THE FEASIBLE MATCHING PROBLEM

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Abstract. Let $G = (V, E)$ be a graph and $R \subseteq E$. A matching M in G is called *R-feasible* if the subgraph induced by the M-saturated vertices does not have an edge of R. We show that the general problem of finding a maximum size R-feasible matching in G is NP-hard and identify several natural applications of this new concept. In particular, we use R-feasible matchings to give a necessary and sufficient condition for the existence of a Systems of Disjoint Representatives in a family of hypergraphs. This provides another Hall-type theorem for hypergraphs. We also introduce the concept of R-feasible (vertex) cover and combine it with the concept of R-feasible matching to provide a new formulation and approach to Ryser's conjecture.

1. Introduction and Generalities

Let $G = (V, E)$ be a graph and $R \subseteq E$. We denote an edge between $u, v \in V$ by uv. A matching M of G is a collection of pairwise disjoint edges. An independent set in G is a collection of vertices of G such that no two of them form an edge. A vertex $u \in V$ is called M-saturated if it is incident with some edge in M. Let $V(M)$ denote the set of M-saturated vertices. Let $G_R = (V, R)$ denote the spanning subgraph of G with edge set R.

Definition 1. *Let $G = (V, E)$ be a graph and $R \subseteq E$. A matching M in G is called* R-feasible *if the subgraph induced by the M-saturated vertices does not have an edge of R. In other words, $V(M)$ is an independent set in G_R.*

If $R = \emptyset$, then we recover the classic definition of a matching. The *Feasible Matching* problem consists of finding a maximum-size R-feasible matching of a graph G, for a given graph G and a given subset R of edges of G. We denote this number by $\nu(G, R)$. Since the M-saturated vertices of an R-feasible matching M of G form an independent set in G_R, we have

$$\nu(G, R) \leq \lfloor \alpha(G_R)/2 \rfloor, \tag{1}$$

where $\alpha(G_R)$ is the maximum size of an independent set of G_R; and the upper bound becomes an equality if G is a complete graph. Inequality (1) hints that in general,

Key words and phrases. Feasible matching; Feasible cover; System of Disjoint Representatives; Ryser's conjecture.

the Feasible Matching (FM) problem is as hard as the Independent Set (IS) problem; which consists of finding a maximum-size independent set, for a given graph H (denoted by $\alpha(H)$). In fact, we have the following theorem.

Theorem 2. *The* FM *problem is* NP*-hard.*

Proof. Let $G = (V, E)$ be a graph and let $R \subseteq E$.

We show that the IS problem can be reduced (in polynomial time) to the FM problem. Let H be a given graph and supposed that we want to find $\alpha(H)$. Let W be a set such that $W \cap V(H) = \emptyset$ and $|W| = |V(H)|$. We construct an auxiliary graph $G = G(H)$ with vertex set $V(G) = W \cup V(H)$ and edge set

$$E(G) = E(H) \cup \{hw : \ h \in V(H) \text{ and } w \in W\}.$$

If we set $R = E(H)$, then it is straightforward to check that $\alpha(H) = \nu(G(H), R)$. Moreover, the construction of G from H can be clearly done in time that is polynomial in $|V(H)|$. Consequently, the FM problem is as hard as the IS problem, which concludes the proof. □

The decision problem corresponding to the FM problem is: "Given a graph G, a subset of edges $R \subseteq E(G)$, and a constant k; is there an R-feasible matching of G of size k?" It follows from Theorem 2 that this decision problem is NP-complete.

For each edge $e \in E$, we associate a binary variable c_e. Then we have the following Integer-Linear-Programming (ILP) formulation of the Feasible Matching (FM) problem:

$$\text{Maximize} \sum_{e \in E} c_e \text{ subject to} \qquad (L_1)$$

$$\sum_{x' \in V;\, xx' \in E} c_{xx'} \leq 1, \quad \text{for each } x \in V \qquad (L_2)$$

$$\sum_{x' \in V;\, xx' \in E} c_{xx'} + \sum_{y' \in V;\, yy' \in E} c_{yy'} \leq 1, \quad \text{for each } xy \in R \qquad (L_3)$$

$$c_e \in \{0, 1\}, \quad \text{for each } e \in E \qquad (L_4).$$

Observe that the linear constraints in (L_3) ensure that the resulting matching is R-feasible. This ILP formulation might be useful in finding $\nu(G, R)$ for small examples.

