specific company's common stock, at a pre-determined price. $\cdot$

