Topic 10: Bonds (*Copyright © 2023 Joseph W. Trefzger*)

A bond is a document (a security) that provides evidence that money has been borrowed, usually for a long time period. Company managers borrow money by issuing long-term bonds because having some debt in the capital structure keeps the company's weighted average cost of capital in check – through the low interest rate paid if debt is used in moderation, and the ability to deduct interest payments when computing taxable income; recall Topic 5. And it is common for the managers to lock into an interest rate that will remain the same for many years so they can make long-term investment plans knowing, with certainty, the cost of the borrowed component within their WACC. (Companies borrow on a short-term basis by getting bank loans, or by issuing securities called "commercial paper" with maturities of one year or less.)

Why are bonds interesting to study? The type of bond we will focus on, with unchanging interest payments, becomes less attractive in a competitive market if the rate of return a lender would expect to earn on a similarly risky new loan increases. Imagine that five years ago XCorp issued $1,000 bonds (it borrowed multiple $1,000 units) and agreed to pay whoever holds each bond 6% of $1,000, or $60, in interest every year for 30 years, after which the $1,000 was to be returned as well. Now five years later a sensible lender would expect an 8% annual return for lending to XCorp for 25 years, either because XCorp has come to be seen as riskier to lend to or because the general level of interest rates across the economy has risen. So the current holder of some of these now-unattractive bonds (each pays $60 in annual interest for 25 remaining years, when a similarly risky new 25-year $1,000 loan would be expected to generate $80 in yearly interest) made a bad bet on where interest rates, specific to XCorp or market-wide, were headed. She can sell the now-unattractive bond in a secondary market transaction only by accepting a price less than the $1,000 principal (or par or face) value.

[Bonds also are issued by government agencies at the federal, state, and local levels in the U.S. and by foreign governmental units, and also by not-for-profit organizations like hospitals and even universities – for example, ISU issued 40-year "municipal revenue" bonds in the early 1970s to raise the $11.8 million to pay for constructing Bone Student Center and Braden Auditorium. Thus the bond market is much bigger than the stock market, which involves the raising of money only by corporations. Bonds issued by the U.S. Treasury (long-term "T-bonds," intermediate-maturity "T-notes," or short-term "T-bills") are recognized for being free of default risk; the U.S. government will always be able to pay its debts, even if it must create more money and pay with inflated dollars. So the U.S. government generally is viewed as the safest entity in the world to lend to, and the interest rate other parties pay to borrow often is measured as some premium over the "risk-free" U.S. government bond benchmark rate.]

I. <u>A Quick Review of Interest Rates</u>

As suggested above, an existing bond's value is largely determined by the interest rate a lender would charge on an equally-risky new loan with similar features. So a bond's theoretical value (the price a rational secondary market buyer would be expected to pay in a competitive market) changes as we see changes in the perceived risk of lending to the issuer in question (which would change the interest rate charged to that borrower) and/or changes in the general level of interest rates (which would change the interest rate charged to all borrowers, including the company or governmental unit in question). In an earlier unit we discussed the theorized general breakdown of an interest rate:

$$r = r^* + IP + DP + LP + MP + FP$$

In this representation:

r = the nominal annual interest rate observed in a money-lending transaction

r^* = the real (not adjusted for inflation) risk-free annual interest rate (while it has been close to 0% in the past few years, various studies over recent decades would show this figure to be typically in the 2.5% – 3.5% per year range)

IP = the premium added to the interest rate to compensate for average annual inflation expected over the lending period [so r* + *IP* should be the *nominal* annual risk-free interest rate r_f, which we can think of as the interest rate the U.S. government would pay on short-term Treasury bills]

DP = the annual premium for default risk (the chance that the borrower will not repay part or all of the principal borrowed, at least on the specified schedule)

LP = the annual premium for liquidity risk (the difficulty the lender might face in trying to sell the security before the maturity date if there is not a ready, active market for it)

MP = maturity risk premium (the added risk of making a long-term financial commitment with an unchanging rate of return that could become unattractive if there is an increase in returns expected on new commitments similar in risk and other features)

FP = foreign exchange risk premium (the risk of converting a foreign currency back to the lender's home currency at the end of the investment period)

(There can be premiums for other perceived risks as well, including the early repayment or "callability" risk that will be discussed later, or the "sovereign" or "country" risk of investing in a foreign locale with market and political traditions that differ from those in the U.S.) That analysis helps explain the annual rate of return expected for lending to a particular company or other borrowing party. But to gain insights on the general level of interest rates across the economy we analyze the relationship, at a given point in time, between *short-term* and *long-term* interest rates. This relationship is called the *term structure* of interest rates. To examine the term structure, find the interest rate the U.S. government is paying today on 3-month, 6-month, 1-year, 5-year, 10-year, etc. T-bills/notes/bonds. (On a particular day the government might be paying 3% per year to borrow for two years but 4% per year to borrow for ten years.) Plot points on a graph that shows interest rate on the vertical axis and time to maturity on the horizontal axis. Connect the dots with a line, and then see if the resulting *yield curve* is, at least for the most part:

1. Upward-sloping (a *normal* yield curve, with long-term rates higher than short-term rates)
2. Downward-sloping (an *inverted* yield curve, with short-term rates higher than long-term)
3. Level (a *flat* yield curve, with short-term and long-term interest rates essentially equal)

There can be other yield curve shapes (*e.g*. humped, with intermediate-term rates higher than short- or long-term), but these three general cases are useful for an introductory discussion.

II. Explaining the Yield Curve

In most periods over recent decades, the yield curve has been upward sloping (hence we call an upward-sloping yield curve "normal"). In the high-inflation period of the early 1980s the curve was inverted. In some recent intervals the yield curve has been fairly flat (or sometimes showing higher interest rates for intermediate-term bonds than for short-term or long-term).

Three theories have been advanced to explain why the term structure would be characterized by a normal, inverted, or flat yield curve on a particular day:

1) *Expectations Theory*: states that any "long-term" (multi-year) rate is the average of the "short-term" (*e.g.*, 1-year) rates expected over the stated period. For example, the 5-year T-note rate should be the average of the 1-year T-bill rates known for the current year and then expected for the subsequent four years. The expectations theory allows us to do some interest rate *forecasting*. Let's say that you want to lend for three years. You have various choices involving bonds with full-year maturities: (1) lock in for three years at 5% per year, (2) lock in for two years at 4% per year, and then lend during year 3 at whatever rate prevails, or (3) lock in only for one year at 2%, and then lend during years 2 and 3 at whatever annual rates prevail.

Question: what is the 1-year interest rate the market expects to observe in year 3? The 1-year rate expected in year 2? The 2-year rate expected in year 2? Recall that the expectations theory tells us that any observed "long-term" interest rate is the average of the known and expected "short-term" interest rates over the observed period. Set up a table:

	Year 1	Year 2	Year 3	Total
3-Yr Lock-in	5%	5%	5%	15%
2-Yr Lock-in + 1-Yr	4%	4%	??	15%
1-Yr Lock-in + 1-Yr + 1-Yr	2%	?	??	15%
1-Yr Lock-in + 2-Yr	2%	???	???	15%

Locking in for three years gives an assured average of 5% per year, or a total of 15% for three years. So per the Expectations Theory, any other plan involving three years also should have a 15% expected total. Thus the 1-year rate the market expects to prevail in year 3 has to be 7%, represented as ?? the second line (and also the third) in the table above, such that 4% + 4% + 7% = 15%. The logic is that if the 1-year rate expected for year 3 truly were, *e.g.*, 10%, lenders could expect to earn 4% + 4% + 10% = 18% with no added risk by simply structuring three years of lending to the government as two transactions, and thus the 3-year bond could not exist unless its interest rate rose to 6% per year. The 1-year rate expected to prevail in year 2 is 6%, such that 2% + 6% + year 3's 7% = 15%. (We could also compute the 6% 1-year rate expected for year 2 by noting that the two 1-year rates of 2% and ?% should total to be the same as the 4% + 4% we know can be earned on the 2-year bond issued today.) And the 2-year rate expected to prevail in year 2 (which would apply to years 2 and 3) is 6.5%, such that 2% + 6.5% + 6.5% = 15%; start with 15%, subtract the 2% that applies to year 1, and divide the difference by 2.

The 6% 1-year interest rate that the market expects to see quoted at the start of year 2, the 7% 1-year rate expected for year 3, and the 6.5% per year rate expected for years 2 and 3 are called *implied forward rates*: forward in that we expect them to apply to a future time period, and while they are not in any way assured to actually happen they are implied, if we place any faith in the Expectations Theory, by the structure of the rates we actually know today.

We have used an additive (arithmetic average) approach to computing implied forward rates. It is more technically correct to use a multiplicative (geometric average) approach. If you earn 5% each year for three years then your compounded total should be not 5% x 3 = 15% but a slightly higher $(1.05)^3 - 1 = 15.7625\%$, and the 1-year rate expected for year 3 should be not 15% – 4% – 4% = 7% but a slightly higher $(1.157625 \div 1.04 \div 1.04) - 1 = 7.0289\%$. Compounding over time definitely has an impact on a money provider's ultimate financial returns, but because interest rate analysis is always subject to some guess work we typically are comfortable with the simpler, more easily understood additive approach.