We now introduce the related definition of *R-feasible cover.*

Definition 3. *Let $G = (V, E)$ be a graph and $R \subseteq E$. A subset $C \subseteq V$ is an R-*feasible cover *of G if for any edge $xy \in (E - R)$ at least one of the following conditions holds:*
(i) $\{x, y\} \cap C \neq \emptyset$;
(ii) there exists an edge $x'y' \in R$ such that $\{x, y\} \cap \{x', y'\} \neq \emptyset$ and $\{x', y'\} \cap C \neq \emptyset$.

If $R = \emptyset$, then we recover the classic definition of a vertex cover. The *Feasible Cover* problem consists of finding the minimum number of vertices in any R-feasible cover for the pair (G, R). We denote this number by $\tau(G, R)$. The Feasible Cover problem is clearly NP-complete since the the classic Vertex Cover problem is NP-complete.

The rest of the paper is organized as follows. In Section 2, we give a Hall-type theorem for feasible matchings and bounds on feasible coverings. In Section 3, we use R-feasible matchings to characterize Systems of Disjoint Representatives and give several other applications. In Section 4, we combine the concepts of R-feasible matching and R-feasible cover to provide a new formulation and approach to Ryser's conjecture. Finally, in Section 5, we discuss some open questions.

2. Some Results on Feasible Matchings and Feasible Coverings

Applications of the results in this section will be given in Sections 3 and 4.

Let $G = (V, E)$ be a graph and $R \subseteq E$. We say that G is R-*bipartite* with bipartite sets X and Y if the graph $G' = (V, E - R)$ is bipartite (in the usual sense) with bipartite sets X and Y. For $W \subseteq X$, let $N(W) = \{y \in Y : \; wy \in E \text{ for some } w \in W\}$.

In what follows, we give a necessary and sufficient condition for the existence of an R-feasible matching of G that saturates all the vertices in X.

Theorem 4. *Let $G = (V, E)$ be a graph and $R \subseteq E$. Suppose that G is R-bipartite with bipartite sets X and Y. Then G has an R-feasible matching that saturates all the vertices in X if and only if X is an independent set in G_R and there exists $S \subseteq Y$ such that S is independent in G_R and for any $W \subseteq X$, we have $|S \cap N(W)| \geq |W|$.*

Proof. Necessity. If M is an R-feasible matching that saturates all the vertices in X, then clearly X is an independent set in G_R and the set S of vertices matched to X in M is an independent set in G_R. Hence, it follows from Hall's theorem [5, 10] that $|S \cap N(W)| \geq |W|$ for any $W \subseteq X$.

Sufficiency. If such a set S exists, then by Hall's Theorem, there is a matching from X to S that saturates all the vertices in X. Since both X and S are independent sets in G_R, such a matching is R-feasible. □

Remark 5. *If $R = \emptyset$ in Theorem 4, then $G = G'$ is a bipartite graph and we obtain Hall's Theorem.*

Let G, R, X, and Y be as in Theorem 4 above. For any subset $X' \subseteq X$, we define the deficiency of X' as follows

$$\operatorname{def}(X') = \min_S \{d \in \mathbb{N} : \; |S \cap N(W)| \geq |W| - d \; \text{ for any } W \subseteq X'\}, \tag{2}$$

where the minimum is over all sets $S \subseteq N(X')$ that are independent in $G_R = (V, R)$ and $\mathbb{N}$ is the set of all natural numbers.

We then obtain the following deficiency version of Theorem 4.

Corollary 6. *Consider a graph $G = (V, E)$ and let $R \subseteq E$. Suppose that G is R-bipartite with bipartite sets X and Y. If X' is an independent subset of X, then G has an R-feasible matching of size $|X'| - \operatorname{def}(X')$.*

Proof. Add $\operatorname{def}(X')$ vertices to Y and connect each of these vertices to every vertex in X' by an edge. Then apply Theorem 4. □

An R-feasible matching of G is called a *perfect R-feasible matching* of G if every vertex of G is M-saturated. The following proposition follows easily form this definition and we omit its proof.

