Topic 5: Cost of Capital (*Copyright © 2024 Joseph W. Trefzger*)

Let's say we have a pile of wooden posts. Some of them are four feet each in length; the rest are nine feet long each. What is the average post length? Here we are averaging a group of fours and a group of nines, so the solution we come up with had better be more than four and less than nine, or we will know that we have made an error. But what is the precise answer?

It depends on how many posts are of each length. If there are equal numbers of the shorter and longer posts, we can think of the average length as a *simple* average. With ten posts, five of each length, we could compute that simple average length using a "brute force" approach as

(4 + 4 + 4 + 4 + 4 + 9 + 9 + 9 + 9 + 9)/10 = 65/10 = 6.5 feet.

[Or seeing that we have equal numbers of each length, we could think of pairs of posts, and find the average length of each pair as (4 + 9)/2 = 13/2 = 6.5 feet.]

Computing a Weighted Average

Brute force is easy to apply when the number of items being averaged is specified, and is fairly small. A more general approach, useful when we are averaging a large or unknown number of observations, is to use the *proportion* of observations in each category as weights in computing a *weighted* average. In the example introduced above we know that 50% of the total are four-foot posts, and the other 50% are nine-footers, so we can compute the weighted average as

(.50)(4) + (.50)(9) = 2.0 + 4.5 = 6.5 feet.

In other words, a simple average is merely a weighted average in which the weights are equal. Now let's assume instead that four of ten posts are four-footers, and the remaining six posts measure nine feet each. Using brute force leads to a computed average of

(4 + 4 + 4 + 4 + 9 + 9 + 9 + 9 + 9 + 9)/10 = 70/10 = 7.0 feet.

(With more of the long posts and fewer of the short ones, the average is greater than in the prior half-and-half case – here the longer posts play a more prominent role, leading to a slightly higher average than the 6.5 generated by the simple averaging process seen earlier.) Using the more generally applicable formula, and noting that 40% of the total are shorter posts and 60% are of the longer variety, we compute the weighted average length as

(.40)(4) + (.60)(9) = 1.6 + 5.4 = 7.0 feet.

Our goal in studying the "Cost of Capital" topic is an application of the weighted average idea. We can illustrate it with an example that looks a lot like the wooden post situation above:

A specified firm always pays for 40% of assets with money from lenders ("debt") [debt/total assets = 40%], and if it were to attempt to borrow money today the lenders would expect to receive a 5.33% annual interest rate. But interest the company pays is deductible from EBIT before income tax is paid on EBT (let's say at a 25% state-plus-federal marginal rate). Because any dollar paid out as interest does not remain in EBT to be taxed, and thus generates a 25-cent income tax savings, the ultimate annual cost of compensating new lenders would be only 75% of the stated interest cost: (.0533)(1 – .25) = (.0533)(.75) = 4.0%.

This particular company always pays for the other 60% of its asset total with money from owners ("equity") [equity/total assets = 60%], and if the firm's managers were to seek money from new common stockholders those new owners would expect an annual rate of return on equity of 9%.

So if the company were to try to raise money for a new investment project, its average annual cost of obtaining money from the two groups of money providers together, or *weighted average cost of capital* (WACC), would be

$$\text{WACC} = (.4)(.04) + (.6)(.09) = .016 + .054 = \underline{\underline{7.0\%}}$$

(remember our average length of wooden posts?).

Why bother to compute this value; what is its significance? The 7% is a very important measure; if the company's managers were striving to generate new wealth for the company's owners (recall that financial managers' goal is to maximize the owners' wealth), they would undertake a new investment, such as purchasing new machinery, only if the expected annual rate of return on the investment were greater than 7%. We therefore sometimes call a firm's annual weighted average cost of capital its "hurdle" rate for new investments; an investment with a periodic rate of return greater than the periodic cost of money creates added wealth for the company's owners.

For example, if the company pays for $2,000 in new equipment with $800 (40% of the total) from lenders and $1,200 (60% of the total) from owners, a 7% return in any given year on the $2,000 invested (.07 x $2,000 = $140) would allow the company's managers to pay

.0533 x $800 = $42.66 in interest initially to the lenders,
but then the firm would save 25% of $42.66 = $10.66 in income tax;
for a net cost of $42.66 – $10.66 = $32 to compensate the lenders.

The remaining $140 – $32 = $108 would be the owners' compensation. Note that this amount represents a $108/$1,200 = 9% return; that is the annual rate of return on equity the owners had expected for taking the risk of providing money with the "back of the line" residual claim.

Earning $140 on the machinery therefore provides for the lenders to get an appropriate annual interest rate, and allows the owners to receive a minimally acceptable annual return on equity, though it adds nothing to the owners' wealth (Economic Value Added, or EVA, would be $0). If we expect the machinery to deliver a percentage return of at least the 7% annual weighted average cost of capital, we can expect to compensate all money providers fairly. Anything earned each year in excess of .07 x $2,000 in additional assets = $140 would increase the value of the owners' investment, and thereby increase their wealth.

The Basic Weighted Average Cost of Capital Formula

In computing the 7% WACC, we treated the annual interest rate that would be expected by new lenders ("cost of debt," k_d), the annual return on equity that would be expected by new owners ("cost of equity," k_e), the marginal annual income tax rate t, and the capital structure (here 40%

debt, 60% equity) all as givens. In our examples, we always will treat the capital structure as a given, though we will discuss some ideas surrounding capital structure in Topic 8.

