

TIME VALUE: PROBLEMS MAIN SET (copyright © 2024 Joseph W. Trefzger)

This problem set covers all of our basic time value of money applications, with a general progression in degree of difficulty as we proceed from problem 1 to problem 24 (note that 22 through 24 are for FIL 404 only). A full understanding of all steps in these problems should indicate solid knowledge of our basic time value of money ideas. Detailed solutions (with the problems repeated) are available in an accompanying file. Be sure you have mastered the easier problems before moving ahead, because the more difficult examples tend to expand on the ideas from the easier ones. Opportunities for additional practice are provided in Problem Set B, which is organized with the same ordering as this main set, and in the largely open-ended Problem Set C.

1. Connie expects to earn a 4.45% average annual interest rate on her savings account. If she makes a \$2,000 deposit today, and then makes no more out-of-pocket deposits, what should her account balance be after six years?
2. What would Amanda pay for an investment that provides no cash flows in years 1 through 9, but will provide a single payment of \$18,750 at the end of year 10, if commitments of similar risk generate a 9% average annual rate of return?
3. Ronald, age 15, just inherited \$100,000 from his late great-uncle Hironymus. His mother, Ronette, says that if Ronald invests the money in a mutual fund he will “probably have a few million dollars” by the time he is 65. What average annual rate of return would Ronald have to earn for his \$100,000 is to grow to \$3,000,000 over 50 years?
4. B.W. makes a \$10,000 investment today. How long will it take for his money to grow to \$40,000 if he can earn a 5% average annual after-tax compounded rate of return on any balance in the account?
5. If Gladys can earn a 3.75% annual rate of return on her account’s growing balance from year to year, how much will she have by the end of year 6 if she makes the series of beginning-of-year deposits described in each of the situations listed below? (Another way to word this problem is: what is the future value of each of the following cash flow streams, with beginning-of-year cash flows and a 3.75% annual compounding rate?)
 - a) \$500 in year 1, \$900 in year 2, \$400 in year 3, \$800 in year 4, \$100 in year 5, \$300 in year 6
 - b) \$500 in each of years 1 through 6 (compute both by compounding individual year payments to the end of year 6, and by grouping these equal payments together with the distributive property and corresponding annuity factor)
 - c) \$500 in year 1, \$506.25 in year 2, \$512.58 in year 3, \$518.99 in year 4, \$525.47 in year 5, and \$532.04 in year 6 (amounts that increase by 1.25% from year to year; compute both by compounding individual year payments to the end of year 6, and by grouping these related payments together with the distributive property and corresponding annuity factor)
6. If Charles can earn a 3.75% annual rate of return on amounts remaining in his account from year to year, how much must he have on deposit today to make the series of year-end withdrawals described in each of the situations listed below? (Another way to word this problem is: what is the present value of each of the following cash flow streams, with year-end cash flows and a 3.75% annual discount rate? Another way is to ask how large a loan a borrower could repay with the payments described?)
 - a) \$500 in year 1, \$900 in year 2, \$400 in year 3, \$800 in year 4, \$100 in year 5, \$300 in year 6
 - b) \$500 in each of years 1 through 6 (compute both by discounting individual year payments to present values, and by grouping these equal payments together with the distributive property and corresponding annuity factor)
 - c) \$500 in year 1, \$506.25 in year 2, \$512.58 in year 3, \$518.99 in year 4, \$525.47 in year 5, and \$532.04 in year 6 (amounts that increase by 1.25% from year to year; compute both by discounting individual year payments to present values, and by grouping these related payments together with the distributive property and corresponding annuity factor)
7. TLM Mutual Funds receives \$4,000 from Sharon, a fund investor, at the end of each year. TLM expects to credit every investor’s account with a 6.5% compounded average annual rate of return. How much should TLM expect to owe Sharon after five years? What if instead she makes her \$4,000 contribution at the beginning of each year? Compute using individual year payment factors, annuity factors, and year-by-year breakdowns of all payments.

8. Beth has been working as a paralegal while saving money to go to law school. She wonders if she now has enough money in the bank to meet her goals. She wants to be able to withdraw \$9,000 each year for 5 years (law school plus two years clerking for a federal judge) to help pay her living costs. She expects her account's declining balance to earn a 7% average annual interest rate. How much money must Beth have on deposit today if she plans to make her withdrawal at the end of each year? What if she plans to take the \$9,000 out at the start of each year? Compute using individual year payment factors, annuity factors, and year-by-year breakdowns of all payments.