Proposition 7. *Let $G = (V, E)$ be a graph and $R \subseteq E$. Then G has a perfect R-feasible matching if and only if $R = \emptyset$ and G has a perfect matching.*

For any connected component U in the graph G_R, the edges in U do not need to be covered in an R-feasible cover of G. This leads to the following definitions. Let $G\backslash R$ be the graph with

$$V(G\backslash R) = \{U \subseteq V : \ U \text{ is a connected component in } G_R\}$$

and

$$E(G\backslash R) = \{UU' : \ U, U' \in V(G\backslash R) \text{ and there exists } u \in U, \ u' \in U' \text{ with } uu' \in E - R\}.$$

For any subset $C \subseteq V$, we define

$$C\backslash R = \{U \in V(G\backslash R) : \text{there exists } c \in C \text{ with } c \in U\}.$$

Proposition 8.

$$\nu(G\backslash R, \emptyset) \leq \tau(G\backslash R, \emptyset) \leq \tau(G, R) \leq 2\nu(G, R).$$

Moreover, if C is an R-feasible cover of G, then $C\backslash R$ is a vertex cover of $G\backslash R$.

Proof. Since every vertex cover of $G\backslash R$ contains a vertex from each edge of a maximum-sized matching in $G\backslash R$, then $\nu(G\backslash R, \emptyset) \leq \tau(G\backslash R, \emptyset)$.

Next we show that $\tau(G, R) \leq 2\nu(G, R)$. Let M be a maximum R-feasible matching in G and let $V(M)$ denote the set of M-saturated vertices. We claim that $V(M)$ is an R-feasible cover for G. In fact, for each edge xy in G such that $\{x, y\} \cap V(M) = \emptyset$, there exists $x' \in V(M)$ such that $xx' \in R$ or there exists $y' \in V(M)$ such that $yy' \in R$. Since otherwise, $M \cup \{xy\}$ together with M is an R-feasible matching, contradicting the assumption that M is a maximum R-feasible matching. Therefore,

$$\tau(G, R) \leq |V(M)| = 2\nu(G, R).$$

Next, we show that $\tau(G\backslash R, \emptyset) \leq \tau(G, R)$. Let C be a minimum R-feasible cover of G. Then it follows from the definition of $C\backslash R$ that $|C\backslash R| \leq |C|$. If $C\backslash R$ is a vertex cover of $G\backslash R$, then $\tau(G\backslash R, \emptyset) \leq |C\backslash R| \leq |C| = \tau(G, R)$. Hence, to finish the proof, it suffices to show that $C\backslash R$ is a vertex cover of $G\backslash R$.

Let XY be an edge in $G\backslash R$. By the definition of $E(G\backslash R)$, there exists $x \in X$ and $y \in Y$ such that $xy \in E(G) - R$. Since C is an R-feasible cover of G, then at least one of the following possibilities holds: (i) $x \in C$; (ii) $y \in C$; (iii) there exists $x' \in C$ such that $xx' \in R$; and (iv) there exists $y' \in C$ such that $yy' \in R$. We only discuss (i) and (iii) since (ii) and (iv) can be handled in a similar manner. If $x \in C$, then $x \in X \in C \backslash R$ by the definition of $C \backslash R$. If there exists $x' \in C$ such that $xx' \in R$, then $x, x' \in X \in V(G \backslash R)$ by the definition of $V(G \backslash R)$. Since $x' \in C$ and $x' \in X$, we have $X \in C \backslash R$ by the definition of $C \backslash R$. □

3. Applications of Feasible Matchings

We point out an application of Theorem 4 to the system of disjoint representatives and several other applications of feasible matchings.

Application 1: Systems of Disjoint Representatives

In this section, we identify a hypergraph with its set of (hyper)edges. Let $\mathcal{A} = \{H_1, ..., H_m\}$ be a family of hypergraphs (i.e., H_i is the set of edges of the i-th hypergraphs, $1 \leq i \leq m$). A System of Disjoint Representatives (SDR) for $\mathcal{A}$ is a function $f : \mathcal{A} \to \bigcup_{i=1}^{m} H_i$ such that $f(H_i) \in H_i$ for all i and $f(H_i) \cap f(H_j) = \emptyset$ whenever $i \neq j$. A necessary and sufficient condition for the existence of an SDR in $\mathcal{A}$ was proved by Aharoni and Haxell [2] using topological methods. Before stating their theorem, we introduce some relevant notation.