But we will not necessarily treat the annual interest rate k_d that new lenders would expect to earn, the annual return on equity k_e that new owners would expect, or the annual dividend percentage return k_p that would be expected by new *preferred stock* buyers (if there is preferred stock financing – a new concept introduced here in Topic 5) as givens. We want to "get inside the heads" of potential investors, to see what annual rates of return they would expect for providing money to the company in question for a specified investment project with a particular expected time horizon as lenders, preferred stockholders, or true owners (common stockholders).

Then, with a known (given) capital structure and marginal income tax rate t, and our computed k_d, k_p [if applicable – we will always assume that there are lenders and true owners, but a particular firm might or might not have preferred stockholders], and k_e figures, we will compute a weighted average cost of capital (WACC, or k_A) with the formula

$$\text{WACC or } k_A = (w_d)(k_d)(1 - t) + (w_p)(k_p) + (w_e)(k_e).$$

Basically we are averaging the k's, using the w's as weights. In this equation:

w_d is the weight, or proportion, of money for a new investment project to be provided by lenders,
w_p is the weight, or proportion, of financing to be provided by preferred stockholders, and
w_e is the weight, or proportion, of financing to be provided by common stockholders.

The three w's must sum to 100%, because we want to account for all of the firm's sources of financing for a proposed new project. We can think of each respective k as relating to

$$\frac{\text{what a new money provider would expect to receive as financial returns each year}}{\text{the amount the money provider initially would invest}}$$

(dollars received back as a percentage of dollars given up). We would need to "get inside the heads" of potential new money providers to infer the annual returns they would expect to earn, based on the investment risks they would feel they were facing. Then keep in mind that we adjust the annual interest rate k_d that lenders would expect to receive to reflect an expected income tax savings, thereby obtaining an *after-tax* annual percentage cost of new debt financing $(k_d)(1 - t)$, because interest is paid with pre-tax dollars (income tax is computed each year after lenders have been paid interest). Returns are deemed to be provided to preferred and common stockholders, however, from net income, which is value that remains after income taxes already have been paid – so there is no associated income tax savings shown with k_p or k_e. (Recall the structure of the income statement.)

Of course, if a firm has no preferred stockholders, then $w_p = 0$ and the entire $(w_p)(k_p)$ term drops out of the equation, such that $w_d + w_e$ totals to 100%, and our WACC formula simplifies to

$$\text{WACC or } k_A = (w_d)(k_d)(1 - t) + (w_e)(k_e),$$

as in our initial example, with

$$\begin{aligned} \text{WACC or } k_A &= (.4)(.0533)(1 - .25) + (.6)(.09) \\ &= (.4)(.04) + (.6)(.09) \\ &= .016 + .054 = \underline{\underline{7.0\%}}. \end{aligned}$$

This weighted average cost of capital is what it would cost the company annually to deliver the required rates of return to all new investors in proportion to their contributions. Because it is the minimum annual return the company would require on new investments, the WACC is the hurdle rate, thus the annual discount rate we typically use in capital budgeting analyses (see Topic 6).

Think about WACC by breaking the computational process into pieces. Essentially we are averaging k_d, k_p, and k_e. But because interest payments the managers make to lenders are deductible (subtracted from EBIT before income tax on EBT is computed), we actually are averaging $(k_d)(1 - t)$, k_p, and k_e. (No income tax savings arises from the returns delivered to preferred and common stockholders, who are paid from the net income that remains after income tax has been paid.) And because the three types of money providers will not contribute equal proportions of the company's financing, the average computed must be a weighted average.

Estimating k_d and k_p: Comparatively Simple

We spoke earlier of "getting inside the heads" of potential money providers, to determine the annual rates of return they would require for bearing the risks of being new lenders, preferred stockholders, or common stockholders in the company being analyzed. Some of the factors that drive investors' expected returns, such as the general level of interest rates and strength of the national economy, are outside the company managers' control. Other factors, such as the risk of investment projects to be undertaken with newly obtained money and the expected payment of dividends, are within the managers' control.

After-Tax Cost of Debt [$(k_d)(1 - t)$]

If the firm were to borrow money (perhaps by issuing new long-term bonds) to pay for part of the equipment needed for a specified investment project, what annual interest rate would the lenders expect to receive? (Technically lenders' "expected" annual rate of return would be lower than the interest rate quoted in the loan agreement, because expected return factors in the possibility that the company borrowing the money might default on the loan, but here we will not distinguish between the contract interest rate k_d and lenders' expected annual rate of return.) The yearly interest rates paid when the company borrowed money in the past tell us almost nothing – those rates reflected conditions of the economy, and/or the risk lenders perceived in lending to the firm, that prevailed at an earlier date but may bear no relationship to the realities of today. (The price you paid for a gallon of orange juice six years ago is not a good indicator of what you would expect to pay for a gallon of orange juice today.)

There are two ways to estimate the annual interest rate the company in question would have to pay if it borrowed money today:

1) *The method we will focus on in this discussion*: look at annual interest rates that competing firms, viewed by lenders as being approximately as risky to lend money to as this company is, have paid on recent borrowings that were structured similarly (the company's investment banking advisors would help the managers with this analysis). In this simple approach, we are

using other, similar firms as the benchmark in estimating *k* (the dollars lenders would expect to receive each year, as a percentage of dollars given up initially). If firms like the one we are analyzing have paid 7.35% annually in recent borrowings, then we would expect it to pay 7.35% per year if *it* borrowed new money – the 5% or 10% annual interest rate paid when it borrowed money two or seven or ten years ago is essentially irrelevant to today's analysis.

2) A more direct method (but one that requires computational tools we have not covered yet): look at the annual rate of return implied by the prices buyers have been paying recently for bonds the company being analyzed issued at earlier times (this rate is known as the *yield to maturity* on its existing bonds). In this more complex approach we use the company itself as the benchmark for thinking about *k* (dollars lenders would expect to receive each year as a percentage of dollars initially given over to the company's managers).