9. As Mike blew out the candles on his 31st birthday cake today, he made a wish: to be able to buy a new Mercedes on his 35th birthday. He expects that a typical Mercedes will cost \$65,000 four years from now, and he currently has no money saved toward making that large purchase. If he can earn a 5.25% compounded average annual rate of return on his growing savings balance, how much must he deposit into his account at the end of each year to accumulate \$65,000 over 4 years? What if he instead made his deposits at the beginning of each year?

10. Curt, the manager and bass player for a central Illinois country/rock group, wants to buy some new amplifying equipment. A bank is willing to lend \$14,000 toward the purchase of the \$18,500 worth of needed equipment. However, because people's tastes in music can change over time the loan officer views the loan as a fairly risky one, and thus quotes an 11.5% annual interest rate.

a. If the loan is to be fully amortized, with equal end-of-year annual payments over 6 years, what should the amount of each payment be? What if beginning-of-year payments instead were to be made?

[PARTS B AND C MAY BE OF INTEREST IF YOU LIKE WORKING THE NUMBERS, BUT ARE NOT ESSENTIAL TO OUR COVERAGE IF YOU WANT TO SKIP THEM.]

b. Assume that Curt inherits some money at the end of year 4, and wants to use it to pay off the loan's remaining balance. If year-end payments are made, how much of the \$14,000 borrowed will still be owed at the end of year 4?

c. Now assume that Curt has saved \$3,000 by the end of year 2, and wants to use it to pay off enough of the loan's principal so that he can make the remaining payments over a 3-year period instead of 4 years. If year-end payments are made, how much (in addition to the regular payment) should he pay the bank at the end of year 2?

11. At his high school graduating class's 10th reunion, Leonard gets drunk and brags to former homecoming queen Ursula Hotbodde that he will be a millionaire by the class's 25th reunion. The next day, while too hung over to go to work, he tries to figure out whether he will be able to live up to that claim. He feels that if he buys clothes only at garage sales, drives a moped, lives rent-free with his elderly aunt, and eats only canned peas from Aldi for the next 15 years he will be able to invest \$18,500 per year in a stock market investment account at Gopher Brokers. What average compounded after-tax annual rate of return must he earn to reach a \$1 million total by the end of the 15th year if he invests \$18,500 at the end of each year? What if instead he invests the \$18,500 at the start of each year?

12. High-tech office supplier Normal Equipment Retail and Distribution (NERD) buys a brand new Copytron Super 5000 photocopier for \$19,799. Gridley Electronic Engraving and Kopies, Inc. (GEEK) agrees to lease the machine for \$3,750 per year for 8 years. GEEK will handle all maintenance, and NERD expects that the machine will have no resale value at the end of the 8-year lease period. What is NERD's expected average annual rate of return on its investment if the lease payment is received at the end of each year? What if it is received at the start of each year?

13. Unlike the typical mutual fund company, which allows investors to open accounts with a few thousand dollars, the Snobby Mutual Fund group wants to deal only with the wealthy, and thus requires a \$400,000 initial investment. Beverly, a highly-paid corporate lawyer, wants to open a Snobby Funds account. If she can make annual year-end deposits of \$22,500 into a savings plan that earns a 7.5% average annual compounded return, how many years will it take for her to amass the needed \$400,000? What if she instead makes her deposit at the beginning of each year?

14. Ne'er-do-well Ben wants to borrow \$13,000 from his more responsible brother Glen. Glen reluctantly agrees, but wants to have a formal, written agreement. Ben agrees to sign a note ("IOU") that calls for a 6% annual interest rate, but states that he will be able to budget only \$850 each year for payments. If Glen agrees to accept \$850 at the end of each year, how many years will it take for Ben to repay him all principal plus applicable interest on the remaining unpaid principal? What if Ben makes his \$850 payment at the beginning of each year?

15. The Federal Ear, Nose, and Back Hair Commission, a government agency dealing with issues affecting America's aging male population, issues bonds. Each bond is an agreement to pay the investor who holds it \$1,000 per year forever. (Such true perpetual bonds probably would not actually be created in today's world, but if they were the issuer likely would be a government agency, not a private company.) If the risk of this investment caused rational people to expect a 6.35% average annual rate of return, what should someone willingly pay for each bond if the \$1,000 were to be received at the end of each year? What if it were to be received at the beginning of each year? What compounded average annual rate of return would the investor earn if she paid \$18,000 for one of these bonds?