2) *Liquidity Preference Theory*: states that both borrowers and lenders prefer to keep their positions liquid, and will pay (or give up) something to achieve liquidity. Whereas under the Expectations Theory we have no reason to expect the yield curve to have any particular slope before we look at specific interest rates on any given day, Liquidity Preference tells us that the yield curve should tend to slope upward, with the average annual long-term interest rate higher than its shorter-term counterpart at any given time.

Why? Because borrowers' liquidity position is enhanced through long-term borrowing (they have access to the borrowed money for a long time and need not worry about having to pay principal back frequently and then negotiate to re-borrow under conditions that might turn out to be unfavorable), and so they willingly pay higher annual interest rates to borrow long-term (or must be quoted lower annual rates to be lured into short-term loans). Lenders' liquidity position is enhanced through short-term loans (they collect principal back quickly, before a healthy borrower or economy can get into trouble, and can decline to re-lend if conditions have arisen that would make re-lending too risky), and so they quote lower annual interest rates to get borrowers to borrow short-term (or must earn higher annual rates to make long-term loans).

So can a downward-sloping yield curve be consistent with the liquidity preference idea? Yes; it simply means that the market expects short-term rates to be much lower in the coming years than they are today, such that the expectations effect overwhelms a still-existing liquidity preference effect. (Note that a downward-sloping yield curve, which is said to be observed when a recession is expected, suggests, per the Expectations Theory, that coming periods' short-term interest rates are expected to be lower than today's.)

3) *Market Segmentation Theory*: states that there are various segments of the market for borrowed funds: perhaps short, intermediate, and long-term, and that what goes on in one segment could be largely unrelated to what occurs in the others. A less extreme form of this argument has been called the *preferred habitat* theory: a borrower might prefer to borrow long-term, but could be lured into short-term loans with a sufficiently low interest rate (while a lender might prefer to lend short-term, but a sufficiently high interest rate could provide the incentive to make long-term loans). According to this theory, supply and demand conditions in short *vs.* long-term debt markets could differ, for example if many businesses were trying simultaneously to obtain short-term ("working capital") loans or if regulations restricted certain market participants to borrowing or lending in a short-term or long-term manner. But because new financial products developed in recent years let investors easily rearrange or trade cash flows (*e.g.*, interest rate swaps, *tranches* of varied maturities in collateralized debt obligations), the idea of unrelated segments of the borrowing/lending market has become less meaningful.

III. A General Discussion of Bonds

A. Types of Long-term Bonds

1) Based on *Priority* of Payment

Parties that lend money to the same company do not necessarily all have equivalent claims. The "front of the line," as we informally call the lender group in class, can be characterized by various tiers of lenders, with those closer to the front being paid in full before those behind them receive any financial returns. Lenders that face higher risks of not being repaid expect to receive higher interest rates. Typical categories we might see are:

- *Collateralized Bonds*: specific assets (real estate, equipment, financial instruments) serve as collateral, and will be sold to generate money to repay holders of these bonds if the company managers can not repay them through successfully running the business.
- *Senior Debentures*: no specific assets serve as collateral to protect holders of these bonds; the firm's ability to generate cash flows is the main assurance of repayment.
- *Subordinated Debentures*: no specific assets serve as collateral, and another group (or multiple groups) of more senior bondholders will receive its full promised return before this subordinated group of lenders receives any interest or return of principal. Subordinated debt has been less commonly used since the 2000s financial crisis.

2) Based on *Method* of Payment

Coupon Bond: the lender gets regular interest payments, and then principal is repaid in full at the maturity date. Long ago bondholders received booklets of paper *coupons*, to be deposited on the indicated dates like we deposit other checks. In recent decades these *bearer* instruments have been almost entirely replaced by a *registered* process, through which the borrowing entity directs interest payments to the brokerage account of the party shown as the holder in company records. (Coupon books could be lost/stolen, IRS might not know who was getting the interest income. The U.S. government last issued paper coupon bonds in 1982. Old bond/stock certificates sometimes have numismatic value; collecting them is called *scripophily*.) Now "coupon" refers to the existence of regular scheduled interest payments, not the booklet of deposit coupons.

Zero-Coupon Bond: the lender lends a *small* amount of money, and then receives no regular interest payments. But at maturity the lender receives a *large* amount of money, to make up for not having received any intermediate-term returns. The benefit to the borrower from this arrangement is the ability to finance a long-term project without the need to start paying back money before the project generates cash. The lender benefits by getting an assured compounded rate of return (the single large payment received at the maturity date constitutes everything the lender will get; with nothing received prior to maturity there is nothing to reinvest, so there can be no reinvestment gain or loss).

Convertible Bond: the lender (bond buyer) has the right to trade the bond in for a pre-determined number of shares of the issuing company's common stock. The lender benefits through the potential to make money on the stock if the stock price later rises. The borrowing firm's primary benefit is that lenders will accept a lower interest rate (perhaps several full percentage points per year lower, even 0% in extreme cases) since they might realize gains if the stock price rises. Holding the interest rate in check with a conversion feature can be especially attractive when the bonds are of low credit quality and the rate otherwise would be quite high. Some, but not all, convertible bonds issued in recent years have been of the speculative or "junk" variety (2018 and 2020 were especially active years for convertible bond issuing), and late 2023 reports showed electric vehicle makers as active convertible issuers. A similar arrangement is to attach "warrants" to a bond issue; the holder of warrants can buy common shares at a pre-determined price without having to trade in the bond. (Then to show how bizarre things can get: Wall Street firms have also created *high* interest rate "reverse convertibles" or "revertibles" on some firms' common stocks; if the stock prices *fell* by 20 or 30% the holder was *required* to trade the bond in for the lower-valued shares.)

Floating Rate Bond: unlike with the fixed-rate bonds we focus on, the interest rate paid to the floating-rate lender is adjusted from time to time to reflect changing market conditions. If the interest rate paid to a lender is always close to what would be paid in a newly negotiated transaction, then the bond should always be worth something close to its par value. One version is the *indexed* bond, with interest that rises over time based on the measured rate of inflation. Another is the *toggle* note, in which a borrowing firm can delay paying interest, perhaps until the maturity date, in return for paying a higher interest rate on the deferrals. A few major U.S. banks have issued "floaters," but these types of bonds are not common (they have been more popular in Europe than in the U.S.); indeed, as we noted, a primary motivation for a firm's managers to borrow long-term is to know the cost of interest with certainty into the distant future.

Sinking Fund Bond: the borrower establishes a program for retiring principal systematically over the bond issue's life, so the lenders have the assurance that they, as a group, will be owed less as the uncertain future progresses. So along with paying applicable interest, the borrowing entity retires some principal each period. The borrower can pay back this principal over time either by a) creating a savings-type "sinking" fund that will, with deposits plus returns, grow by the maturity date to the principal total owed or b) buying back some bonds from current holders in secondary market transactions on a regular basis so that the amount of principal the issuer owes keeps dropping. This arrangement provides some safety for lenders; if the long-term borrower's financial strength starts declining later at least it will not still have as much money to repay.

B. Bond Concepts and Terminology: An Intuitive Discussion

If you considered lending money to a local small business owner, you would first want to check on the borrower's financial strength and good character, by looking over the small firm's relatively simple financial statements and asking knowledgeable parties about the business owner's reputation. Then you would want to have an enforceable agreement on what the borrower would do with your money. You might want that agreement to restrict certain actions on the borrower's part (*e.g.*, you might not want the borrower to be able to borrow large amounts from other lenders before you have been repaid, with interest). Then you would want to be able to check on the borrower's compliance with the agreements by visiting the business every so often. And you would want to keep checking on the borrower's financial strength on an ongoing basis, after making the loan.

If instead you considered lending money to a large, faraway business – by purchasing its bonds – you would want to go through the same steps. However, you could not easily negotiate a lengthy agreement with a large, distant company; check on its reputation and its complex finances; and then show up frequently to monitor it for compliance with your agreement. And paying an expert to do these things just for you would not be cost effective if you were a small bond investor. But the marketplace has created an efficient means of having these services provided (if it did not, then money providers would not lend by buying an issuer's bonds).

Specifically, a company's *investment banker* advises it on restrictions that should be contained in an enforceable agreement if money providers are to be comfortable lending to the company. A party with legal expertise is responsible for seeing that the borrower complies with the restrictions spelled out in the agreement. And a party with financial expertise reports whether the borrower's reputation and financial condition (and promises spelled out in the agreement) are strong enough to merit lending the company money at a particular interest rate (either directly, or by purchasing a previously-issued bond from another investor). It issues an initial report when the lender first wants to borrow, and then revises its evaluation periodically if the borrower's ability to pay in a timely manner appears to have changed. [The same steps are taken when a state or local government agency wants to borrow money by issuing bonds.]

C. Bond Terminology

Now let's look more closely at the ideas and terms discussed in the informal story above.

Indenture: the complex legal document that spells out the protections provided to the lenders. Contained within the indenture are a series of:

Restrictive Covenants: each covenant is a restriction on the borrower, as stated in the indenture (a *negative* covenant might be that debt/total assets can not rise above a specified level even if the firm is acquired by another company; a *positive* covenant might be that the times-interest-earned ratio must be maintained at some minimum specified level).