For those who would like an example (ignore it if you wish; we will deal with yield to maturity in detail in Topic 10), assume that five years ago this company issued 25-year bonds with $1,000 principal values and a 9% annual interest rate. So whoever now is the holder of one of these bonds will get .09 x $1,000 = $90 per year in interest for the remaining 20 years, and then also get the $1,000 principal back when the bond matures. Recent buyers have willingly been paying $1,170.15 to buy one of these previously-issued bonds (with 20 years of payments remaining) from other investors. Note that only with a 7.35% annual discount rate do we compute the present value of the expected dollar receipts to be $1,170.15:

$$(\$90)\left(\frac{1-\left(\frac{1}{1.0735}\right)^{20}}{.0735}\right)+(\$1{,}000)\left(\frac{1}{1.0735}\right)^{20}$$

$$= (\$90)(10.311836) + (\$1{,}000)(.242080)$$
$$= \$928.07 + \$242.08 = \$1{,}170.15\ . \ \checkmark$$

The fact that recent buyers have required a 7.35% annual yield to maturity is what has caused each bond to sell for a $1,170.15 price. (We would have found the 7.35% through a trial-and-error process that will be discussed in Topic 10, on bond valuation.) The yield to maturity is the annual rate of return that knowledgeable parties recently have required for taking on the risks of lending to the company in question as it is today; they factor that return into the transaction through the price they bid (what they give up) to get the promised annual interest payments (what they expect to get back). Our logic is that if an alert person currently requires a 7.35% annual return for taking the place of a current lender, we would expect that same alert individual (or any like-minded party) to require a 7.35% annual return for taking a newly-created position as a lender as the company managers try to raise new money for new investment projects – as long as new lenders would perceive the risk of being a lender going forward to be about the same as the risk the lending marketplace has recently attributed to lending to the firm its current state. In our WACC discussion, we will treat this yield to maturity as a given, but it is interesting to recognize that we would find it by observing the prices currently being paid for bonds the company in question issued in the past – as noted earlier, using the company being analyzed, rather than similar firms that borrowed recently, as the benchmark in estimating *k*.

After we have estimated potential lenders' required annual interest rate (through one of the two methods noted above – it's fine to think in the simpler terms of what companies like the one in question have had to pay in recent borrowings), we must make one final adjustment in computing the firm's ultimate expected annual cost of delivering returns to new lenders. Recall that interest is paid with pre-tax dollars (paying a dollar of interest leaves one less dollar on which to pay income tax, so it costs less than 100 cents for the company manages to deliver $1.00 in interest to lenders). Therefore we compute

$$\text{After-tax annual cost of debt financing} = (k_d)(1 - t),$$

with t representing the firm's marginal income tax rate. (We use the *marginal* income tax rate because the company managers are trying to decide whether to go forward with a new investment opportunity.) If new lenders would require a 7.35% annual interest rate, and if the firm being analyzed is in a 26% marginal income tax bracket, then borrowing money would cost $k_d(1 - t) = (.0735)(1 - .26) = (.0735)(.74) = 5.44\%$ annually. New lenders would expect, and would be paid, 7.35% interest each year for bearing the risks of lending to the company for a new investment project. But because the government picks up part of the borrowing cost (taxable income EBT is computed as operating income, or EBIT, minus interest paid), it would cost the company managers only 5.44% each year to deliver that 7.35% required annual return.

<u>Cost of Preferred Stock (k_p)</u>

We have not addressed preferred stock in earlier discussions. Preferred shares get preferential treatment, relative to common shares, in receiving dividends; preferred stockholders get their dividends before common stockholders receive their dividends. (In earlier discussions we suggested that lenders stand at the front of a figurative line of people who provide money for the company managers to buy assets with and owners stand at the back of that line; preferred stockholders would stand in the middle of that line.) Preferred stock is a *hybrid* form of financing, with some features that make it seem like debt but others that make it seem like equity. (It *behaves* largely like a form of debt financing, but is seen *legally* as a form of equity.) When companies issue preferred stock (not all companies do), typically they indicate in advance the dividends that will be paid, specifying a planned yearly dividend per share as a steady annual percentage (*e.g.*, 6.76%) of a stated per-share par value (*e.g.*, $100), for an expected annual dividend (.0676 x $100 = $6.76). [Dividends actually are paid quarterly, and in Topic 13 on stock valuation we will work specifically with quarterly payments in determining existing preferred shares' theoretical values, but here we are dealing with the much less certain process of estimating what *might* happen with *potential* new preferred shares, so in this analysis we estimate by combining four quarterly payments into an annual total.]

Unlike bonds (which might have 10 or 20 year remaining lives), preferred stock typically has an infinite expected life. So we treat preferred stock as a perpetuity: whoever holds one of the shares described above expects to receive a $6.76 annual dividend each year indefinitely.

If the firm has a 6.76%, $100-par preferred stock issue outstanding, we can infer that investors were happy with a $k_p = 6.76\%$ annual return, for facing the risks of being preferred stockholders to the company as it existed at the time, when those preferred shares were issued a number of years ago. But what would new preferred stockholders expect for helping finance a new investment project today? As with bonds, we can estimate new preferred stockholders' likely required annual rate of return by looking at the price people currently are paying for the right to collect the stated stream of yearly dividends on each existing preferred share (dollars they expect to get back as a percentage of dollars they give up initially).