16. Glenda wants to borrow money to start a business, but she expects that eight years will pass before the venture is successful enough for her to be able to make loan payments. After that time, however, she expects to be able to comfortably make payments of \$35,000 per year for twelve years (years 9 through 20). She finds a non-traditional lender willing to extend a loan with a 10.5% annual interest rate, no payments in years 1 through 8, and then annual payments in years 9 through 20. How much should this lender be willing to lend if the borrower agrees to make a \$35,000 annual payment at the end of each indicated year? What if she pays \$35,000 at the beginning of each year?

17. When the 200th anniversary of Reggieville's founding occurs in 19 years, part of the celebration will involve creating a new Bicentennial Museum. Town planners expect the facility to require a \$400,000 annual budget in perpetuity, and they hope the town founder's wealthy descendant, Reginald Redbird VI, will create an account today to provide for the museum's ongoing operations. If any money invested for the museum can be expected to earn a 4.15% average annual rate of return, how much should Redbird contribute now to fund the future museum? How much of an endowment would the museum need to receive from Redbird today to fund an operating budget that would start at \$400,000 in year 19 and then increase by 1.65%, on average, every year going out indefinitely?

18. Every year for 11 years Raymond will deposit \$2,200 in an account that earns a 3.85% average annual rate of return. But at the end of year 11 he will not close the account; rather, he will leave the accumulated balance to earn returns for an additional 6 years. What will the account balance be at the end of year 17 if the deposits are made at the end of each year? At the beginning of each year? What if instead he deposits \$2,200 in each of years 1 through 11, then \$3,200 in each of years 12 through 17, and then \$4,200 in each of years 18 through 36?

19. Edith just finished medical school at age 28. She hopes to spend three years devoting her medical expertise for humanitarian purposes before she turns 35, but knows that she must amass some savings first. Her current plan is to work at a hospital for 4 years, and then spend the next 3 years volunteering with Doctors Without Borders. Because Doctors Without Borders can provide only a small salary, she will have to withdraw money from her savings in each of those 3 years to help meet her living costs.

a. If she can earn a 4.25% average annual rate of return on any money that remains in her savings account from year to year, and if she saves \$12,000 at the end of each of her 4 working years, how much can she withdraw at the end of each of her 3 volunteer years? What if instead she makes her withdrawal at the beginning of each year?

b. If Edith can earn a 4.25% average annual rate of return on any money that remains in her savings account from year to year, and if she saves \$12,000 at the beginning of each of her 4 working years, how much can she withdraw at the end of each of her 3 volunteer years? What if instead she makes her withdrawal at the beginning of each year?

c. If Edith expects to earn a 4.25% average annual rate of return on any money that remains in her account from year to year, and if she wants to withdraw \$20,000 at the end of each of her 3 volunteer years, how much must she save at the end of each of her 4 working years? What if instead she makes deposits at the beginning of each year?

d. If Edith expects to earn a 4.25% average annual rate of return on any money that remains in her savings account from year to year, and if she wants to withdraw \$20,000 at the beginning of each of her 3 volunteer years, how much must she save at the end of each of her 4 working years? What if instead she makes her savings deposit at the beginning of each year?

e. If Edith saves only \$12,000 at the end of each of her 4 working years, what annual rate of return will she have to earn if she wants to be able to withdraw \$20,000 at the end of each of her 3 volunteer years?

f. Assume that Edith can earn a 4.25% average annual rate of return on any money that remains in her savings account from year to year, and that she saves \$12,000 at the end of each of her 4 working years. She then joins Doctors Without Borders, and immediately decides that she would like to stay with the organization for 3 additional years. If she reduces her annual withdrawal to \$9,600, will her savings sustain her for 6 years of volunteer service?

20. Doug, who is 47, is doing some planning toward meeting important financial goals.

a. He plans to deposit \$750 at the end of every three-month period into a mutual fund account that holds his Roth IRA retirement savings plan. If the average rate of return he expects to earn is a 10% annual percentage rate (APR), but with quarterly compounding, how much will he have in his IRA after 13 years, when he turns 60? What if he makes beginning-of-quarter deposits? What if instead he deposits \$700 per quarter for the first 3 years, \$750 per quarter for the next 6 years, and \$800 per quarter for the last 4 years? What Effective Annual Rate (EAR) of return will he be earning?

b. Assume that Doug will retire at age 60. He will then use the IRA balance to supplement his primary sources of retirement income (company pension and Social Security). He will want to make level withdrawals at the end of every six-month period from the account, and to be confident that he will not outlive his money (he projects that he will live to age 94 ½). If he expects to earn a 10.2% average annual percentage rate of return on any money that remains in the account from year to year, how much should he be able to withdraw every half-year beginning six months after his 60th birthday, and ending on his 94th birthday?