Trustee: the party (perhaps the trust department of a major bank) responsible for making sure that the borrower (the corporation, government agency, or other entity issuing the bonds) complies with the conditions of the indenture. The trustee is paid by the borrowing organization, but has a fiduciary obligation to the lending bondholders. So the trustee can take legal action, against the party paying it, for violating the indenture.

Bond Rating: a professional analyst's opinion on the borrower's ability to repay. A rating depends on a combination of quantitative and judgmental factors, among which are the firm's financial strength and the existence of any collateral assets or other assurances of repayment (such as a guarantee from a weak company's stronger parent company).

The rating agencies generally assign ratings akin to grades in school. The grades consist of letter combinations, with refinements through +/– signs and further refinements through comments on the outlook (state of Ohio bonds were *upgraded* in 2011 from AA + negative to AA + stable). Ratings range from AAA (top investment quality) to C and D (long-term bonds and short-term commercial paper, respectively, that already are in default). A higher rating is assigned to the extent that the rating agency sees a lower chance that the borrower will default on the particular bond issue, and a lower amount of loss to the lenders if a default occurs. Lenders expect to earn a higher return on a bond that carries a lower rating, of course. So a borrower hopes its bonds will get a high initial rating so that lenders will accept a lower interest rate.

Bonds with riskier ratings of BB or less are *below investment grade*, also known as *junk* or *speculative* or *opportunity* bonds. Junk bonds have played a larger role in the corporate bond market since the mid-1980s; in the aftermath of the last decade's financial crisis more than 40% of outstanding corporate bonds had "junk" status. Junk bonds traditionally came about when a once-strong company became weak and its bonds were *downgraded* ("fallen angels"), but in recent times we have seen some bonds issued that were junk from their inception (as when issued by an upstart company or one with an already-high debt ratio).

The three largest rating agencies (Moody's, Standard & Poor's, and Fitch) became controversial during the financial crisis because they had assigned AAA ratings to bonds collateralized by low-quality ("subprime") home mortgage loans that ultimately suffered repayment problems, leading to huge losses for large banks and investment firms that had bought them. An interesting irony was that the large banks bought AAA-rated mortgage-backed securities because federal regulations restricted some of their investment choices to instruments with AAA ratings from the three largest raters. New rules created under 2010's Dodd-Frank legislation removed that requirement, and as a result some new entrants gained a stronger role in the bond rating arena (notably Chicago-based Morningstar's DBRS unit, corporate security consulting firm Kroll, and longtime insurance industry data provider A.M. Best). S&P received additional notoriety when it downgraded U.S. Treasury debt from AAA to AA+ in August 2011.

Coupon Rate: the stated annual interest rate that the borrower (company or government unit that issued the bond) agrees to pay when the money is borrowed. So it represents the annual interest rate that lenders required when the bond was issued. The coupon rate times the par value gives us the annual interest payment that, for a traditional fixed-interest rate bond, does not change over the bond's life. If that same borrower were to issue new bonds today, it might have to pay a different coupon rate, either because investors perceive more or less risk in lending to the borrower than they saw in the past, or else because the general level of interest rates across the economy has changed. [Recall that some bonds do not have regular scheduled coupon interest payments; they are the "zero-coupon" bonds mentioned earlier.]

Par Value: the face value of the bond; typically the amount of principal that the borrower will pay to the lender when the bond matures. For coupon-paying U.S. corporate bonds, the tradition is for par value to be $1,000 per bond (a large firm is likely to issue a huge number of these $1,000 pieces, borrowing hundreds of millions or perhaps even billions of dollars at one time, both to meet its large need for financing and to spread the bond issuance's high legal and administrative costs over a larger base). So here is what happens in the typical case: a company borrows $1,000 (the par value) from an investor, agreeing to pay a coupon rate of (let's say) 9% per year for 20 years. Thus whoever holds that bond (the original buyer or whoever that investor might sell the bond to in a secondary market transaction) gets 9% x $1,000 = $90 in interest every year for 20 years, and then gets the $1,000 back along with the final $90 interest payment. Because the $90 annually never changes, the bond's market value will change from the $1,000 par value *if* $90 no longer represents the annual interest payment new lenders would expect for lending $1,000 to the borrower in question.

Default: the borrower's failure to make a payment of interest or principal on the date it is due.

Call Provision: an indenture provision allowing the borrower to repay the principal earlier than the maturity date. (Just as people refinance their mortgage loans when interest rates fall, a company is likely to exercise the call provision if interest rates applying to its own borrowing or across the broader market decline; they use new money borrowed at a low annual interest rate to retire existing debt carrying a higher annual interest rate.) A *callable* bond usually carries a higher coupon interest rate than an otherwise-similar non-callable bond. Typically there is a multiple-year waiting, or *call protection*, period during which the borrowing entity can not call the bond, and the lender also is likely to receive extra money in the form of a *call premium* if the bond is called. (A bond also might be conditionally callable, as when an event such as a tax law change increases the issuer's cost of paying the interest.) In our examples we will treat any call premium as being an unchanging amount, but a real call premium might decline over time once an initial call date has been reached, or it might be a "make-whole" call premium that gets larger as market interest rates fall (because the investor must then reinvest the proceeds at a low interest rate and also possibly pay capital gain taxes by receiving a call premium on top of the principal originally lent).

Put Provision: an indenture stipulation that the lender can sell the bond back to the borrower (require the repayment of the owed principal before maturity). The interest rate paid generally is lower if the bond can be put. A put provision might make a bond issue attractive if buyers seemed likely to worry about interest rate risk (i.e., that interest rates will rise) or the issuing firm's future financial strength (putting the bond forces the borrower to repay the lender before bad conditions get worse). A put also might be triggered by a merger of the issuing firm with a less financially secure acquirer.

IV. Bond Valuation

Now that we have discussed the qualitative aspects of bonds we can look at the numbers in a more meaningful context. Recall the general asset value equation, introduced in our Topic 6 discussion. Under traditional financial analytical tools, the value of a financial asset can be computed as the present value of the cash flows the asset is expected to generate for its owner over time. We can relate all of the value estimation computations we will work with in Topic 10 (bonds) and Topic 13 (preferred and common stock) to the general valuation equation:

$$V_{Asset}\ (= CF_0) = CF_1\left(\frac{1}{1+r}\right)^1 + CF_2\left(\frac{1}{1+r}\right)^2 + CF_3\left(\frac{1}{1+r}\right)^3 + CF_4\left(\frac{1}{1+r}\right)^4 + \cdots + CF_n\left(\frac{1}{1+r}\right)^n$$

This general equation, which should guide our thinking in all asset valuation applications (in class discussions we often informally call it the "big idea" for Topics 10 and 13) is just a restatement of the Net Present Value equation:

$$NPV = CF_0\left(\frac{1}{1+r}\right)^0 + CF_1\left(\frac{1}{1+r}\right)^1 + CF_2\left(\frac{1}{1+r}\right)^2 + CF_3\left(\frac{1}{1+r}\right)^3 + \cdots + CF_n\left(\frac{1}{1+r}\right)^n$$

An individual's or organization's purchase of bonds or shares of stock is an "investment" that can be analyzed in NPV terms, just like a company's purchase of costly new equipment. The asset value equation is just a slight alteration of the NPV equation we worked with in the Topic 6 capital budgeting discussion; in computing the value of an asset that is expected to generate subsequent cash flows, we solve for the period 0 cash flow – the price a buyer should willingly pay today – that would cause NPV to be \$0, such that CF_0 equals the total present values of all subsequent expected cash flows 1 – n to deliver a compounded rate of return of r% per period.

So whereas in our capital budgeting exercises we typically were given CF_0, the investment made today, and had to determine the amount of wealth that investment would create for the firm's owners, in asset valuation problems we set NPV = \$0 and solve for CF_0. We solve for asset value based on a \$0 NPV because we assume that in a competitive market, with buyers trying to pay as little and sellers trying to charge as much as possible, the agreed-on price will provide the buyer with an annual rate of return just sufficient to compensate for the attendant risks, nothing less or more. Recall that a \$0 NPV project is just barely acceptable; it provides an expected annual return just equal to the annual cost of compensating money providers.

While the general asset valuation equation shown above will always guide our thinking, we will not always have to manually sum all of the projected cash flows' individual present values. If some or all of those cash flows are expected to follow a nice pattern (*e.g.*, level annuities or perpetuities), we can compute the combined PVs of some or all of the expected cash flows with a shortcut computational technique. For example, someone who buys a coupon-paying bond expects every subsequent cash flow to consist of just the coupon interest payment, except for the final payment date, when the amount received will be a last coupon payment plus the return of the principal originally lent. So the general valuation equation (what we are thinking) as applied to coupon-paying bonds (how we physically compute what we are thinking about) can be restated more specifically to reflect what happens with bonds as

$$V_B = \begin{matrix}\text{Coupon}\\\text{Payment}\end{matrix}\left(\frac{1}{1+r}\right)^1 + \begin{matrix}\text{Coupon}\\\text{Payment}\end{matrix}\left(\frac{1}{1+r}\right)^2 + \cdots + \begin{matrix}\text{Coupon}\\\text{Payment}\end{matrix}\left(\frac{1}{1+r}\right)^{n-1} + \left[\begin{matrix}\text{Coupon}\\\text{Payment}\end{matrix} + \begin{matrix}\text{Ending}\\\text{Amount}\end{matrix}\right]\left(\frac{1}{1+r}\right)^n$$

This dollar amount probably is best thought of as the bond's *theoretical* value, what it would sell for in a market in which parties that want to buy a particular type of bond are reasonably able to find willing sellers. Traders (large-scale buyers and sellers) report that the bond market can become *illiquid*, with some bonds that have features particularly sought out by buyers becoming difficult to find, and that under those conditions transactions can close at prices above what observers might objectively view as the theoretically correct values. In this introductory coverage, when we discuss a bond's value we mean the present value of the expected cash flow stream that constitutes the traditionally computed theoretical value. (A price above the recently perceived theoretical value perhaps could be thought of as a newly computed theoretical value that reflects a lower discount rate, which might incorporate a negative liquidity premium since a buyer will be especially confident in the ability to resell.)