If investors recently have been paying $84.50 per share for this 6.76% dividend preferred stock, then by paying $84.50 for the right to collect $6.76 per year they have forced the transaction to deliver an expected annual rate of return of $6.76/$84.50 = 8%. If the company would issue new preferred stock, it seems reasonable to assume that the purchasers would expect that same k_p = 8% annual return, as long as they believed that the company's managers would be no more and no less able in conducting new projects than they have been, on average, in overseeing the company's existing operations. After all, someone who buys existing preferred shares (as with existing bonds or existing shares of common stock) is getting a small financial claim on all of the company's existing investment projects. (We do not have to go through trial-and-error steps with preferred stock, because of perpetuities' computational simplicity.)

Recall that we do not adjust our k_p estimate for an expected income tax savings, because the dollar returns paid to preferred stockholders (those dollars are classified as *dividends*) are viewed by governmental authorities as coming from *net income*, which remains after income tax already has been paid. It is too late for the company managers to get an income tax break when they pay money to preferred stockholders. Thus while we feel that it would cost the firm only 5.44% each year to deliver 7.35% annual interest returns to new lenders, we would expect it to cost 8% per year for the managers to deliver 8% annual dividend returns to buyers of new preferred shares; Uncle Sam would not pick up part of the cost through lower income taxes assessed.

Estimating k_e: Systematic Analysis Needed

Our "dollars expected back as a percentage of dollars given up" concept of *k* is fairly simple to implement for potential lenders or preferred stockholders. Because of the comparative ease of "getting inside the heads" of those who might provide new money at the front or middle of the figurative line, we devote little systematic effort to estimating k_d or k_p (remember to adjust k_d for expected income tax savings). Since buyers of bonds or preferred shares have been told what they will get back each year (dollars to be paid in interest and preferred dividends are specified in advance), we can easily compare what buyers expect to get back annually to the observed prices paid for existing bonds or preferred stock shares in any transaction. We then assume that potential investors will expect the risk of providing money going forward to be similar to the risk perceived by recent buyers of the firm's bonds or preferred stock, and thus new money providers would expect annual percentage returns similar to those we know recent investors expected.

But this concept of *k* as dollars expected back each year relative to dollars first invested can be difficult to apply with common stockholders, because those "back of the line" investors are not told in advance, and thus can not know with any certainty, what they will get as dollar returns each subsequent year. Recall that common stockholders, as the true owners of a corporation, have the residual claim: they simply receive as financial returns the value that remains each year after all other parties with financial claims have been compensated. So we may lack the ability to "get inside the heads" of potential new common stockholders by drawing inferences from prices paid for common shares in recent transactions. After all, while we know what recent share buyers gave up (as we also know with respect to recent purchases of the company's existing bonds and preferred stock), we may not have a solid feel for what recent common stock buyers were expecting to get back on an annual basis. As a result, we must have systematic methods for "getting inside the heads" of those who might become common stockholders. How can an analyst estimate potential new common stockholders' expected annual returns? There are three possibilities, the first two of which we will devote meaningful attention to.

Constant Dividend Growth Model: A Behavior-Based Approach

Our favored method is the *constant dividend growth model,* sometimes called *discounted cash flow model*. In this approach, we make a direct attempt to estimate *k* as dollars expected back each year as a percentage of dollars initially given up. Clearly we can verify the prices that knowledgeable parties have paid to buy existing common shares in recent transactions, so the important question is: can we also identify the dollar returns they expect to get back each year? (No one can tell common stockholders in advance what they will receive as financial returns, but buyers of common shares in recent transactions did have *expectations* of returns they would get.)

We theorize that one part of the return a common stockholder expects to receive each year is a dividend (as with preferred shares, we think in terms of the total of four quarterly payments) that is some proportion of the value of each share, for an annual *dividend yield*. Then, if the share buyer expects managers to make wise investments with the portion of net income retained (*not* paid out as dividends), that individual expects "Wall Street" to recognize that retaining those earnings has made the company stronger, such that each existing share represents an unchanged fractional claim on the residual of a bigger, more viable company, and each existing share should grow in resale value in the coming year. We can represent this idea as

$$k_e = \frac{D_1}{P_0} + g.$$

Someone who buys an existing common share now (at the end of period 0) at today's price, P_0, expects to earn a financial return in the coming year that combines the coming year's expected dividend, D_1, as a percentage of today's price P_0, with the percentage growth rate g in the stock's per-share value expected over the coming year. In other words, net income belongs to the firm's owners, but the managers decide how to divide that net income into payments of dividends for those common stockholders' immediate use and the retention of earnings to purchase assets that will make the company stronger – and thus able to pay even more dividends in later years. This concept of an owner's return consisting of dividends and the benefit of reinvested earnings is one we have noted since our discussion of balance sheets and income statements back in Topic 2.

So this equation relating k_e to expected dividends plus growth shows what any buyer of a firm's common shares expects as a total rate of return over the following year, based on what that party expects D_1/P_0 and g together to be. But it works as a method for getting inside potential new common stockholders' heads if, and only if, we as analysts feel we know what buyers of existing shares *in general* expect D_1 and a related g individually to be. For the constant dividend growth model to be valid to use in estimating k_e for a proposed project, we must be confident that:

- we know the expectations of D_1 and g held by a significant proportion of the parties that constitute the market for the company's common shares (managers within the company, professional investment analysts who follow the firm's activities, portfolio managers
- at pension funds or mutual funds that buy a lot of the company's common stock), *and*

- those parties are largely in agreement both on the magnitude of growth rate *g* and in the view that this annual growth rate will persist at the same level for many years into the future (recall that the approach is called the *constant* dividend growth model).