21. Cierra obtains a student loan that is to be repaid in equal monthly installments.

a. If she borrows \$8,000 at a 6.72% Annual Percentage Rate (APR) of interest for a five-year term, and the contract calls for her to pay at the end of each month, how much will each payment be? What if the payments instead are to be made at the beginning of each month (even though loan payments usually are made at the end of each period)?

b. If she can afford to make a \$185 monthly payment over a five-year term, and the contract calls for her to pay at the end of each month, how much principal can Cierra afford to borrow? What if the payments instead are to be made at the beginning of each month?

c. If Cierra borrows \$8,000 at a 6.72% APR and can afford to pay back only \$107.96 at the end of each month, how long will it take for her to repay all principal and the applicable interest? What if instead she makes the \$107.96 payment at the beginning of each month?

d. If Cierra borrows \$8,000 and pays back \$163.75 at the end of each month over a five-year period, what monthly periodic rate of interest is she paying? What corresponding annual percentage rate (APR) of interest would the lender be charging? What would be the corresponding Effective Annual Rate (EAR) of interest?

e. If Cierra's monthly-payment loan carries an EAR of 7.1864%, what is the monthly periodic rate? What is the corresponding APR that the lender would be likely to quote as the interest rate on her loan?

22. [FIL 404 only.] Pete, who is 38, decides to start a long-overdue retirement savings plan. He will make a series of annual deposits, but they will not be equal in amount. Instead, each deposit will exceed the previous one by 2.25%, which is the average annual inflation rate that he expects to observe over future decades.

a. If he expects to earn a 5.75% compounded average annual rate of return on his account's growing balance, and if he makes year-end deposits starting with \$3,000 in the current year, how much money will Pete have in 27 years when he turns 65? What if instead he makes 27 beginning-of-year deposits starting with \$3,000?

b. If Pete expects to earn a 5.75% compounded average annual rate of return on his account's growing balance, and he wants to have \$350,000 when he turns 65 in 27 years, how large should the first of his growing deposits be if he makes his deposits at the end of each year? How large should the first deposit be if instead he makes beginning-of-year deposits that grow by 2.25% annually?

c. If Pete makes year-end deposits starting at \$3,000 and growing by 2.25% annually, how high an average annual rate of return must he earn for his account balance to reach \$500,000 by the time he turns 65 in 27 years? What if instead he makes beginning-of-year deposits starting at \$3,000 and growing by 2.25% annually?

d. If Pete expects to earn a 5.75% compounded average annual rate of return on his growing balance, and if he makes a series of year-end deposits starting with \$3,000 and growing by 2.25% annually, how many years will it take for his account to grow to \$200,000? What if instead he makes beginning-of-year deposits starting at \$3,000 and growing by 2.25% annually?

e. Now assume that Pete wants to have \$350,000 when he turns 65 in 27 years, but instead of making deposits for all 27 years he plans to make equal annual deposits for only 20 years, and then just let the accumulated balance earn an average of 5.75% annually until he turns 65. How much must he deposit at the end of each of the 20 years to reach his goal? What if instead he made his twenty deposits at the beginning of each year? If he wanted his annual deposits to grow each year by 2.25%, how much should his first deposit be (compute for both end-of-year and beginning-of-year deposit cases)?

f. Now assume that Pete wants to have \$350,000 when he turns 65 in 27 years, but instead of making annual deposits he decides to put money into the account semiannually (every 6 months). If he can earn a 5.75% Annual Percentage Rate (APR) of return on his growing balance, and plans to increase his deposits by 1.125% (half of 2.25%) every 6 months, how much should he plan to deposit at the end of each semiannual period? What if he instead makes the deposits at the beginning of each semiannual period? What Effective Annual Rate (EAR) of interest would he be earning on his account's growing balance?