A. Bond Valuation Based on Annual Coupon Interest Payments

What should you be willing to pay today for the right to collect \$100 at the end of year 1, \$100 at the end of year 2, and \$100 + \$1,000 = \$1,100 at the end of year 3 if the appropriate discount rate (your required annual rate of return) is 0%, 8%, 10%, or 12%? Based on the equation for computing a bond's theoretical value, as shown above, the respective answers are

$$V_B = \$100\left(\frac{1}{1.00}\right)^1 + \$100\left(\frac{1}{1.00}\right)^2 + \$1{,}100\left(\frac{1}{1.00}\right)^3 = \$100 + \$100 + \$1{,}100 = \$\underline{1{,}300},$$

$$V_B = \$100\left(\frac{1}{1.08}\right)^1 + \$100\left(\frac{1}{1.08}\right)^2 + \$1{,}100\left(\frac{1}{1.08}\right)^3 = \$92.59 + \$85.73 + \$873.22 = \$\underline{1{,}051.54},$$

$$V_B = \$100\left(\frac{1}{1.10}\right)^1 + \$100\left(\frac{1}{1.10}\right)^2 + \$1{,}100\left(\frac{1}{1.10}\right)^3 = \$90.91 + \$82.64 + \$826.45 = \$\underline{1{,}000}, \text{ and}$$

$$V_B = \$100\left(\frac{1}{1.12}\right)^1 + \$100\left(\frac{1}{1.12}\right)^2 + \$1{,}100\left(\frac{1}{1.12}\right)^3 = \$89.29 + \$79.72 + \$782.96 = \$\underline{951.97}$$

(Each of these simple computations is an example of a bond with a 10% annual coupon rate, \$1,000 par value, and three years remaining until maturity.) Note that if we discount the expected payments at an annual rate lower than 10%, the resulting value is greater than \$1,000. (At a 0% discount rate the bond's value is simply the unadjusted \$100 + \$100 + \$1,100 = \$1,300 that the lender will receive in total; a 0% discount rate indicates that the lender is just as happy to receive \$100 in three years as to receive it in one year.) If we discount at a rate higher than 10%, the value is less than \$1,000. And if we discount at 10%, V_B = \$1,000.

Why? Let's say the bonds originally were issued two years ago with five-year lives. At that time, the lenders (the original buyers of the bonds) were happy to receive 10% interest (\$100 per year on each $1,000 invested) based on the risks they perceived that they were taking. If, under today's market and economic conditions, people would willingly lend to this company for three years at a 10% annual interest rate, then the price paid for this bond in secondary market transactions should continue to be the $1,000 face, or par, value. (See the 10% computation above.) Another way to think about things is that a bond paying 10% interest ($100) in a 10% environment just meets the market standard, and thus is worth its par value.

But what if the issuing company has weakened, or interest rates across the economy have risen, such that people would commit to lending the company money for three years today only if they could expect a 12% annual return? Then if they spent $1,000 on a newly issued bond they would expect to receive $120 per year in interest. They will buy a bond that pays only $100 per year in interest – but only if they pay a price of $951.97 (see the 12% computation above), which forces the subsequent receipt of $100, $100, and $1,100 to represent a 12% compounded annual rate of return. Another way to think about things is that a bond paying 10% interest ($100 per year for the ongoing use of $1,000) in a 12% environment (where $120 should be paid each year) is unattractive, and therefore will be expected to sell at a discounted price so what the buyer gets back represents a 12%, not 10%, annual return on what she gave up.

What if the issuing company has become stronger, or interest rates across the economy have declined, such that people would be happy to lend to the company for three years today for an expected 8% annual return? Then if they spent $1,000 on a newly issued bond they would expect to receive only $80 per year in interest. Someone will sell them a bond that pays a higher $100 per year in interest – but will charge a price of $1,051.54 (see 8% computation above), for which the subsequent receipt of $100, $100, and $1,100 represents an 8% compounded annual rate of return. Another way to think about things is that a bond paying 10% interest ($100 per year for the ongoing use of $1,000) in an 8% environment (where only $80 should be paid each year) is attractive, and therefore will be expected to sell at a premium price so that what the buyer gets back represents an 8% annual return on what she gave up.

When dealing with a bond that has a long period of expected interest payments, it is more efficient to work with an application of the bond version of the general valuation equation that groups the unchanging coupon interest payments as an annuity and then treats the return of the par value to be received at the maturity date as a single dollar amount. In other words, the value of a long-lived bond has two components: the *present value of the annuity* represented by any regular interest payments, plus the *present value of the single dollar amount* represented by the return at maturity of the $1,000 par value (the amount originally lent). For the 8% discount rate case above, for example, we could have computed

$$V_B = \$100\left(\frac{1-\left(\frac{1}{1.08}\right)^3}{.08}\right) + \$1{,}000\left(\frac{1}{1.08}\right)^3 = \$257.71 + \$793.83 = \$\underline{1{,}051.54}$$

Consider an example with a long remaining period until maturity, such that discounting individual cash flows would be terribly tedious. This bond has a 10% annual coupon interest rate and 20 years remaining to maturity. It might have been issued five years ago with a 25-year life, ten years ago with a 30-year life, twenty years ago with a 40-year life, or any other such possibility. (All that matters to us is what will be received in the future, *i.e.*, how many years of payments remain; how many years already have passed since the issue date is immaterial). We can compute its value if the required annual rate of return is 8% as

$$V_B = \$100 \left(\frac{1-\left(\frac{1}{1.08}\right)^{20}}{.08}\right) + \$1{,}000 \left(\frac{1}{1.08}\right)^{20} = \$981.81 + \$214.55 = \$\underline{1{,}196.36}$$

The value if the required annual rate of return is 12% is

$$V_B = \$100 \left(\frac{1-\left(\frac{1}{1.12}\right)^{20}}{.12}\right) + \$1{,}000 \left(\frac{1}{1.12}\right)^{20} = \$746.94 + \$103.67 = \$\underline{850.61}$$

It should not be surprising that the value, if the required annual rate of return is 10%, is

$$V_B = \$100 \left(\frac{1-\left(\frac{1}{1.10}\right)^{20}}{.10}\right) + \$1{,}000 \left(\frac{1}{1.10}\right)^{20} = \$851.36 + \$148.64 = \$\underline{1{,}000}$$

So when a 10% coupon bond is issued, the original buyer pays $1,000 (traditional par or face value of American companies' corporate bonds) and is promised that whoever holds the bond will receive $100 per year in interest, plus a return of the $1,000 principal at maturity. That $100 per year plus $1,000 at maturity is a contractual obligation; changing market conditions will not change the amount of interest paid each year or the principal returned at maturity. The borrowing entity has locked into a known cost of using the $1,000 each year for the long term.

But the *value* V_B *of the right to collect* those amounts changes with conditions affecting the borrowing entity or the broader lending markets. So the original buyer lends the company $1,000, and the last holder (if the original buyer later sells it) gets that $1,000 back. But the price paid for the bond by intermediate buyers will likely be something other than $1,000, to reflect current conditions. After all, if the cash flows stay constant while the world around us changes, something has to give – and that "something" is the price at which we would expect the bond to be sold to a subsequent holder in a secondary market transaction.

B. Bond Valuation Based on Semiannual Payments

In the U.S., corporations traditionally have paid interest on bonds semiannually, instead of annually. (Firms located outside the U.S. often pay annual interest, as in our examples above.) With semiannual interest, the annual coupon interest payment is split in half and paid in two equal installments, one every six months. But then in computing the value of the stream of interest payments, we are dealing with a number of time periods equal to *twice* the number of years, and we must discount at a rate that is the semiannual periodic required rate of return. That semiannual r is equal to half of the required annual rate, *if* the annual rate we talk about is stated in annual percentage rate (APR) terms.

