

# Direction of Price Changes in Third-Degree Price Discrimination

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There is a long tradition in economic literature of comparing welfare under discriminatory pricing with nondiscriminatory pricing employing the usual Marshallian measure. For example, Richard Schmalensee (1981), by assuming independent demand and constant marginal cost, clearly shows that if welfare were to increase under discrimination, then the total output with discrimination must be higher than the total output without discrimination. Hal Varian (1985) analyzes the welfare question in a more general setting. He develops bounds that serve as a sufficient condition for welfare to increase. In addition, these bounds provide important insights in predicting the welfare change under different demand and cost functions. Marius Schwartz (1990) proves that, for any cost function, if discrimination decreases output, it will decrease welfare, as well.

There are two reasons why the welfare question has been so difficult to answer. 1) Based on his analysis Schmalensee (1981 p. 245) concludes that "If all demand functions are strictly concave or convex and if the  $p_i$ 's [prices in each submarket after discrimination] are not nearly equal, there is apparently no simple, general way to tell if monopolistic discrimination will raise or lower total output."<sup>1</sup> Given this absence of

a simple test, it is not possible to verify whether the necessary condition (i.e., increase in output under discrimination) is satisfied or not. 2) Even if one were able to develop a test that could predict when the total output would increase under discrimination, it may not be very useful in answering the basic question of welfare change, because an increase in total output is only a necessary condition and hence does not guarantee an increase in welfare. Thus, to answer the welfare question, a new approach is needed for theoretical analysis of third-degree price discrimination.

Instead of concentrating on the output effects of discrimination, this paper focuses on the price effects of discrimination. A basic result that has remained unquestioned in the literature is that when there are two classes of buyers, discrimination raises price for one class and lowers it for the other. However, in an interesting paper focusing on price discrimination in intermediate good markets, Michael Katz (1987 p. 156) concludes that "Under reasonable conditions, intermediate good price discrimination leads to higher input price being charged to *all* buyers, a result that never would arise in a corresponding model of a final good market." According to Katz this surprising result is a striking example of the difference between intermediate and final good markets. We not only show that discrimination in the final good market can also raise prices for all buyers (the result denied by Katz),

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<sup>1</sup>Joan Robinson (1933 pp. 192-5) proposed a test based on the adjusted concavities of the submarkets' demand functions at the nondiscriminating monopoly price to determine whether total output rises or falls after discrimination. However, Melvin Greenhut and Hiroshi Ohta (1976), with the help of an example, have clearly demonstrated that Robinson's proposed test is not valid under all conditions and, hence, seems to have little real value.

but further show that it can also *lower* the prices for all buyers. It is important to emphasize that when prices move in the same direction in both markets, the welfare effect of discrimination may be quite large compared to the more typical situation when price rises in one market and falls in the other. When both prices move in the same direction, the welfare effects are predictable. When both prices go down, consumer's surplus increases because of the price movement, while profit increases due to discrimination; thus, welfare must go up. When prices go up, the total output is reduced, which causes welfare to decrease.

### I. Main Results

We first give two examples to demonstrate that price discrimination can either increase prices in both markets or decrease prices in both markets. We do not restrict the demand functions to be strictly concave or convex. We only require that they are continuous and twice differentiable with negative slope throughout their range. We also do not restrict the profit functions to be strictly concave or restrict marginal revenue curves to be declining continuously (e.g., Schmalensee, 1981 p. 243). John Formby, Stephen Layson, and James Smith (1982) clearly demonstrate that the assumption of continuously declining marginal revenue may be quite restrictive, and "...demand conditions leading to upward sloping marginal revenue may indeed be pervasive" (Formby et al., 1982 p. 306). Based on their examples they conclude that, "...very simple analytical demand curves may have non-trivial upward sloping marginal revenue curves and that multiple profit equilibria for firms cannot be easily dismissed" (p. 309). We use polynomial demand functions to illustrate these results for the following reasons. First, a polynomial demand function relaxes the assumption of strict concavity or convexity, and at the same time, it also relaxes the assumptions of declining marginal revenue and concavity of profit functions. Second, any sufficiently smooth demand function can be approximated with a polynomial using Taylor's expansion with

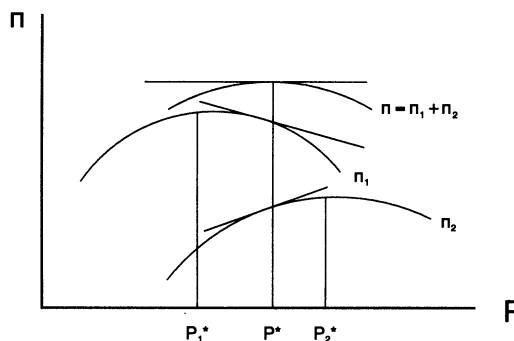


FIGURE 1. OPTIMAL PRICES WITH CONCAVE PROFIT FUNCTIONS

any desired degree of accuracy. Thus, a polynomial demand function, being more general, can provide an analytical framework for identifying other types of demand functions where similar results may hold. For the sake of illustration, we consider only two markets and constant marginal cost. Both markets are served with and without discrimination.