Let H be the set of edges of some hypergraph. A subset $F \subseteq H$ is *pinned* by another set K of edges if every edge in F has a non-empty intersection with some edge in K. For any subfamily $\mathcal{B} \subseteq \mathcal{A}$, let $\cup\mathcal{B} = \bigcup_{H \in \mathcal{B}} H$.

Theorem 9 (Aharoni-Haxell [2])**.** *A family of hypergraphs $\mathcal{A}$ has a system of disjoint representatives if and only if for each subfamily $\mathcal{B} \subseteq \mathcal{A}$ there exists an assignment of a matching $M_{\mathcal{B}}$ in $\cup\mathcal{B}$ which satisfies the condition that $M_{\mathcal{B}}$ cannot be pinned by fewer than $|\mathcal{B}|$ edges from the set of edges $\bigcup\{M_{\mathcal{C}} : \mathcal{C} \subseteq \mathcal{B}$ and $M_{\mathcal{C}}$ is a matching in $\cup\mathcal{C}\}$.*

We will use Theorem 4 and the following construction to devise another necessary and sufficient condition for the existence of an SDR in a family of hypergraphs.

Construction 1: *Given a family of hypergraphs $\mathcal{A} = \{H_1, \ldots, H_m\}$, we let $G = G(\mathcal{A})$ denote the auxiliary graph of $\mathcal{A}$ with vertex set $V = X \cup Y$ and edge set $E = E_0 \cup R$, given below.*

$$X = \{1, 2, ..., m\}, \quad Y = H_1 \cup \ldots \cup H_m,$$
$$E_0 = \{ih_i : 1 \leq i \leq m \text{ and } h_i \in H_i\},$$
$$R = \{h_i h_j : h_i \in H_i,\ h_j \in H_j,\ \text{and } h_i \cap h_j \neq \emptyset\}.$$

It follows from Construction 1 that there is a one-to-one correspondence between an SDR of $\mathcal{A}$ and an R-feasible matching of $G = G(\mathcal{A})$ saturating X. Note that the auxiliary graph $G(\mathcal{A})$ given in Construction 1 is R-bipartite and X is an independent set in G_R.

Applying Theorem 4, we have the following characterization of the existence of an SDR of a family of hypergraphs.

Corollary 10. *Let $\mathcal{A}$ be a family of hypergraphs and let $G = G(\mathcal{A})$ be its auxiliary bipartite graph with bipartite sets X and Y and edge set $E = E_0 \cup R$ (as given by Construction 1). Then $\mathcal{A}$ has a system of disjoint representatives if and only if there exists $S \subseteq Y$ such that S is independent in G_R and for any $W \subseteq X$, we have $|S \cap N(W)| \geq |W|$.*

Application 2: Hypergraph Matchings.

Consider an $r-$uniform hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ which satisfies the following property:
$(*)$ $\mathcal{V}$ can be partitioned into two sets A and B such that for any $e \in \mathcal{E}$, we have $|e \cap A| = 1$ and $|e \cap B| = r - 1$.

Note that the above condition implies that $\mathcal{H}$ is 2-colorable, i.e., we can assign 2 colors to its vertices such that none of its edges is monochromatic. Let

$$\mathcal{C} = \{U \subseteq \mathcal{V} :\ e \cap B = U \text{ for some } e \in \mathcal{E}\},$$

$$E_0 = \{aU :\ a \in A,\ U \in \mathcal{C},\ \{a\} \cup U \in \mathcal{E}\},$$

and

$$R = \{UW :\ U, W \in \mathcal{C} \text{ and } U \cap W \neq \emptyset\}.$$

We define an auxiliary graph $G = G(\mathcal{H})$ with vertex set $V = A \cup \mathcal{C}$ and edge set $E = E_0 \cup R$. Then, there is a one-to-one correspondence between matchings in the hypergraph $\mathcal{H}$ and an R-feasible matching of G. In particular, the maximum size of a matching in $\mathcal{H}$ is the maximum size of an R-feasible matching in G.