For the 20-year bond with the 10% annual coupon interest rate illustrated above, there would be semiannual coupon payments of $100/2 = $50 each, and they would be received 40 times (twice each year for 20 years). If lenders require an 8% annual stated rate of return (non-compounded APR) we should discount at an 8% ÷ 2 = 4% semiannual rate, for an indicated value of

$$V_B = \$50\left(\frac{1-\left(\frac{1}{1.04}\right)^{40}}{.04}\right) + \$1{,}000\left(\frac{1}{1.04}\right)^{40} = \$989.64 + \$208.29 = \$\underline{1{,}197.93}$$

If lenders require a 12% annual rate of return in APR terms we should discount at a 12% ÷ 2 = 6% semiannual rate, for an indicated value of

$$V_B = \$50\left(\frac{1-\left(\frac{1}{1.06}\right)^{40}}{.06}\right) + \$1{,}000\left(\frac{1}{1.06}\right)^{40} = \$752.31 + \$97.22 = \$\underline{849.53}$$

Of course, at a 10% annual required return in APR terms, meaning a 5% semiannual discount rate (it carries a 5% semiannual cash flow in an environment that requires a 5% semiannual return), the theoretical value, *i.e.*, the price we would expect to see in a competitive market, is

$$V_B = \$50\left(\frac{1-\left(\frac{1}{1.05}\right)^{40}}{.05}\right) + \$1{,}000\left(\frac{1}{1.05}\right)^{40} = \$857.95 + \$142.05 = \$\underline{1000}$$

A few points to note:

- When the interest rate that would be applied to a new loan with similar features rises (falls), the prices that would be paid in a competitive market for previously-issued bonds fall (rise). So from one viewpoint, a bond investor likes to buy bonds and then watch the general level of interest rates fall (so any bonds she holds will rise in resale value).

  But things are not quite that simple; after all, if you invest actively in coupon bonds you always have interest payments coming in, and if market interest rates decline then *those* inflows have to be reinvested at the new, lower rates for the same level of risk. So actually there is a tradeoff between bond value and reinvestment gains/losses. This situation can be examined through a concept known as *duration*, which is covered in detail in investments courses. An additional factor called *convexity* addresses the idea that the relationship between a bond's value and the required periodic rate of return is not linear. Consider a $1,000 par value bond with annual interest payments and a 20-year original maturity. On the day it is issued it sells for its par value, because the 8% annual coupon interest rate reflects the 8% annual rate of return lenders would expect to earn at that time. But then two years later market conditions have caused the expected annual return for an 18-year loan with similar risk and other features to change by 1% (referred to as 100 "basis points" in bond market terminology). Note that the bond's expected price, or theoretical value, drops to

$$V_B = \$80\left(\frac{1-\left(\frac{1}{1.09}\right)^{18}}{.09}\right) + \$1{,}000\left(\frac{1}{1.09}\right)^{18} = \$700.45 + \$211.99 = \$\underline{912.44}$$

(a change of – $87.56 ÷ $1,000 = – 8.756%) if the required annual return rises to 9% but increases to a proportionally higher

$$V_B = \$80\left(\frac{1-\left(\frac{1}{1.07}\right)^{18}}{.07}\right) + \$1{,}000\left(\frac{1}{1.07}\right)^{18} = \$804.73 + \$295.86 = \$\underline{1{,}100.59}$$

(a $100.59 ÷ $1,000 = 10.059% change) if the required annual return declines by the same 100 basis points to 7%. This convexity relationship, with value growing by more after a decline in required periodic returns than it falls after an equivalent increase, generally holds true for the type of coupon bonds we deal with in this introductory course.

- The rates-go-up-bond-prices-fall (or the opposite) effect is greater for longer-term bonds than for shorter-term. As the maturity date approaches, the bond will have a value pretty close to $1,000 even if market rates are substantially different from the bond's coupon rate.

- For a bond always to be worth $1,000, the *yield curve* must shift such that, for example, the appropriate 20-year return is 10% when the bond is issued and the appropriate 5-year rate is 10% fifteen years later. We typically ignore this issue in an introductory coverage.

- In this introductory course we always assume that we are analyzing a bond exactly six months or a year before the next coupon interest payment date (*i.e.*, immediately after an interest payment has been made). In practice the issuer's obligation to pay interest builds up every day, such that if a bond is sold 17 days after a coupon payment was made the buyer should pay the seller approximately a price consistent with the computations shown above, plus something extra for the 17 days of "accrued" interest that rightly belongs to the seller but will be included in the first coupon payment received by the buyer. (Those who find this topic interesting can see a more complete discussion in section F below.)

- The bond market tends to be dominated by large institutions; individuals generally do not directly purchase long-term bonds. For example, life insurance companies are major buyers of long-term bonds issued by U.S. corporations. Individuals generally gain exposure to the bond market by purchasing shares in mutual funds that directly buy long-term bonds.

- The computations shown in this discussion are consistent with the time value of money principles we first saw in Topic 4. But we sometimes oversimplify; the bond market has many idiosyncrasies and traditions. For example, "real world" corporate bond analysis sometimes is based on an assumed 360-day year, and further complications can arise when a bond matures on a weekend or holiday such that repayment must be delayed for a day or two. Dealing with those complications is beyond the scope of this introductory coverage.

C. The Yield to Maturity (and Related IRR Measures)

In the examples above we knew the lender's required periodic return and used that information in computing a bond's theoretical value. But what if instead we know the price investors have been paying for a bond, which is the present value of its remaining stream of cash flows, and we want to figure out the rate of return implicit in that price?

That rate is known as the bond's *yield to maturity* (YTM). It is actually the same concept as the IRR we saw in capital budgeting analysis; bond valuation is just an NPV application in which we solve for CF_0, and yield to maturity is just bond market terminology for internal rate of return. In fact, for a bond with multiple periods of expected cash flows we must solve for the YTM through an iterative trial and error process, just as we did with IRR examples in the capital budgeting discussion. (One component of the yield to maturity is the *current yield*, which is the annual coupon interest payment divided by the bond's current market price.)

Consider two earlier examples, one with annual coupon payments and one with semiannual. A bond with a 20-year remaining life and 10% annual coupon rate, with interest paid annually, currently sells for $1,196.36. What is its yield to maturity? We try different discount rates in the equation

$$\$1{,}196.36 = \$100\left(\frac{1-\left(\frac{1}{1+\mathrm{r}}\right)^{20}}{\mathrm{r}}\right) + \$1{,}000\left(\frac{1}{1+\mathrm{r}}\right)^{20}$$

When we use 8% we find

$$\$1{,}196.36 = \$100\left(\frac{1-\left(\frac{1}{1.08}\right)^{20}}{.08}\right) + \$1{,}000\left(\frac{1}{1.08}\right)^{20} = \$981.81 + \$214.55 = \$\underline{1{,}196.36}\ \checkmark$$

So 8% is the yield to maturity. It should make intuitive sense that if we find a bond's $1,196.36 theoretical value by discounting the expected cash flows at an 8% annual rate, then someone who pays $1,196.36 and receives those cash flows gets an 8% annual rate of return. Now consider a bond with a 20-year remaining life and a 10% annual coupon rate, but with *semi*annual interest payments, currently selling for $849.53. What is its yield to maturity? We try different semiannual discount rates in the equation

$$\$849.53 = \$50\left(\frac{1-\left(\frac{1}{1+r}\right)^{40}}{r}\right) + \$1{,}000\left(\frac{1}{1+r}\right)^{40}$$

When we use a 6% semiannual discount rate we find

$$\$849.53 = \$50\left(\frac{1-\left(\frac{1}{1.06}\right)^{40}}{.06}\right) + \$1{,}000\left(\frac{1}{1.06}\right)^{40} = \$752.31 + \$97.22 = \$\underline{849.53}\ \checkmark$$

The semiannual discount rate that solves the equation is 6%. But recall that when we talk about rates of return, including yields to maturity, we talk in annual terms. So 6% is not the yield to maturity. But neither is 12%. The word *yield* usually signifies that the annual rate figure we talk about incorporates the impact of compounding that occurs within the year (an effective annual rate, or EAR, measure). The lender's true effective annual rate of return, or yield to maturity, from earning 6% semiannually in this case thus is $(1.06)^2 - 1 = 12.36\%$; .06 x 2 = 12% is an annual return measured in simpler APR terms. (If a bank pays 4% stated annual interest on savings but compounds semiannually, the saver's true EAR rate of return is $(1.02)^2 - 1 = 4.04\%$.)

In our discussions we will consistently treat yield to maturity, and the structurally similar yield to call and holding period yield, to be discussed later, as EAR applications. (Some textbooks, and even professional analysts, would call 12% here the "yield to maturity," recognizing that such a value is not the true effective rate of return; such sources might call 12.36% the "*effective* yield to maturity.")

A few points to note:

- We compute bond values and yields to maturity with the exact same equation. In computing value (similar to NPV analysis in our capital budgeting unit) we know the expected cash flows and the discount rate, and must find the value; whereas in computing

YTM (similar to IRR analysis in our capital budgeting unit) we know the expected cash flows and the bond's price or value, and have to work backwards to find the discount rate.

- If we are given the yield to maturity on a bond with *annual* coupon payments, then that is the rate we use in discounting the bond's cash flows to find its value. But if given the yield to maturity on a bond with *semiannual* payments, we have to undo the compounding to find the semiannual discount rate. (Recall that we are treating the yield to maturity as a compounded EAR measure.) For example, if told that the YTM is 12.36%, we find the semiannual discount rate as $\sqrt[2]{1.1236}$ – 1 = .06, or 6% (consistent with our example above). If the yield to maturity is 7.6%, then the semiannual discounting rate would be $\sqrt[2]{1.076}$ – 1 = .037304, or 3.7304% [because $(1.037304)^2 - 1 = 7.6\%$].