Figures 1 and 2 provide geometrical explanation for our results. If  $\pi_1(p)$  and  $\pi_2(p)$  are profit functions for submarkets 1 and 2, respectively, and  $\pi(p) = \pi_1(p) + \pi_2(p)$  is the uniform-price profit function, then at the single uniform profit-maximizing price,  $p^*$ ,  $\pi'(p) = \pi'_1(p) + \pi'_2(p) = 0$ . Note in Figure 1 that, at  $p^*$ ,  $\pi'_1(p) < 0$  and  $\pi'_2(p) > 0$ . Therefore, if  $\pi_1$  and  $\pi_2$  both are concave,  $p_1^* < p^* < p_2^*$ , the usual case. If, however,  $\pi_2$  has two local maxima, then it is possible that  $p_1^* < p^*$  and  $p_2^* < p^*$  or  $p_1^* > p^*$  and  $p_2^* > p^*$ . The former case is illustrated in Figure 2 (for expository purposes, the shapes of the graphs of three profit functions are more exaggerated for Example 1).

*Example 1:* Discrimination lowers prices in both markets.

Market 1 demand:

$$Q_1 = -0.25p_1^3 + 2.0001p_1^2 - 5.5p_1 + 10$$

Market 2 demand:

$$Q_2 = -0.2561p_2^3 + 2.7p_2^2 - 9.5p_2 + 12$$

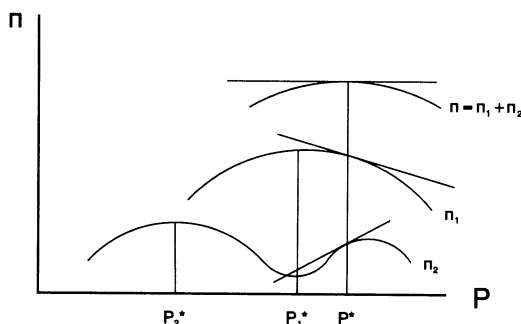


FIGURE 2. OPTIMAL PRICES WITH UNRESTRICTED PROFIT FUNCTIONS

Combined demand:

$$Q = -0.5061p^3 + 4.7001p^2 - 15p + 22$$

Marginal cost:

$$c = 0.1$$

Profit  $\pi(p)$  without discrimination:

$$\begin{aligned}\pi(p) &= -0.5061p^4 + 4.7001p^3 - 15p^2 + 22p \\ &\quad - 0.1\{-0.5061p^3 \\ &\quad + 4.7001p^2 - 15p + 22\} \\ &= -0.5061p^4 + 4.75071p^3 \\ &\quad - 15.47001p^2 + 23.5p - 2.2\end{aligned}$$

Profit maximization yields<sup>2</sup>

$$\begin{aligned}p^* &= 3.859496, \quad Q^* = 5.0232297, \\ \pi^* &= 18.884816, \quad Q_1 = 4.1931976, \\ Q_2 &= 0.8300321.\end{aligned}$$

Profit  $\hat{\pi}(p_1, p_2)$  with discrimination:

$$\begin{aligned}\hat{\pi}(p_1, p_2) &= -0.25p_1^4 + 2.0001p_1^3 - 5.5p_1^2 + 10p_1 \\ &\quad - 0.2561p_2^4 + 2.7p_2^3 - 9.5p_2^2 + 12p_2 \\ &\quad - 0.1\{-0.25p_1^3 + 2.0001p_1^2 - 5.5p_1 \\ &\quad + 10 - 0.2561p_2^3 + 2.7p_2^2 - 9.5p_2 + 12\}\end{aligned}$$

<sup>2</sup>All these values represent global optima. Detailed numerical computations for the two examples are available from the authors upon request.