Application 3: Network Channel Assignment with Interference.

Consider a wireless network with a set of customers A and set of (signal) towers B. We put an edge between $a \in A$ and $b \in B$ if customer a can connect to tower b. At any point of time, we want to find a maximum-size matching in the bipartite graph G_0 with partite sets A and B and edge set E_0. In practice, we have a multichannel network, i.e. a tower b can accept a number m_b of connections. We can incorporate this condition by creating m_b copies of the vertex b in B and find a maximum-size matching in the resulting graph. Moreover, let R be the set of all edges b_1b_2 such that there is an interference between the channels used by towers b_1 and b_2. Then given the graph G with $V(G) = V(G_0)$ and $E(G) = E_0 \cup R$, we are interested in finding the maximum-size R-feasible matching, which corresponds to a maximum size interference-free channel assignment.

Application 4: Scheduling with constraints.

Consider a scheduling problem in which we have a set B of musical bands to be scheduled at time periods T for a music festival. Assuming that each band can only play at some specified times, we can try to find a maximum-size assignment of bands to time periods. Moreover, we add a red edge between two bands if they cannot both be scheduled to perform. Similarly, we may have some restrictions about the time periods represented by blue edges. This problem can be modeled by the problem of finding an R-feasible matching, where R is the set of red and blue edges.

4. Application to Ryser's conjecture

For a hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$, the vertex cover number, denoted by $\tau(\mathcal{H})$, is the minimum size of a vertex set that intersects every edge. The matching number $\nu(\mathcal{H})$, is the maximum size of a set of pairwise-disjoint edges. A hypergraph is called r-uniform if all its edges have size r. A hypergraph is called r-partite if its vertex set can be partitioned into r parts, and every edge contains precisely one vertex from each part. Note that an r-partite hypergraph must be r-uniform. Since every edge in a hypergraph $\mathcal{H}$ intersects some vertex of $V(M)$, where M is a maximum matching in $\mathcal{H}$, then

$$\tau(\mathcal{H}) \leq |V(M)| = r\nu(\mathcal{H}) \tag{3}$$

for any r-partite hypergraph $\mathcal{H}$.

Ryser conjectured that for $r \geq 2$, every r-partite hypergraph $\mathcal{H}$ satisfies

$$\tau(\mathcal{H}) \leq (r-1)\nu(\mathcal{H}). \tag{4}$$

For $r = 2$, Conjecture 4 holds by König's theorem [4]. However, for $r \geq 3$, this conjecture turns out to be very difficult. For $\nu(\mathcal{H}) = 1$ and $3 \leq r \leq 5$, Conjecture 4 has been proved by Tuza [8, 9]. For $r = 3$ and all values of $\nu(\mathcal{H})$, it has been proved by Aharoni [1]. Füredi [3] proved the so-called fractional version of the conjecture, i.e., $\tau^*(\mathcal{H}) \leq (r-1)\nu(\mathcal{H})$, where $\tau^*(\mathcal{H})$ is the *fractional covering number* of $\mathcal{H}$.

For an R-bipartite graph $G = (V, E)$ with R-bipartite sets $V = X \cup Y$, we define a weight function $w : V \to \{1, r-1\}$ such that $w(v) = 1$ for $v \in X$ and $w(v) = r - 1$ for $v \in Y$. Let $\tau_w(G, R)$ be the *minimum weight* R-feasible cover of the vertex-weighted graph (G, w) and let $\nu(G, R)$ be the maximum size R-feasible matching in the unweighted graph G.

Construction 2: *Given an r-partite hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the union of r disjoint parts V_i, $1 \leq i \leq r$, let*

$$X = V_1,$$

$$Y = \{y : \ e \cap \cup_{i=2}^{r} V_i = y \text{ for some } e \in \mathcal{E}\},$$

$$E = \{xy : \ x \in X, \ y \in Y, \ \{x\} \cup y \in \mathcal{E}\},$$

and

$$R = \{yy' : \ y, y' \in Y \text{ and } y \cap y' \neq \emptyset\}.$$

We define an auxiliary graph $G = G(\mathcal{H})$ with vertex set $V = X \cup Y$ and edge set $E \cup R$.