23. [FIL 404 only] Andrea just received a large inheritance. She wants to give most of this money to charity, but because she also wants to quit her job and do full-time volunteer work for the next 9 years she must use part of the inheritance to fund her living expenses. Specifically, she will keep some of the inherited money to open an account, and then will withdraw yearly amounts that increase by 1.6% per year (her estimate of the average annual inflation rate during her 9-year expected period of volunteer work).

a. If she expects to earn a 4.8% average annual rate of return on any money remaining in her account from year to year, how large an account must she open today to fund a series of year-end withdrawals that start at \$25,000? What if instead she wants to make beginning-of-year withdrawals?

b. If Andrea opens a \$200,000 account, if she expects to earn a 4.8% average annual rate of return on any money remaining in her account from year to year, and if she makes year-end withdrawals, how much should the first of her stream of growing withdrawals be? What if instead she makes beginning-of-year withdrawals?

c. If Andrea opens a \$200,000 account, and she wants to make nine withdrawals that start at \$30,000 and then increase by 1.6% per year, what average annual rate of return does she have to earn on her account's declining balance if she makes her withdrawal at the end of each year? What if she makes beginning-of-year withdrawals?

d. Assume that Andrea opens a \$200,000 account, but then realizes that she may come to enjoy her volunteer work so much that she will want to stay with it for more than nine years. Of course, doing so will require her to live on less money each year. For how many years could she continue to make year-end withdrawals, starting with \$20,000 and growing with inflation by 1.6% annually, if she can earn a 4.8% average annual rate of return on any money remaining in her account from year to year? What if instead she makes beginning-of-year withdrawals?

e. Now assume that Andrea opens a \$200,000 account, and then starts making preparations to quit her job. But her boss asks if she would not reconsider and stay for three more years, to oversee completion of the Forbes project to which she has devoted so much of her career. After briefly thinking it over, she agrees; she will simply wait three years before starting her nine-year volunteer program. If she can earn a 4.8% average annual rate of return on any money remaining in her account from year to year, and if she will make a series of nine year-end withdrawals that grow by 1.6% per year after three years have passed (so withdrawals will come in years 4 through 12), how much should the first of her stream of growing withdrawals be? What if withdrawals instead are at the start of each year?

f. Again assume that, after opening a \$200,000 account, Andrea agrees to wait the three years before starting her volunteer period. But now assume she realizes that it is difficult to budget when your income arrives in one large annual chunk, so instead she wants to make quarterly withdrawals. If she can earn a 4.8% Annual Percentage Rate (APR) of interest on any money remaining in her account from year to year, and if she wants her withdrawals to increase by .4% per quarter, how much can she expect to withdraw at the end of each quarter in years 4 through 12? What if instead she makes the withdrawals at the beginning of each quarter? What Effective Annual Rate (EAR) of return would she be earning on her account's declining balance?

24. [FIL 404 only] ISU's wealthiest professor, Joe Solberg, donates 20 acres of land to the campus for a small park. He also wants to give enough money to provide for park maintenance (mowing, clean-up, snow removal) forever into the future. ISU finance director Regina McRedbird tells him that grounds-keeping costs go up by 2.4% per year, on average.

a. If Solberg feels that the university will be able to earn a 7.2% average annual rate of return on money it invests, and if maintaining 20 acres of open space currently costs about \$17,000 per year, how much must he contribute today to fund the growing maintenance budget? Compute for both the case of end-of-year maintenance payments and that of beginning-of-year outlays.

b. Assume instead that Solberg simply contributes \$350,000 along with the land, specifying that the money is to be used toward maintaining the park forever. If the university can earn a 7.2% average annual rate of return on money it invests, how much can it apply to the park's growing maintenance budget in the first year if it accounts for the money at the end of each year? What about the more realistic case of needing the money for park maintenance at the start of each year?

c. Assume again that Solberg contributes \$350,000. If the university wants to spend amounts that start at \$19,500 and then increase by 2.4% per year, what average annual rate of return must it earn on invested money? Compute for both the end-of-year payments case and the beginning-of-year payments case.

d. Now assume that Solberg gives the land and \$350,000, but the university tells him that the land will be used by the ISU Agriculture program to grow experimental crops for six years before it is converted into a park. If the university can earn a 7.2% average annual rate of return on money it invests, how much can it spend on maintenance in year 7 (the first year in a perpetual stream of outlays that will grow by 2.4% annually) if it pays such expenses at the end of each year? What if it pays for park maintenance at the beginning of year 7 and each subsequent year?

e. Assume again that Solberg gives the land and \$350,000, and that the university will use the land for agricultural experiments for six years before converting it to a park. However, now the university wants to restate its budget to show monthly payments for park maintenance. If it expects to earn a 7.2% Annual Percentage Rate (APR) of return on money it invests, and if the amount it needs to spend is expected to rise by .2% per month, how much can it spend on maintenance at the end of the first month of year 7? What if it pays for park maintenance at the beginning of each month? What Effective Annual Rate (EAR) of return will ISU be earning on its money invested?