ISMAA Annual Meeting
March 30-31, 2012
Abstracts
Plenary Sessions

Michael Axtell. A Picture's Worth a Thousand Words: Zero-Divisor Graphs and Rings. Friday, 12:50 p.m.

Zero-divisor graphs are a quite recent development in the study of commutative rings. In the last ten years, they have been extensively studied and have yielded several surprising results. A good deal of the work has been conducted by undergraduates. This talk will provide an overview of the subject, with a particular focus on the role played by undergraduate researchers. In addition, some recent results in the area will also be discussed.

Joshua Holden. *Braids, Cables, and Cells: Representing Art and Craft with Mathematics and Computer Science.* Friday, 7:30 p.m.

The mathematical study of braids combines aspects of topology and group theory to study mathematical representations of one-dimensional strands in three-dimensional space. These strands are also sometimes viewed as representing the movement through a time dimension of points in two-dimensional space. On the other hand, the study of cellular automata usually involves a one- or two-dimensional grid of cells which evolve through a time dimension according to specified rules. This time dimension is often represented as an extra spacial dimension.

Therefore, it seems reasonable to ask whether rules for cellular automata can be written in order to produce depictions of braids. The ideas of representing both strands in space and cellular automata have also been explored in many artistic media, including drawing, sculpture, knitting, crochet, and weaving, and we will touch on some of these.

Holly Gaff. *Impact of migration patterns on tick-borne disease expansion.* Saturday, 8:30 a.m.

Mathematical models can be used to explore tick-borne pathogen dynamics, quantify risk of tick-borne disease and identify optimal strategies to reduce that risk. Questing ticks have been collected from May 2009 to the present at ten separate locations in southeastern Virginia. This study identified the invasion of two new tick species from the south. One tick species has exhibited a continuous, diffusive type invasion while the other has a series of long distance dispersal events. An agent-based model has been parameterized using these data to explore the future disease dynamics in the region as a result of each type of invasion.

Annalisa Crannell. *Math and Art: The Good, the Bad, and the Pretty.* Saturday, 12:10 p.m.

How do we fit a three-dimensional world onto a two-dimensional canvas? Answering this question will change the way you look at the world, literally: we'll learn where to stand as we view a painting so it pops off that two-dimensional canvas seemingly out into our three-dimensional space. In this talk, we'll explore the mathematics behind perspective paintings, which starts with simple rules and will lead us into really lovely, really tricky puzzles. Why do artists use vanishing points? What's the difference between 1-point and 3-point perspective? Why don't your vacation pictures don't look as good as the mountains you photographed? Dust off those old similar triangles, and get ready to put them to new use in looking at art!

General Sessions

M. Vali Siadat with Euguenia Peterson, Cyrill Oseledets, Ming-Jer Wang, Guo Quan (Jack) Zhang. *What are the bestways to stimulate students' intellectual curiosity and affect deep learning? What is the role of modern technology?*.Richard J. Daley College. Friday, 2:15 p.m.

In this presentation we discuss the impact of an innovative teaching strategy (Keystone) along with other conventional approaches on student learning in developmental mathematics. Our three-year long research study measured student outcomes on a national standardized test (COMPASS) as well as on departmentally-constructed internal examinations. The study also investigated the impact of learning technologies on traditional and non-traditional teaching approaches and measured their effects on student learning outcomes.

Dan Hrozencik with Timothy D. Comar and Graham Atkinson *Undergraduate Research Projects Focusing on the Comparison of Boolean and Continuous Dynamics of Gene Networks.* Chicago State University. Friday, 2:15 p.m.

An active area of current research centers on the development of mathematical models used to test the workings of gene networks. The standard models are either continuous (differential equations) or discrete (Boolean). Recent research indicates that these different models lead to vastly different conclusions regarding the operation of n-gene networks, where n = 2, 3 or 4. The authors outline several projects they have designed to introduce undergraduate students to these models and to compare their results.