Observe that there is a one-to-one correspondence between matchings in the hypergraph $\mathcal{H}$ and an R-feasible matching of G. In particular, the maximum size of a matching in $\mathcal{H}$ is the maximum size of an R-feasible matching in G, i.e.,

$$\nu(\mathcal{H}) = \nu(G, R). \tag{5}$$

Proposition 11.

$$\tau(\mathcal{H}) \leq \tau_w(G, R).$$

Proof. Let T be a minimum weight R-feasible cover of G. Let $T_{\mathcal{H}}$ be the set of vertices in $\mathcal{H}$ obtained by recovering vertices in T, i.e., every vertex in $T \cap X$ will be kept and every vertex in $T \cap Y$ will be recovered to $r-1$ vertices in $\cup_{i=2}^{r} V_i$. We claim that $T_{\mathcal{H}}$ is a vertex cover of $\mathcal{H}$. Indeed, for every edge $e = \{v_1, v_2, \cdots, v_r\} \in E(\mathcal{H})$ with $v_i \in V_i$, the corresponding pair $\{v_1, y\}$, where $y = \{v_2, \cdots, v_r\}$, is an edge in G. Since T is a cover of G, then either $v_1 \in T$, or $y \in T$, or there is $y' \in T$ such that $y \cap y' \neq \emptyset$. In each case, edge e intersects $T_{\mathcal{H}}$ and the claim is verified. Moreover, we clearly have $|T_{\mathcal{H}}| = \tau_w(G, R)$. Hence

$$\tau(\mathcal{H}) \leq |T_{\mathcal{H}}| = \tau_w(G, R).$$

□

Proposition 12.

$$\tau_w(G, R) \leq r\nu(G, R).$$

Proof. Let M be a maximum R-feasible matching in G. We claim that the vertex set $V(M)$ is an R-feasible vertex cover for G. In fact, for each edge xy in G with $\{x, y\} \cap V(M) = \emptyset$, either there exists $x' \in V(M)$ such that $xx' \in R$, or there exists $y' \in V(M)$ such that $yy' \in R$. Since otherwise, $M \cup \{xy\}$ is an R-feasible matching, contradicting the assumption that M is a maximum R-feasible matching. Since every vertex in $X \cap M$ has weight 1 and every vertex in $Y \cap M$ has weight $r-1$, then

$$\tau_w(G, R) \leq r|M| = r\nu(G, R).$$

□

Propositions 11, 12, and equation (5) imply the bound in (3). So if G and R are such that Proposition 12 can be improved to $(r-1)\nu(G, R) \geq \tau_w(G, R)$, then Ryser's conjecture (inequality (4)) holds for corresponding r-partite hypergraphs. This yields a new approach to Ryser's conjecture that consists of finding families of subgraphs G and R for which Ryser's conjecture holds, as opposed to the current approach (see [1, 8, 9]) of finding r or ν for which the conjecture holds.

5. Concluding Remarks

After Theorem 2, we have established that the decision version of Feasible Matching problem is in general NP-complete. This leads to the following related complexity question.

Question 13. *Let $G = (V, E)$ be a graph, $R \subseteq E$, and let $G_R = (V, R)$ be the spanning subgraph of G with edge set R. Characterize the subgraphs G_R for which there exists a polynomial time algorithm that finds a maximum-size R-feasible matching in G.*

If R does not contain P_3 (the path on 3 vertices) as an induced graph, then R is a disjoint union of cliques. In this case, it is easy to see that $\nu(G, R) = \nu(G\backslash R, \emptyset)$, where $\nu(G\backslash R, \emptyset)$ is the (classic) maximum size matching in the graph $G\backslash R$. Since this latter number can be computed in polynomial time, $\nu(G, R)$ can also be computed in polynomial time in this case. What about other classes of graphs G_R?

Finally, it would be interesting to define a general framework within which one can define and study a hybrid problem obtained by combining a polynomial-time problem and an NP-hard problem in a suitable way. In this sense, the maximum size feasible matching problem is a hybrid problem that results from the maximum size matching problem and the maximum size independent set problem. We will pursue these questions in subsequent projects.

Acknowledgements: The authors thank the referees for carefully reading the manuscript and for their detailed comments which helped improve the paper.

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