We theorize that common stockholders willingly forego current dividends in favor of earnings retention only if they expect the resulting bigger, stronger firm to pay even more in dividends in later periods. Therefore, dividends expected over the long term determine the price a buyer is willing to pay for a share of common stock. So *if, but only if, parties that drive the market for a firm's common stock generally expect dividends to grow by a specific annual percentage rate consistently into the future, they should expect the per-share value to grow each year by that same percentage*. Thus if a dividend D_0 was paid during the most recent year 0 (a time-based subscript of 0 indicates today, or a time period that just ended today), and if parties that make up a major part of the market expect a yearly dividend growth rate of about g to persist, year upon year, far into the distant future, then we can estimate D_1 as $D_0(1 + g)$, and our equation becomes

$$k_e = \frac{D_1}{P_0} + g = \frac{D_0(1+g)}{P_0} + g.$$

Only with an estimate of the coming year's growth rate in dividends can an analyst who knows D_0 estimate D_1, and only if that same growth rate is expected to persist year after year into the distant future will the share price be expected to grow each year by that same percentage. In a real world case the values P_0 and D_0 would be easy to find. Let's say the current price of XCorp's common stock is P_0 = \$40 per share, and the most recent year's dividend total was D_0 = \$2.86 per share. The missing piece of the puzzle, the only value not readily identifiable, is g.

We communicate with parties that have considerable impact on the market for this firm's common shares and learn that almost all of them expect dividends to increase by approximately 5% annually, year upon year, into the distant future. Thus recent buyers of this firm's common shares generally expect the dividend to grow by g = 5% to $D_1 = D_0(1 + g)$ = \$2.86 (1.05) = \$3.00 in the coming year. And since expected dividends (and expected dividend changes) drive the stock price (and price changes), a view that dividends will continue growing consistently by about 5% per year into the distant future also should be reflected in a 5% increase in value for each share of the firm's common stock every year.

With confidence that we have identified 5% as a g estimate that is widely accepted and expected to persist over time, we as analysts can conclude that the typical buyer who pays \$40 today expects a .075 x \$40 = \$3 dividend and a .05 x \$40 = \$2 "capital gain" over the next year, and similar percentage benefits in all following years, for an expected annual rate of return of

$$k_e = \frac{D_1}{P_0} + g = \frac{D_0(1+g)}{P_0} + g = \frac{\$2.86(1.05)}{\$40} + .05 = \frac{\$3.00}{\$40} + .05 = .075 + .05 = .125 \text{ or } 12.5\%,$$

and we will have succeeded in directly identifying k as what share buyers today expect to get back each year relative to what they are giving up. The only sensible conclusion to draw, under the condition of constant expected future dividend growth of approximately 5% per year, is that a typical buyer paying \$40 per share today expects a 12.5% annual rate of return for bearing the risks of being a common stockholder in this firm as it currently exists, since we feel confident that most buyers expect a yearly dividend of 7.5% ($3/$40 in year 1), and also expect something close to 5% constant annual growth in dividends, and thus in annual share value growth as well.

We then further conclude that if the company were to raise new money for new investment projects, new common stockholders would expect annual returns of 12.5% as long as they expect the risk of providing money going forward to be similar to the risk perceived by recent buyers of the company's common stock. [Be sure to identify whether you have been given D_0 or D_1; you ultimately need an estimate of D_1, the dividend expected in coming year 1, to complete the computation, so if all you are given is D_0 then remember to compute D_1 as $D_0\,(1 + g)$.]

If investors expecting 5% ongoing annual growth in dividends and share value would be happy with an annual rate of return less than 12.5%, they would cheerfully pay more than $40 per share today for the stock (paying $60 would give them an expected annual return of $3/$60 + .05 = 10%). If they required a return greater than 12.5% per year, they would insist on paying less than $40 per share (paying $30 would provide an expected annual return of $3/$30 + .05 = 15%). Paying $40 indicates an expected annual return of exactly 12.5% – but if, and only if, we think the buyers expect *g* to persist into the distant future at a level remaining close to 5% annually.

The constant dividend growth model (which we will use in Topic 13 as a tool for estimating a common stock's value) is our favored method of estimating k_e because it directly relates dollars expected back to dollars given up, based on recent stock purchasers' actions and perceived expectations (a weakness is that it does not *directly* account for risk). Yet it is appropriate to use only if we feel a specific dividend is generally expected to be paid in the coming year, and can identify an expected growth rate *g* for both dividends and per-share value that we feel represents a market consensus that is expected to remain fairly stable over a very long, multi-year period.

A quick aside on estimating the growth rate g (not essential to our discussion): We can compute the growth rate *g* for use in the constant dividend growth model in various ways. One way to estimate a constant expected annual growth rate for future periods is to ask some "Wall Street" analysts who follow the company in question (and its industry), and perhaps average their estimates. Another is to find a simple *arithmetic* average based on recent past growth. Let's say that dividends have been paid over the most recent five years, and the growth rates observed have been 7%, 9%, 4%, 1%, and 14%. Just compute (7% + 9% + 4% + 1% + 14%)/5 = 35%/5 = 7%. Yet another is to compute a geometric average with the equation (looks scarier than it really is):

$$g_A = \left(\prod_{i=1}^{n}(1 + g_i)\right)^{1/n} - 1$$

with g_A = average annualized growth over a specified time period,
Π representing the operator for a product (like Σ is used to indicate a summation),
i = an individual year whose growth is included,
g_i = the growth rate observed in a particular year, and
n = the last year in the sequence (*i.e.*, total number of years included).
For n = 5 and $g_1 - g_5$ as indicated above, start by computing

$$[(1.07)(1.09)(1.04)(1.01)(1.14)]^{1/5} = 1.3965929^{1/5},$$

or the fifth root of 1.3965929, or 1.3965929 to the power of .2 (those all mean the same thing; it just depends on how you and your calculator like to attack the problem) = 1.0690893. That value is one plus the annualized growth rate, so subtract one (as shown above) to get the average "constant" growth rate of 6.90893%, or about 7%. With these particular numbers, the arithmetic and geometric averages came out just about exactly equal, though these two averages would not be roughly equal in all cases. The geometric average is, arguably, a better measure than the arithmetic average, since the geometric average deals with *compounded* growth.