- The yield to maturity is the internal rate of return that a bond buyer creates for herself through the price she pays for the bond. A price greater than $1,000 gives the buyer a periodic return less than what would be suggested by the bond's periodic coupon payment (perhaps the company has become financially stronger since the bond was issued, or interest rates in general are lower now), while a price less than $1,000 gives a periodic return greater than what the periodic coupon payment would indicate (perhaps the firm has weakened since the issue date, or interest rates across the economy have gone higher).

  You might recall that we used our observation of the yield to maturity for "getting inside the heads" of potential lenders when computing a weighted average cost of capital in Topic 5. After all, what better way is there to gauge the lending public's view of a company than to see what rate of return new lenders require when taking over current lenders' positions by buying their existing bonds in secondary market transactions?

- Two concepts related to yield to maturity are *yield to first call* and *holding period yield*. We work with this further refined version of the general valuation equation as applied to bonds:

$$V_B = \begin{matrix}\text{Coupon}\\\text{Payment}\end{matrix}\left(\frac{1-\left(\frac{1}{1+\mathrm{r}}\right)^{\mathrm{n}}}{\mathrm{r}}\right) + \begin{matrix}\text{Ending}\\\text{Amount}\end{matrix}\left(\frac{1}{1+\mathrm{r}}\right)^{\mathrm{n}}$$

  Here V_B is the value of a bond that pays interest annually or semiannually, Coupon Payment is the total amount of interest received each annual or semiannual period, r is the annual or semiannual percentage rate of return currently required by lenders, n is the number of annual (years) or semiannual (half-years) periods that the bond is expected to be (or was) held, and Ending Amount is what the bond holder will (or expects to, or already did) receive when the investment in the bond terminates ($1,000 par in Yield to Maturity case). Any of these variables can be an unknown (typically it is either V_B or r). As noted earlier, if V_B is the unknown we have a bond valuation problem (and find an NPV solution).

But if r is the unknown, then we have a *yield* problem (and are solving for IRR). Which type of yield? If n is the number of periods until the bond matures and Ending Amount is the bond's $1,000 par value, then r is the *Yield to Maturity* (or the semiannual value we compound to get YTM, as discussed earlier). But if n is the number of periods until the bond can be called and Ending Amount is the $1,000 par plus a *call premium* (which could be zero, but typically is a positive amount), then r is the *Yield to (First) Call* (or the semiannual value we compound to get YTC). And if n is the number of periods that a particular investor expects to (or did) hold the bond and Ending Amount is the market value the bond is expected to (or did) sell for at the end of the specified holding period, then r is the *Holding Period Yield* (or the semiannual value we compound to get HPY).

Recall, from above, the bond just purchased for $849.53. It has a 10% annual coupon interest rate but with semiannual interest payments, and 20 years (40 half-years) remain until it matures at the $1,000 par value. The equation for computing the 12.36% yield to maturity, as was shown above, is:

$$\$849.53 = \$50\left(\frac{1-\left(\frac{1}{1+r}\right)^{40}}{r}\right) + \$1{,}000\left(\frac{1}{1+r}\right)^{40}$$

But let's say also that this bond can be called after just five years (10 half-years), and that if it is called (a likely event if lenders' required rates of return drop considerably from their current level so that the company could refinance the bonds at a lower interest rate) then the bondholder will receive a price of $1,100 (par value, plus another full year of interest as a call premium). So someone buying the bond for $849.53 today does not expect to get $50 every six months for twenty years followed by a repayment of the par value, but rather to get $50 every six months for five years and then $1,100. That situation has to involve some positive rate of return (pay $849.53 and then get back $50 x 10 + $1,100 = $1,600 total over time); this rate of return is known as yield to call (sometimes identified more specifically as yield to *first* call because it is a measure of the bondholder's return if the bond is called as soon as the call protection period ends). We set the equation up as

$$\$849.53 = \$50\left(\frac{1-\left(\frac{1}{1+r}\right)^{10}}{r}\right) + \$1{,}100\left(\frac{1}{1+r}\right)^{10}$$

and solve for *r* with trial and error; it turns out to be 7.927816%. But this is a semiannual periodic rate, so to find the yield to first call (an effective annual rate application) we must compute $(1.07927816)^2 - 1 = 16.4841\%$.

Now move forward in time. Let's assume that the bond was not called five years after the current holder bought it (perhaps interest rate levels actually rose further in subsequent years). After buying the bond for $849.53, the holder in question ended up holding it for thirteen years (26 half-years) and then selling it for a market price of $823.12. That situation has to involve some positive rate of return (pay $849.53 and then get back $50 x 26 + $823.12 = $2,123.12 total over time); this rate of return is known as the holding period yield. We set the equation up as

$$\$849.53 = \$50\left(\frac{1-\left(\frac{1}{1+r}\right)^{26}}{r}\right) + \$823.12\left(\frac{1}{1+r}\right)^{26}$$

and solve for r with trial and error; it turns out to be 5.831735%. But this is a semiannual periodic rate, so to find the holding period yield (an effective annual rate application) we must compute $(1.05831735)^2 - 1 = 12.0036\%$.

D. The Realized Compound Yield (an MIRR Measure)

Notice that since YTM (or YTC or HPY) is an internal rate of return application, in computing it we do not take into account the effect of the expected reinvesting of cash flows (such as interest payments) between the date they are received and the end of the bond's life or other holding period. But bond analysts have a modified internal rate of return (MIRR) measure that includes the effect of expected reinvestment, just as the MIRR in capital budgeting explicitly includes the effects of expected reinvestment. Let's say you just bought the 20-year, 10% coupon bond described above for $849.53, and plan to hold it until it matures at its $1,000 par value. The annual return represented by the cash flows themselves is, as shown, the YTM or IRR of 12.36%.

But when you look back after 20 years, will your overall compounded return have been exactly 12.36%? Not if you reinvest each interest payment when you receive it and earn a semiannual average compounded return that differs from $\sqrt[2]{1.1236} - 1 = 6\%$. Let's say that right after you buy the bond for $849.53, the level of interest rates across the market changes, such that for the risks of lending to this company over a long period you should expect a semiannual return of 7% instead of 6% (or the 5% that was appropriate when the bond was first issued). Assume further that the appropriate reinvestment rate can be expected to remain at 7% on all cash flows for the remainder of this bond's 20-year remaining life (so you expect a fairly flat yield curve in the future). Then, by the end of year 20, your forty end-of-half-year $50 interest payments ($2,000 total) should have grown with the 7% semiannual rate of return (along with also getting back the $1,000 in principal at the end of half-year 40), to

$$\$50\,(1.07)^{39} + \$50\,(1.07)^{38} + \cdots + \$50\,(1.07)^{2} + \$50\,(1.07)^{1} + \$1{,}050\,(1.07)^{0}$$

$$= \$50\,(1.07)^{39} + \$50\,(1.07)^{38} + \cdots + \$50\,(1.07)^{2} + \$50\,(1.07)^{1} + \$50\,(1.07)^{0} + \$1{,}000\,(1.07)^{0}$$

$$= \$50\,[(1.07)^{39} + (1.07)^{38} + \cdots + (1.07)^{2} + (1.07)^{1} + (1.07)^{0}] + \$1{,}000\,(1.07)^{0}$$

$$= \$50\left(\frac{(1.07)^{40} - 1}{.07}\right) + \$1{,}000\,(1.07)^{0} = \$9{,}981.76 + \$1{,}000 = \$10{,}981.76$$

in your hands 20 years, or 40 half-years, from now. (Recall that the future value of a level ordinary annuity factor is consistent with *n* payments, here 40, but only *n* – 1 applications of interest, here 39 as seen in the year-by-year computations.) So you pay \$849.53 initially and then expect to have accumulated \$10,981.76 forty half-years from the purchase date. Those dollar values represent a return of

$$\$849.53\,(1 + r)^{40} = \$10{,}981.76$$
$$(1 + r)^{40} = 12.926860$$
$$\sqrt[40]{(1+r)^{40}} = \sqrt[40]{12.926860}$$
$$1 + r = 12.926860^{1/40} = 12.926860^{.025} = 1.066074$$
$$r = .066074 \text{ or } \underline{6.6074\%}$$

This rate is the overall compounded semiannual return with a specific reinvestment assumption directly factored in. Annualized it is the $(1.066074)^2 - 1 = \underline{\underline{13.6514\%}}$ *realized compound yield* (RCY), or MIRR. Note that this value is somewhere between the 12.36% YTM or IRR and the $(1.07)^2 - 1 = 14.49\%$ annualized reinvestment rate, just as an MIRR measure always is.

E. Zero-Coupon Bonds

With coupon bonds that make regular periodic interest payments, the original lender (bond buyer) lends \$1,000. She, or whomever she might later sell the bond to, receives regular interest each period (typically semiannually), and then receives the \$1,000 back, along with the final interest payment, at maturity. But another way to deliver returns to a lender would be for the borrower to borrow a small amount of money, then to pay no regular interest over the bond's life, but to pay a large amount back at maturity (to make up for having paid no regular returns). If no coupon payments are made, then the bond is of the *zero-coupon* variety.

A zero-coupon bond is a simple instrument to analyze: computationally it is just the present value of a single dollar amount (there is no interest payment annuity). Its theoretical value, the price we would expect it to sell for in a competitive market, is just the PV of the face value to be received at maturity. (A zero-coupon bond's face or maturity value might be something other than \$1,000, but for consistency in our discussions we will treat "zeros" as having the same \$1,000 face values that usually characterize coupon bonds issued by U.S. firms.)