Vince Matsko. *Teaching Mathematics as Art.* IMSA. Friday, 2:45 p.m.

What is Mathematics? Each mathematician has his or her response to this question. Given the way mathematics is currently taught, how might a high school or college graduate respond to the same question? Likely few, if any, would consider mathematics as art. Paul Lockhart, in his thought-provoking essay, *A Mathematician's Lament,* suggests that this is a serious shortcoming of mathematics education. Unless we teach mathematics as a creative endeavor, few students will graduate from high school or college with a genuine appreciation for mathematics. As a result, we must continue to endure the consequences of being educators in a culture discouragingly apathetic to the art of mathematics.

Jeremy Alm. *Rigor in College Algebra?!* Illinois College. Friday, 3:30 p.m.

College Algebra courses are notoriously problematic, for reasons that need not be enumerated here. In this talk, I will discuss my (controversial!) attempt to address the problem by striking at the root, the arithmetic of rational numbers: fractions! By giving fraction arithmetic a proper treatment, the transition from arithmetic to algebra is made smoother for students. College Algebra can and should be brought up to the standards of 21st-century mathematics. (Objections and arguments are encouraged!)

Ken Clements with Neirda Ellerton. *Learning Mathematics at Illinois State Normal University.* Illinois State University. Friday, 3:30 p.m.

Illinois State Normal University (ISNU) was the state's first public university, and during its first 50 years (1857-1907) its Mathematics Department was regarded as the University's strongest department. Important development in mathematics education occurred at the University, which could have been linked to the fact that three early Heads of ISNUs Mathematics Department (Richard Edwards, John Cook, and David Felmey) would be subsequently be appointed as ISNU Presidents. Another Head of Department, Thomas Metcalf, achieved an unenviable reputation as an outstanding teacher. This presentation will discuss the level and quality of mathematics taught at ISNU, the lack of relevant qualifications of Mathematics Department Faculty, the participation of women in ISNU mathematics classes, and the methods of teaching that were used at the University.

Neirda Ellerton with Ken Clements. *Misconceptions About Young Abraham Lincoln's School Mathematics Achievements.* Illinois State University. Friday, 4:00 p.m.

When he was aged between 15 and 17, living in rural Indiana, in the 1820s, young Abraham Lincoln prepared at least one ciphering book in which he set out answers to mathematical questions. This presentation will summarize what is known about his efforts to learn mathematics, and about the content of the mathematics that he learnt. Direct quotations from entries made by Lincoln will be presented. It will be argued that there is no evidence that Lincoln was strong mathematically, or that he was any more diligent than his fellow students in his efforts to master the mathematics that he wrote in his cyphering book.

Todd Oberg. *A Calculus of Mathematics Certification.* Illinois College. Saturday, 9:45 a.m.

Since this past fall there have been changes at ISBE with regard to certification, and there will be more changes to come over the next two-three years. This session will summarize the changes that have already occurred and will provide a sense of changes yet to come based on information from ISBE. While general information will be provided, the emphasis will focus on issues related to Mathematics. If time allows, there will be an opportunity for discussion.

Andrew Leahy. *Using SMS Texts to Communicate with Students in Mathematics Courses.* Knox College. Saturday, 9:45 a.m.

New technology changes the way that faculty interact with students. E-mail and course websites are two obvious examples in recent years of ways that have emerged for faculty to communicate with their students. However, both require that students have access to a computer or a web-enabled portable device–technology that isn't quite ubiquitous yet. On the other hand, many students are almost constantly using their cell phones to exchange SMS text messages about their personal lives. This talk will describe a simple software gateway we've written to bridge the web and SMS texting worlds, which allow us to communicate important and timely information about our mathematics courses to enrolled students via SMS text messages on their cell phones.

Ellen Ziliak. *Coset Enumeration and Modified Coset Enumeration an Approach to Group Extensions* Benedictine University. Saturday, 9:45 a.m.