A final approach might be to look at the actual dividends at the beginning and the end of some stated time period; let's say, for example, the year 1 dividend was $5 per share and the year 6 dividend was $5.76. So we find the compounded rate by which $5 grew to $5.76 over 5 years:

$$\$5 \text{ x } (1 + g)^5 = \$5.76$$
$$1 + g = (\$5.76/\$5)^{1/5} = (1.152)^{1/5} = (1.152)^{.2}$$
$$1 + g = 1.028704, \text{ so } g = .028704, \text{ or about } 2.9\%.$$

But in any of these cases, it is important to ask whether we are getting a meaningful answer. After all, the *constant* dividend growth model is valid to use only if we feel that growth in dividends is expected by market participants to occur at a fairly constant annual rate in the future. If past dividend growth has been erratic, why would we think parties with a serious impact on the market for the stock would project future growth to be a steady percentage? Indeed, if we could not infer a stable estimate of expected growth g in dividends (and by extension the stock's price), we probably would want to estimate k_e with the following approach.

Security Market Line Model: A Logical Approach

As noted, our preferred method of estimating k_e, the constant dividend growth model, is valid to use only if we feel there generally is a consensus view that an expected dividend growth rate g will persist at about the same constant level for many years into the future [such that we can start by estimating the coming year's expected per-share dividend D_1 as D_0 $(1 + g)$]. The problem is that what keeps the stock market functioning is that different people have different expectations of where a particular corporation's dividends are headed. A transaction occurs because one individual who knows the company well thinks the future dividend stream will be unfavorable and thus wants to sell the shares she holds, while another equally knowledgeable party thinks future dividends will rise attractively and thus he wants to buy shares today. So cases in which we can confidently apply the constant dividend growth model are likely to be quite limited.

The *security market line* (SML) *model* is a theoretical, or logic-based, approach to "getting inside the heads" of potential common stockholders. It is our chosen method when we do not feel there is a consistent, consensus estimate of growth rate g in dividends and share prices (or perhaps no expectation that *any* dividends will be paid for many years into the future), such that we can not think of k directly in terms of dollars a new common stockholder would expect to get back each year relative to dollars initially given up. The logic on which we rely is that no one would invest in riskier instruments without expecting higher rates of return, so that a new common stockholder would expect to receive the "risk-free" annual rate k_{rf} (think perhaps of the annual "Treasury Bill" interest rate that the federal government would pay when it borrows for a short-time period), plus an adjustment for risk. The formula is

$$k_e = k_{rf} + \textit{Risk Adjustment}$$

$$= k_{rf} + (k_m - k_{rf})\,\beta.$$

Thus in this model we do adjust explicitly for perceived risk. In the "risk adjustment" term we adjust for two risk features:

- the degree to which stock market investors generally expect to earn annual returns above the risk-free rate ($k_m - k_{rf}$, which we call the *market risk premium*), and
- a measure of the risk of the stock in question (indicated by the Greek letter beta, or β), with this risk measured as the volatility in that stock's historical annual returns relative to the volatility in historical average yearly returns earned on the stock market overall. (The beta we will work with is more specifically called "levered" or "equity" beta. In higher-level finance courses you might factor out the volatility of returns caused by the risk of required payments to lenders, or *financial risk*, from the volatility caused by the *business risk* of the firm's chosen asset base, to get an "unlevered" or "asset" beta. Unlevered beta can be useful in computing the price an acquiring firm should willingly pay for *all* of another company's common shares, since the proportion of debt financing in the optimal capital structure for the merged organization might differ from the debt proportion that was deemed optimal for the acquired company as a separate business.)

k_m is the average annual future return we think investors expect on the stock market overall, a value we typically estimate by taking measurements over some chosen past time period (those who study the stock market have been projecting, of late, a future average annual k_m of approximately 8%, while historically it has been cited in the 10 – 12% annual range). β is 1 for a stock of average risk, greater than 1 for a stock that is riskier than average to hold, and less than 1 for a stock with a history of being less risky (returns less volatile from year to year) than the overall stock market, on average. If k_{rf} = 2.5%, k_m = 9%, and XCorp's β is 1 (in this introductory coverage you would be given the β value), then we have

$$k_e = k_{rf} + (k_m - k_{rf})\,\beta$$
$$= .025 + (.09 - .025)\cdot 1 \quad = \quad .025 + (.065)\cdot 1 = .09.$$

See the unsurprising result? We theorize that investors should expect a stock of average risk to have an annual return equal to the market's 9% average expected annual return. A sensible individual would expect 2.5% annually on the least risky investment imaginable, and then would expect a single dose of the 6.5% extra (premium) that people generally expect for accepting the risks of being company owners. A stock with less than average risk (a $\beta < 1$) should have an expected annual return less than the market's average expected annual return of 9%; if beta is .5 holders of this stock should expect an annual return of 5.75% (the risk-free rate plus half of a dose of the market risk premium):

$$k_e = .025 + (.09 - .025)\cdot .5 \quad = \quad .025 + (.065)\cdot .5 = .0575.$$

Finally, investors should expect a stock with greater-than-average risk (a $\beta > 1$) to provide an annual rate of return greater than the market's 9% average expected annual return; a 1.5 beta should yield a 12.25% expected annual return (the risk-free rate plus a 1.5 dose of the extra return expected for taking the risk of being in the stock market):

$$k_e = .025 + (.09 – .025) \cdot 1.5 \quad = \quad .025 + (.065) \cdot 1.5 = .1225.$$