For example, XCorp wants to raise a large sum of money to build a new manufacturing facility. It issues zero-coupon bonds, promising to pay \$1,000 to each bond holder in 15 years. At what price should each of these “zeros” initially sell if lenders’ required annual rate of return on the day the bonds are issued is 14%? We still use our bond valuation equation:

$$V_B = \begin{matrix}\text{Coupon}\\\text{Payment}\end{matrix}\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right) + \$1{,}000\left(\frac{1}{1+r}\right)^n$$

But with a coupon payment of \$0 the first term ultimately drops out and we have

$$V_B = \$0\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right) + \$1{,}000\left(\frac{1}{1+r}\right)^n = \$1{,}000\left(\frac{1}{1+r}\right)^n = \$1{,}000\left(\frac{1}{1.14}\right)^{15} = \$140.10$$

Because there are no coupon payments and we are analyzing the bond on a day when there are exactly n years (not, *e.g.*, n years and 37 days) until maturity, as we conveniently do in an introductory coverage, it does not matter whether we discount on an annual or semiannual basis – as long as we use the true annual yield to maturity or its semiannual counterpart (for 14% annually it would be $\sqrt[2]{1.14} - 1 = .0677$, or 6.77% semiannually):

$$V_B = \$1{,}000\left(\frac{1}{1+r}\right)^n = \$1{,}000\left(\frac{1}{1.0677}\right)^{30} = \$140.10$$

If buyers still sought a 14% effective annual rate of return five years later, with only 10 years remaining until maturity, each of the bonds would be expected to sell for

$$V_B = \$1{,}000\left(\frac{1}{1.14}\right)^{10} = \$1{,}000\left(\frac{1}{1.0677}\right)^{20} = \$269.74$$

(whereas if five years later buyers would seek only a 12% effective annual or $\sqrt[2]{1.12} - 1 =$ 5.83% semiannual return for becoming 10-year lenders to XCorp, each of the bonds would be expected to sell for

$$V_B = \$1{,}000\left(\frac{1}{1.12}\right)^{10} = \$1{,}000\left(\frac{1}{1.0583}\right)^{20} = \$321.97;$$

a reduced time remaining to maturity would drive the value up to \$269.74, while discounting the lone expected \$1,000 payment at the lower 12% rate would raise it farther to \$321.97). We can solve for the yield to maturity on a zero-coupon bond without resorting to trial and error; it is a simple rate of return equation. In our example immediately above, someone buys the bond for $321.97 and expects to receive $1,000 ten years later. The compounded annual rate of return (yield to maturity) is found as

$$\$321.97 = \$1{,}000\left(\frac{1}{1+r}\right)^{10}$$
$$\$321.97\,(1 + r)^{10} = \$1{,}000$$
$$(1 + r)^{10} = \$1{,}000 \div \$321.97 = 3.10587943$$
$$\sqrt[10]{(1+r)^{10}} = \sqrt[10]{3.10587943}$$
$$1 + r = 3.10587943^{.1} = 1.12 \quad \text{so } r = .12 \text{ or } 12\%$$

“Zeros” can work well for a borrower doing an investment project that requires a long time to be up and running. (The borrower may want to be able to delay paying cash out until the project has cash coming in.) They can work well for a lender that does not want to have to bother with reinvesting periodic interest payments. After all, if interest is not received, it can not be reinvested. So the overall return – the realized compound yield in bond speak – is equal to the yield to maturity. (Just as in our capital budgeting coverage a project’s MIRR was equal to its IRR if only one cash inflow was anticipated after the investment initially was made.) But a drawback for lenders is that even though interest is not received year by year, it is taxed each year as income by the U.S. government. (What actually is taxed is the bond’s yearly increase in value as the maturity date approaches, and there is less time to have to wait to collect the $1,000 or other face value.) For this reason, some financial advisors suggest that zero-coupon bonds are best held by untaxed investors, or in untaxed vehicles (like retirement accounts).

F. Refunding a Bond Issue

Let’s say that XCorp has $50 million of bonds outstanding with an 8% annual coupon interest rate (semiannual payments) and a remaining life of 15 years. Because the company is now perceived by investors as stronger than it was five years ago when the bonds were issued (with an original 20-year life), XCorp could today issue $50 million of new 15-year bonds and pay an annual coupon rate of only 6%. Should the company “refund,” and replace the existing 8% annual coupon rate bonds with new 15-year, 6% annual coupon rate bonds?

First, replacing $50,000,000 of 8% (4% semiannually) debt with $50,000,000 of 6% (3% semiannually) debt would reduce the company’s interest payments every six months from .04 x $50,000,000 = $2,000,000 to .03 x $50,000,000 = $1,500,000, for a $500,000 savings every six months. If the company is in a 36% marginal income tax bracket, this savings is reduced to ($500,000)(1 – .36) = $320,000 every six months (paying interest generates income tax benefits for a company, so paying less interest reduces the income tax benefits from borrowing). What is the value of a $320,000 savings every six months for fifteen years (30 times) [if we use the company’s current cost of borrowing as the discount rate]?

PMT x FAC = TOT

$$\$320{,}000\left(\frac{1-\left(\frac{1}{1.03}\right)^{30}}{.03}\right) = \$6{,}272{,}141.23$$

So the company should refund the issue if its administrative and related costs (such as a call premium paid to investors whose existing bonds are retired) are no higher than $6,272,141.23. This example is somewhat oversimplified, but it should serve to illustrate the main points.

Material Primarily for FIL 404 Students

G. Bond's Value Between Interest Payment Dates

In every previous example we computed a bond's value with an exact whole number of years remaining until maturity, as though the bond would be bought or sold immediately after an annual (or even-numbered in semiannual cases) interest payment had been made. But a bond might be sold at some other date, and then computing its value – the PV of everything we expect to receive – becomes a little more complex. (Below we compute values based on the time value of money principles developed throughout the course, but real world bond analysis would proceed slightly differently because of bond market idiosyncrasies noted earlier. Here we use a 365-day year, as is used in U.S. government bond analysis but not with corporate bonds, which traditionally are analyzed based on a 360-day year. So what is shown below is more a thought model than a precise "how-to" presentation.)

Consider a bond with a 10% annual coupon interest rate (semiannual payments of 10%/2 x $1,000 = $50 each), and a 6.5% required semiannual return [$(1.065)^2 - 1 = 13.4225\%$ effective yield to maturity]. It will mature in four years and eleven months; one month has passed since the last interest payment was received, and whoever holds the bond (the current holder or a new buyer) must wait five months until the next semiannual interest payment will be received. First, think in terms of half-years, since the bond's payments come in semiannually. Right after the *next* interest payment is made, there will be 4.5 years, or 9 half-years, until maturity. At that time (five months from now), the bond should be worth

$$\$50\left(\frac{1-\left(\frac{1}{1.065}\right)^{9}}{.065}\right) + \$1{,}000\left(\frac{1}{1.065}\right)^{9} = \$900.16$$

Now think in terms of months, since there are five months until the $900.16 value will be relevant. (We would actually want to use days instead, since the number of days in a month can vary, and to be accurate should account for every day's appropriate rate of return. But months gives less cluttered numbers, so we will approximate by illustrating the idea with months since we have a whole number of months left.) The annualized compounded return required is, as shown, $(1.065)^2 - 1 = 13.4225\%$. What *monthly* periodic return compounds to a 13.4225% annual effective yield to maturity? It is just $\sqrt[12]{1.134225} - 1 = 1.134225^{.0833333} - 1 = 1.05511\%$ [such that $(1 + r)^{12} - 1 = (1.0105511)^{12} - 1 = 13.4225\%$].

If you buy this bond today, you can expect to get a \$50 interest payment in five months and also realize the benefit of having something worth \$900.16 on that day. What is it worth now to expect to have \$50 + \$900.16 = \$950.16 in five periods if the periodic required rate of return is 1.05511%? The answer is

$$\$950.16\left(\frac{1}{1.0105511}\right)^5 = \$901.58$$

So, based on this slightly simplified analysis, you should pay \$901.58 today, and then in five months you will get \$50 in interest and could (if you chose to) sell the bond for \$900.16. More correctly, you would count *days* until the next interest payment (you certainly would not try to use a month-based approximation if the analysis were done in the middle of some month), and then discount the \$950.16 for the appropriate number of days at the *daily* periodic rate of $\sqrt[365]{1.134225} - 1 = 1.134225^{.0027397} - 1 = .03451\%$ [such that $(1 + r)^{365} - 1 = (1.0003451)^{365} - 1 = 13.4225\%$]. If the five months remaining until the next interest payment contained 153 days (April through August), then the value today (based on our time value approach, which is "big picture" correct but only approximates actual bond market practice) should be

$$\$950.16\left(\frac{1}{1.0003451}\right)^{153} = \$901.30$$

We have to know the annual compounded rate of return to be able to find the required periodic rate for any period less than a year: semiannual, monthly, weekly, daily, *etc*.

Now we can see why it so important to correctly compute the yield to maturity on a semiannual bond as the EAR of $(1 + \text{semiannual rate})^2 - 1$, and not as the too-simple APR (2 x semiannual periodic rate) that textbooks often use. Incidentally, in the bond market's colorful terminology "dirty price" is a bond value measure that includes the interest that has built up since the last coupon date, while "clean price" is a value measure that does not include the built-up interest.