Profit maximization yields

$$\begin{aligned}p_1^* &= 3.809910 & p_2^* &= 1.097442 \\ Q_1^* &= 4.2521695 & Q_2^* &= 4.4876276 \\ \pi_1^* &= 15.775169 & \pi_2^* &= 4.476147 \\ p^* &> p_1^* > p_2^* & \text{ and } \pi^* < \hat{\pi}^* = \pi_1^* + \pi_2^*.\end{aligned}$$

Note that the output in market 2 increases substantially due to discrimination.

*Example 2:* Discrimination raises prices in both markets.

Market 1 demand:

$$Q_1 = -0.25p_1^3 + 2.0001p_1^2 - 5.5p_1 + 6$$

Market 2 demand:

$$Q_2 = -0.25p_2^3 + 2.655p_2^2 - 9.5p_2 + 12.5$$

Combined demand:

$$Q = -0.5p^3 + 4.6551p^2 - 15p + 18.5$$

Marginal cost:

$$c = 0.1$$

Profit  $\pi(p)$  without discrimination:

$$\begin{aligned}\pi(p) &= -0.5p^4 + 4.6551p^3 - 15p^2 + 18.5p \\ &\quad - 0.1\{-0.5p^3 + 4.6551p^2 - 15p + 18.5\}\end{aligned}$$

Profit maximization yields

$$\begin{aligned}p^* &= 1.158687, \quad Q^* = 6.5916246, \\ \pi^* &= 6.978467, \quad Q_1 = 1.9235665, \\ Q_2 &= 4.6680581.\end{aligned}$$

Profit  $\hat{\pi}(p_1, p_2)$  with discrimination:

$$\begin{aligned}\hat{\pi}(p_1, p_2) &= -0.25p_1^4 + 2.0001p_1^3 - 5.5p_1^2 + 6p_1 \\ &\quad - 0.25p_2^4 + 2.655p_2^3 - 9.5p_2^2 + 12.5p_2 \\ &\quad - 0.1\{-0.25p_1^3 + 2.0001p_1^2 - 5.5p_1 \\ &\quad + 6 - 0.25p_2^3 + 2.655p_2^2 \\ &\quad - 9.5p_2 + 12.5\}\end{aligned}$$

Profit maximization yields

$$p_1^* = 3.013776 \quad p_2^* = 1.170637$$

$$Q_1^* = 0.7474162 \quad Q_2^* = 4.616279$$

$$\pi_1^* = 2.177805 \quad \pi_2^* = 4.942359$$

$$p^* < p_2^* < p_1^* \quad \text{and} \quad \pi^* < \hat{\pi}^* = \pi_1^* + \pi_2^*.$$

Note that the output in market 1 decreases significantly due to discrimination.

We now provide sufficient conditions under which the single nondiscriminatory price will always be lower than the maximum of the prices in the submarkets but higher than the minimum of the prices in the submarkets. These results are stated as a theorem and its corollaries.

Consider a monopolist selling a product in a single market. Let  $Q = Q(p)$  be the demand function, where  $p$  is the price. Let  $\pi$  be the profit function, and let  $C(Q)$  be the total cost function. Then,

$$\pi(p) = pQ(p) - C(Q).$$

Now suppose the monopolist segments the market into  $n$  distinguishable submarkets, where  $n \geq 2$  is a positive integer. Let  $Q_i(p)$  be the demand function in  $i$ th market,  $i = 1, 2, \dots, n$ . If the monopolist discriminates, the total profit is

$$\begin{aligned} \hat{\pi}(p_1, p_2, \dots, p_n) &= \sum_{i=1}^n \pi_i(p_i) \\ &= \sum_{i=1}^n p_i Q_i - C\left(\sum_{i=1}^n Q_i\right) \end{aligned}$$

where  $\pi_i$  is the profit in the  $i$ th submarket. Let  $(p_1^*, p_2^*, \dots, p_n^*)$  be the price vector maximizing the total profit  $\hat{\pi}(p_1, p_2, \dots, p_n)$ .

If no discrimination occurs, the profit equals  $\pi(p) = \hat{\pi}(p, p, \dots, p)$ .