Not surprisingly, the security market line (SML, also called capital asset pricing model, or CAPM) approach is far from perfect. It is a versatile technique, in that it can be applied even when we do not believe that the market expects a consistent future g (as when the firm's past dividend payments have not followed a consistent growth pattern, or if no dividends have been paid in the recent past or are expected in coming years). And while anyone can verify the β, k_m, and k_{rf} values we use (*vs*. a guess about D_1 in the DCF model), they are historical measures that may not be relevant, looking ahead, in today's market. (Expected interest rates change over time, and β relates how periodic returns to the company's common stockholders have compared historically to the stock market's average periodic returns. Past returns, relative to the overall market, for a company whose business model has been changing over time may be a weak indicator of how future returns would be expected to mirror the market. An analyst looking at a company with many different kinds of investment projects might try to compute a cost of capital, and thus cost of equity, for each individual project, using the β for a "pure play" company that produces only one product or service as a proxy for a particular project's β.)

Bond Yield Plus Risk Premium: A Last-Ditch Effort

The *bond yield plus risk premium* approach is based on the idea that, since a particular firm's common stockholders face more risk than do its bond holders, we might try to estimate potential common stockholders' expected annual returns simply by adding an appropriate amount to the annual interest rate that new bond holders would expect. But what is the appropriate add-on? One study found that common stockholders tend to receive, on average, 3% to 5% more per year than bond holders receive. So by this analysis, we might estimate k_e as 7.35% + 4% = 11.35%.

A serious problem with this approach is that we can not confidently generalize about what the appropriate premium should be in any particular case. For example, a specific firm's lenders might be given protections that cause them to be happy with fairly low interest rates, whereas the market would still perceive that the owners faced considerable risk. Therefore we should view the bond yield plus risk premium approach as a very imperfect default to select only if we have no confidence in the estimates that serve as inputs in our first (the behavior-based constant dividend growth) or second (the logic-based security market line) choices.

So now we have considered three methods to try to "get inside the heads" of potential common stockholders for XCorp. Let's say we feel confident that a substantial portion of the market expects future dividends to grow by approximately g = 5% per year steadily, such that we view the 12.5% result under the constant dividend growth model as our best estimate of k_e. [The constant dividend growth model and security market line model should, in theory, provide the same answer. But inefficiencies in the marketplace – uncertainties regarding the values of variables in the two models – may well cause the answers to differ.] With the estimates shown above for $k_d(1 – t)$, k_p, and k_e, and given capital structure percentages (let's say 30% debt + 10% preferred + 60% common stock = 100% of the financing), we can compute a WACC:

$$\begin{aligned} \text{WACC or } k_A &= (w_d)[(k_d)(1 – t)] + (w_p)(k_p) + (w_e)(k_e) \\ &= (.30)[(.0735)(1 – .26)] + (.10)(.08) + (.60)(.125) \\ &= (.30)(.0544) + (.10)(.08) + (.60)(.125) \\ &= .01632 + .008 + .075 = .09932, \text{ or } \underline{\underline{9.932}}\% \text{ (approx. 10\%)} \end{aligned}$$

(in averaging 5.44%, 8%, and 12.5% we should get an answer > 5.44% and < 12.5%, skewed a bit toward the higher value since most of the money is costlier common equity). This company's managers should reject any proposed investment project with an expected annual return less than approximately 10%.

Some Points to Note

1) Risk Adjustment: The WACC, as we have discussed it ("getting inside potential investors' heads"), reflects our perception of the investing public's recent view of the firm we are analyzing, based on the risks they would have perceived when recently providing money for the company's existing operations – which consist of all investment projects undertaken in the past and still in existence. That measure also would tend to be relevant in anticipating the annual percentage cost of money for new projects that would be seen as similar in risk, on average, to the existing projects.

But what if the company were raising money for a new investment that people would view as *more risky* than the average existing project? (Maybe something involving an untried technology, a developing part of the world, or other matters unfamiliar to the company's management.) Then we would have to adjust the earlier-computed WACC measure for the higher level of perceived risk. Making this kind of adjustment is hard to do; there is no specific "how to" formula. It is likely to be a somewhat arbitrary, "gut-feeling" addition to the measure of WACC for a typical project. But making this risk adjustment is essential; recall that we are computing what it would cost the company managers to deliver the appropriate risk-adjusted returns to their money providers, based on the annual returns we think they would require.

If people expected the firm's managers to be doing riskier-than-typical things with the money, they might well require (on average) returns that would cost the firm about *20*%, rather than 10%, to deliver. If so, the managers would not want to accept the risky project if its expected rate of return were 15% (even though a 15% return would be attractive for a project of typical risk). [A project of less-than-typical risk would call for a *reduction* in the WACC measure.] Another way to state this idea is that the WACC is an *opportunity rate*: its magnitude depends on where the money will *go* (what money providers expect it will be used it for), not on where the money came from. [Though we might want to think of the degree of risk in a *portfolio* context, with perceived risk depending on the project's own features, how the project relates to the firm's other activities, and how the firm's overall returns relate to those of the economy overall.]

2) Don't Let the Symbols Confuse You: Different publications may use different symbols to mean the same thing. For example, instead of showing w_d (debt's proportional weighting) the weighted average cost of capital equation printed in some outlet might show *D/V* (debt as a fraction of the company's value, which is the same thing). And instead of *k* (percentage cost), the equation might show *r* or *R* (rate of return, which is the same thing), for a representation as

$$\text{WACC} = (D/V)(R_D)(1 - t) + (P/V)(R_P) + (E/V)(R_E).$$

The version we used earlier in this discussion has the benefit of reminding us through the *w*'s that we are computing a *weighted* average.