H. Duration and Immunization

Earlier we discussed how the realized compound yield (RCY) on a bond investment is affected by the rate at which coupon payments are reinvested. For a bond that will not be held until it matures, there actually is a tradeoff between the better (worse) reinvestment situation and the lower (higher) bond value that accompanies a higher (lower) rate. This tradeoff is addressed with a weighted measure of time called *duration*. (This time measure is specifically called Macauley duration after the 1930s economist who developed it; another popular measure called modified duration is a percentage-based adjustment to Macauley duration.) Consider a \$1,000 par value bond with a 15% annual coupon interest rate, semiannual interest payments, and 5.5 years remaining until maturity, priced today at $1,085.01 to provide a 13.2% effective yield to maturity. We compute the Macauley duration of approximately four years as follows:

Period	Present Value of Cash Flow		PV as Proportion of Bond Value	Period x Proportion	
1	$ 75 $\left(\frac{1}{1.064}\right)^{1}$ =	$ 70.49	$ 70.49/$1,085.01 = .064966	1 x .064966 =	.064966
2	$ 75 $\left(\frac{1}{1.064}\right)^{2}$ =	$ 66.25	$ 66.25/$1,085.01 = .061058	2 x .061058 =	.122117
3	$ 75 $\left(\frac{1}{1.064}\right)^{3}$ =	$ 62.26	$ 62.26/$1,085.01 = .057386	3 x .057386 =	.172157
4	$ 75 $\left(\frac{1}{1.064}\right)^{4}$ =	$ 58.52	$ 58.52/$1,085.01 = .053934	4 x .053934 =	.215736
5	$ 75 $\left(\frac{1}{1.064}\right)^{5}$ =	$ 55.00	$ 55.00/$1,085.01 = .050690	5 x .050690 =	.253449
6	$ 75 $\left(\frac{1}{1.064}\right)^{6}$ =	$ 51.69	$ 51.69/$1,085.01 = .047641	6 x .047641 =	.285845
7	$ 75 $\left(\frac{1}{1.064}\right)^{7}$ =	$ 48.58	$ 48.58/$1,085.01 = .044775	7 x .044775 =	.313426
8	$ 75 $\left(\frac{1}{1.064}\right)^{8}$ =	$ 45.66	$ 45.66/$1,085.01 = .042082	8 x .042082 =	.336655
9	$ 75 $\left(\frac{1}{1.064}\right)^{9}$ =	$ 42.91	$ 42.91/$1,085.01 = .039551	9 x .039551 =	.355956
10	$ 75 $\left(\frac{1}{1.064}\right)^{10}$ =	$ 40.33	$ 40.33/$1,085.01 = .037172	10 x .037172 =	.371717
11	$1,075 $\left(\frac{1}{1.064}\right)^{11}$ =	$543.31	$543.31/$1,085.01 = .500746	11 x .500746 =	5.508208
	Total Bond Value	$1,085.01	1.000000	Duration	8.000230

The Macaulay duration is about 8 half-years, or 4 years. The significance of that measure is that if we buy a bond with a 4-year duration, we will be approximately *immunized* over a 4-year (8 half-year) holding period. Notice what happens, for example, if we buy this bond today and then hold it for 4 years. If the reinvestment rate is 7.8% semiannually (for an EAR of $1.078^2 - 1 = 16.2084\%$), the coupon receipts will grow, with a 7.8% periodic reinvestment rate over 8 periods, to

$$\text{PMT x FAC = TOT}$$

$$\$75\left(\frac{(1.078)^{8}-1}{.078}\right) = \text{TOT}$$

$$\$75\ (10.560078) = \$\underline{792.01}$$

By the end of year 4, $792.01 will have accumulated with reinvested interest, and the bond should be able to be sold, with three half-years remaining until maturity, for a price of

$$V_B = \$75\left(\frac{1-\left(\frac{1}{1.078}\right)^{3}}{.078}\right) + \$1{,}000\left(\frac{1}{1.078}\right)^{3}$$

$$= \$75\ (2.586426) + \$1{,}000\ (.798259) = \$\underline{992.24}$$

So by the end of the 4-year holding period, the bond holder will have $792.01 through interest received and reinvested, and can sell for $992.24, for a total $1,784.25 value at the end of half-year 8. Investing \$1,085.01 today and then having \$1,784.25 after 8 periods corresponds to a periodic rate of return computed as:

$$\text{BAMT}\ (1 + r)^n = \text{EAMT}$$
$$\$1{,}085.01\ (1 + r)^8 = \$1{,}784.25$$
$$(1 + r)^8 = 1.644455$$
$$\sqrt[8]{(1 + r)^8} = \sqrt[8]{1.644455}$$
$$(1 + r) = 1.644455^{1/8} = 1.644455^{.125} = 1.064150\ ,$$

for an r = .064150 semiannual return and an annualized realized compound yield (RCY) of $(1.064150)^2 - 1 =$ 13.2%. The 16.2084% rate's impact on the reinvested cash flows (higher) and on the bond's resale value (lower) almost exactly offset each other if the holding period is equal to the bond's 4-year Macaulay duration, so the RCY is equal to the YTM of 13.2%.

If the reinvestment rate is 4.9% semiannually (for an EAR of $1.049^2 - 1 = 10.0401\%$), the coupon receipts will grow, with a 4.9% periodic reinvestment rate over 8 periods, to

$$\text{PMT x FAC} = \text{TOT}$$
$$\$75\left(\frac{(1.049)^8 - 1}{.049}\right) = \text{TOT}$$
$$\$75\ (9.515022) = \$\underline{713.63}$$

By the end of year 4, \$713.63 will have accumulated with reinvested interest, and the bond should be able to be sold, with three half-years remaining until maturity, for a price of

$$V_B = \$75\left(\frac{1 - \left(\frac{1}{1.049}\right)^3}{.049}\right) + \$1{,}000\left(\frac{1}{1.049}\right)^3$$
$$= \$75\ (2.728359) + \$1{,}000\ (.866310) = \$\underline{1{,}070.94}$$

By the end of the 4-year holding period, the bond holder will have \$713.63 through interest received and reinvested, and can sell for \$1,070.94, for a total \$1,784.57 value at half-year 8's end. Investing \$1,085.01 today and then having \$1,784.57 after 8 periods (a tiny rounding difference from our answer above) corresponds to a periodic rate of return computed as:

$$\text{BAMT}\ (1 + r)^n = \text{EAMT}$$
$$\$1{,}085.01\ (1 + r)^8 = \$1{,}784.57$$
$$(1 + r)^8 = 1.644750$$
$$\sqrt[8]{(1 + r)^8} = \sqrt[8]{1.644750}$$
$$(1 + r) = 1.644750^{1/8} = 1.644750^{.125} = 1.064150\ ,$$

for an r = .064174 semiannual return and an annualized RCY of $(1.064174)^2 - 1 =$ 13.2%. The 10.0401% rate's impact on the reinvested cash flows (lower) and on the bond's resale value (higher) almost exactly offset each other if the holding period is equal to the bond's 4-year Macaulay duration, so the RCY is equal to the YTM of 13.2%.

Here the holding period was equal to the bond's Macaulay duration, so the RCY is equal to the YTM no matter what the reinvestment rate (the bond investment has been immunized). If the holding period exceeds the Macaulay duration, a reinvestment rate greater than the YTM gives a RCY greater than the YTM; a reinvestment rate less than the YTM gives a RCY less than the YTM. If the holding period is shorter than the Macaulay duration, a reinvestment rate greater than the YTM gives a RCY less than the YTM; a reinvestment rate less than the YTM gives a RCY greater than the YTM. (A coupon bond's Macaulay duration is always shorter than its remaining maturity; a zero-coupon bond's maturity is equal to its Macaulay duration. With no coupon payments to reinvest for better or for worse, a zero-coupon bond's RCY is equal to its YTM.) Thus an aggressive bond investor might try to predict interest rate movements, and then purchase bonds with Macaulay durations greater than some targeted holding period if he believed that reinvestment rates would fall OR purchase bonds with Macaulay durations less than that targeted holding period if he believed that reinvestment rates would rise.

What occurs is that if Macaulay duration is greater than the holding period and market rates fall (rise), the loss (gain) on reinvestment is more than (less than) compensated for by the gain (loss) in the resale price that accompanies lower (higher) market rates. If Macaulay duration is less than the holding period and market rates rise (fall), the gain (loss) on reinvestment more than (less than) compensates for the loss (gain) in resale price that accompanies higher (lower) market rates. Modified duration (an adjusted version of Macaulay duration) measures a bond's price sensitivity to changing interest rates, so if you think interest rates are going to rise you want to have bonds with relatively shorter durations (so their values will fall only a little), whereas if you think rates will fall you want bonds with relatively long durations (so their values will rise by a lot).

The immunization technique shown above is somewhat oversimplified; immunizing actually requires rebalancing frequently, buying new bonds with durations equal to the remainder of the original investment period (a year from now the remainder of the holding period will be 3 years, but the bond illustrated will have a duration at that time of about 3.5 years – Macaulay duration does not change in a linear, step-by-step manner with remaining time to maturity). •