**THEOREM 1:** *If for each  $i = 1, 2, \dots, n$ ,  $\pi_i(p_i)$  is, for  $p_i > 0$ , a continuous function with a global maximum at  $p_i^*$  such that, for  $p_i < p_i^*$ ,  $\pi_i(p_i)$  is strictly increasing and,*

*for  $p_i > p_i^*$ ,  $\pi_i(p_i)$  is strictly decreasing (so that the profit function is single-peaked), then  $(p_1^*, p_2^*, \dots, p_n^*)$  maximizes  $\hat{\pi}(p_1, p_2, \dots, p_n)$ . If  $p^*$  maximizes  $\pi(p) = \hat{\pi}(p, p, \dots, p)$ , then*

$$\begin{aligned} &\min\{p_1^*, p_2^*, \dots, p_n^*\} \\ &\leq p^* \leq \max\{p_1^*, p_2^*, \dots, p_n^*\}. \end{aligned}$$

**PROOF:**

For  $p < \min(p_1^*, p_2^*, \dots, p_n^*)$  consider  $\hat{\pi}(p, p, \dots, p) = \pi(p) = \sum_{i=1}^n \pi_i(p)$ . For  $p_i < p_i^*$ , each  $\pi_i(p_i)$  is an increasing function, so that for  $p < \min(p_1^*, p_2^*, \dots, p_n^*)$ ,  $\pi(p)$  is increasing. Similarly, for  $p > \max(p_1^*, p_2^*, \dots, p_n^*)$ ,  $\pi(p)$  is decreasing. Also,  $\pi(p)$  is a continuous function on the compact interval  $[\min(p_1^*, p_2^*, \dots, p_n^*), \max(p_1^*, p_2^*, \dots, p_n^*)]$ ; thus, it obtains a maximum there. The maximum is global since the values of  $\pi(p)$  for  $p$  not in  $[\min(p_1^*, p_2^*, \dots, p_n^*), \max(p_1^*, p_2^*, \dots, p_n^*)]$  are smaller than for  $p$  in the interval. Since  $p^* \in [\min(p_1^*, p_2^*, \dots, p_n^*), \max(p_1^*, p_2^*, \dots, p_n^*)]$ , this proves the theorem. Note that we make no assumption about the demand and cost functions, but only about the profit function.

Corollaries 1 and 2 are immediate.

**COROLLARY 1:** *If each profit function  $\pi_i$  is concave, then the conclusion of Theorem 1 holds.*

**COROLLARY 2:** *If each  $Q_i$ , for  $i = 1, 2, \dots, n$ , is concave (this includes the case of linear demand functions) and  $c$  is constant, then the conclusion of Theorem 1 holds (for  $p_i > c$ ).*

**COROLLARY 3:** *If the demand functions for each market are of the constant-elasticity type and marginal cost is constant, then the conclusion of Theorem 1 holds.*

**PROOF:**

Consider the  $i$ th market for  $i = 1, 2, \dots, n$ . We have  $Q_i(p_i) = p_i^{-\eta_i}$ , where  $\eta_i > 1$  is a

constant. Then

$$\pi_i(p_i) = (p_i - c)p_i^{-\eta_i}.$$

If  $c = 0$ , this function does not have a maximum, and in fact  $\lim_{p_i \rightarrow 0^+} = +\infty$ , so this case is insignificant. If  $c > 0$ , the function has a unique maximum at  $p_i^* = c[\eta_i/(\eta_i - 1)]$ , is increasing for  $p_i < p_i^*$  and decreasing for  $p_i > p_i^*$ . Thus, the assumptions of Theorem 1 are met.

Note that the profit functions  $\pi_i$  in the constant-elasticity case are not concave; their concavities change at

$$p_i = c \frac{\eta_i + 1}{\eta_i - 1}.$$

## II. Concluding Remarks

In a recent paper, as a corollary to their proposition 2, Jerry Hausman and Jeffrey MacKie-Mason (1988) conclude that, "...if marginal cost is constant, then with more than one market served under uniform pricing, at least one discriminatory price *must* be higher than the uniform price, so that a Pareto improvement is not possible" (p. 256 and fn. 9, p. 257). Our paper shows that this conclusion in general is not true. If demand functions are not restricted to a particular class, the examples presented above clearly demonstrate that third-degree price discrimination may either lower or raise price in all submarkets. Also, the traditional result that discrimination increases price in some markets and lowers it in the other markets can be obtained.

We emphasize that discrimination results in a Pareto improvement *only* when both discriminatory prices are lower than the uniform price. Hausman and MacKie-Mason (1988) show that, when both markets are served under uniform price, scale economies are necessary for Pareto improvement. They rule out the possibility of Pareto improvement in the absence of scale

economies if both markets are to be served with a uniform price. We show that this need not be the case. Pareto improvement can occur even when marginal cost is constant. Thus, Hausman and MacKie-Mason's conclusion that discrimination can result in a Pareto improvement could be generalized to include the constant marginal cost as well.

Although in this paper we have shown that both prices can move in the same direction as a result of discrimination, the mathematical and economic conditions under which it is true remain unexplored.

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