3) <u>Book *vs*. Market Values</u>: We actually should describe a firm's capital structure with regard to the *market* values of the debt and equity securities the firm has issued or would expect to issue soon, because the market values reflect the economic values that the money providers have at risk. But the book values of claims shown on the balance sheet's right-hand side sometimes are used as approximations of true market values when the weighted average cost of capital is computed. So if the book value of a corporation's equity (the common stockholders' paid-in capital plus the earnings the managers have retained) is $1 million and the managers have borrowed $1 million, then the capital structure is 50% debt w_d and 50% equity w_e in book value terms. But if the shares of stock could be sold in arm's-length transactions today for a total of $1.05 million (the "market capitalization") and the bonds that represent the $1 million borrowed could all be purchased for just 95% of their face value or $950,000 (a result we might see if interest rates in the borrowing/lending market rise), then the capital structure in market value terms would be $950,000/$2,000,000 = 47.5% debt w_d and $1,050,000/$2,000,000 = 52.5% equity w_e. (In this introductory coverage the market *vs*. book distinction does not matter, because we treat the capital structure as a given in all of our examples rather than computing it.)

4) <u>Inclusion of All Debt or Long-Term Debt Only</u>: The term "capital" technically refers to long-term sources of money. Therefore, from one viewpoint, we should consider only the cost of long-term debt in computing the company's cost of capital. An underlying assumption (at least an implicit one) in this approach is that long-term assets are paid for with long-term sources of money, and short-term sources finance only short-term assets. While this presumed matching has some beneficial features (and is likely the approach you would see in a more advanced Finance course with more detailed coverage), we also want to recognize that the company's ability to borrow on a long-term basis can not entirely be separated from its use of short-term debt. My preference for this introductory coverage is to view the cost of capital as the weighted average cost of all sources of money (right-hand side of the balance sheet), and to think of the pre-tax cost of debt as the weighted average of the interest rates expected by all lenders.

5) <u>Break points and the Cost of New Common Stock</u>: We have treated the cost of getting money from investors as relating only to their required risk-adjusted returns, and have ignored the administrative/brokerage costs of selling new securities. For bonds and preferred stock these costs tend to be low (in percentage terms), and we typically ignore them for convenience.

But the cost of issuing and selling new common stock can be substantial. The k_e cost of equity figure we computed above might more correctly be called the *cost of retained earnings*, (thus k_e is sometimes shown as k_{re}), since it includes only the compensation that common stockholders would expect for bearing the "back of the line" risks (if the company managers can get the owners' contribution to buying new assets simply by retaining earnings, then the cost of getting that money is just the cost of delivering the owners' expected returns; no substantial administrative costs would arise).

If, on the other hand, the managers have to pay their investment bankers to sell new common stock to the investing public, then the annual cost of obtaining the money includes both the return expected by the owners *and* the payment to the investment bankers. Let's say that the recent price per share for the common stock of the company in question has been $40, that new shares if issued would be expected to sell for $40 each, and that new common stockholders would expect dividends of $3 per share in the coming year. But the investment banker helping the company sell the shares to the investing public would keep $2.50 (a $2.50 *flotation cost*, or

F) from each share to cover its costs, so the firm's managers would end up with the use of only \$37.50. The expected annual *cost of new common stock* thus would be

$$k_{ncs} = \frac{D_1}{P_0 - F} + g = \frac{\$3.00}{\$40 - \$2.50} + .05 = \frac{\$3.00}{\$37.50} + .05 = 13\%.$$

Recall the yearly cost of getting money from owners by retaining earnings is expected to be only

$$k_e = \frac{D_1}{P_0} + g = \frac{\$3.00}{\$40} + .05 = 12.5\%.$$

So if the managers plan to invest so many dollars that they can not expect to get the owners' share simply by retaining earnings, then new common stock will have to be sold, and we might want to use k_{ncs} instead of k_e in computing the proposed new investment project's WACC.

In fact, this rising cost of equity money as more is invested in new projects leads to *break points* (upward jumps) in the WACC. This rising WACC schedule, along with a declining *investment opportunity schedule* (subsequent opportunities are not as well suited to the firm's expertise as it keeps accepting more projects, so successive projects have successively lower expected annual returns), can provide an indication of how much in total the company should invest.

6) Relationship to EVA: When we discussed the Economic Value Added (EVA) measure of company performance in our discussion of financial statement analysis, we computed EVA as [EBIT (1 – t)] – dollar cost of delivering fair returns to investors. In that discussion we referred simply to a "percentage cost" multiplied by total assets, because we did not want the distraction of unneeded terminology, but now we know that this percentage cost is a WACC-type measure.

Actually in our EVA discussion we were multiplying total assets by an average cost of capital, computed on an after-the-fact basis for the entire company, to get a measure of the wealth actually generated by all of the company's operations in a single period. In capital budgeting analysis (in upcoming Topic 6), on the other hand, we use an anticipated average cost of capital (WACC), computed on a before-the-fact basis for a single proposed new project that would be incorporated into the company's operations, to measure the present value of wealth that the specific project is expected to generate over multiple periods. Thus while the two percentage figures technically are not the same, our knowledge of WACC should help us to see the earlier EVA discussion in a more meaningful light.

7) Regulated Monopolies: A regulated monopoly, such as an electric power company, is supposed to be able to charge a price no higher than is needed to cover all of its costs, including a fair rate of return to the providers of money. Because that fair return is embodied in the weighted average cost of capital, regulated monopolies and state government regulatory bodies must devote considerable attention to measuring/monitoring regulated monopolies' WACCs.