In this talk I will discuss an approach to doing arithmetic in group extensions using a method called augmented Coset Enumeration. A group extension is a group built from two smaller groups namely the normal subgroup and the quotient group. I will then illustrate some of the difficulties of using this method when computing with groups of a certain size. Finally I will describe a new modified method developed using Cayley Graphs which is a graphical representation of the multiplication table for a group. This new method can be used to modify the traditional coset enumeration method and this modification should lead to more efficient computation in a group extension.

David Barker with Saad El-Zanati, Wendy O'Hanlon. *Implementing Mathematics Research Experiences into Teacher Preparation Programs* Illinois State University. Saturday 10:15 a.m.

Preservice teachers rarely have the opportunity to engage in work that leads to mathematical discoveries and the subsequent opportunities to communicate these discoveries to colleagues. The Research Experiences for Undergraduates (REU) Site for Secondary Mathematics Teachers at Illinois State University was designed in response to the national need for highly qualified mathematics teachers and the call for prospective mathematics teachers to experience scholarship in mathematics. Pre-service teachers are recruited to take part in the development of original research from unsolved mathematics problems. The goals of the program are to expand the prospective and practicing mathematics teachers' views of mathematics as a dynamic endeavor, to provide opportunities for mathematical discovery, and to enhance their content and pedagogical content knowledge. This presentation will describe the REU program, sample research problems, components designed to help teachers' translate their research experience to the classroom, and suggestions for implementation. In addition, data will be provided that characterizes the changes in teacher beliefs as a result of experiencing mathematics research.

Ryan Bunge with S. I. El-Zanati, J. Mudrock, C. Vanden Eynden, and W. Wannasit. On λ-fold Rosa-type Labelings and Cyclic Graph Decompositions. Illinois State University. Saturday 10:15a.m.

A labeling (or valuation) of a graph G is an assignment of integers to the vertices of G subject to certain conditions. A hierarchy of graph labelings was introduced by Rosa in the late 1960s. Rosa showed that certain basic labelings of a graph G with n edges yielded cyclic G-decompositions of K_{2n+1} while other stricter labelings yielded cyclic G-decompositions of K_{2nx+1} for all natural numbers x. Until recently, labelings of the latter type were defined only for bipartite and almost-bipartite graphs. We report on recent progress in this area and show how these concepts extend to λ-fold labelings. We show that some λ-fold labelings of a graph G with n edges lead to cyclic G-decompositions of the λ-fold complete graph ${}^{\lambda}K_{\frac{2n}{\lambda}+1}$ while others lead to cyclic G-decompositions of ${}^{\lambda}K_{\frac{2n}{\lambda}x+1}$ for every positive integer x.

Jim Olsen. Common Core Math Standards and Teacher Education Session. Western Illinnois University. Saturday, 11:00 a.m.

The new Common Core Math Standards (CCMS) are being implemented in Illinois' K-12 schools this year (and have been adopted in 46 of the 50 states). In this session we will look at the CCMS (Mathematical Practices and content standards) and how they will affect teacher education at the college level. The new (grade 3-11) state assessments over these new standards are slated for 2014-15. The assessments are being developed by the PARCC Consortium (of which Illinois is a member). In addition, we will look at some resources and materials that are available, and being developed, which we can use (and use in teacher education courses) to understand the CCMS and help our

students get ready to teach them. I'd also like to take a few minutes to discuss the Illinois Basic Skills Test and have people share what they are doing to prepare their teacher education students for the test. This is one of a couple teacher education sessions. Todd Oberg will also do a session and will look at changes coming in Illinois teacher certification and licensure.

Wesley Calvert. *Algebra and Logic of Random Variables.* Southern Illinois University. Saturday, 11:00 a.m.

Perhaps surprisingly, randomness is one of the most computationally useful things in the world. An algorithm that uses some randomness and produces a very probably correct solution is often much faster than the best known classical method. But classical deterministic computation has a very close relationship with logic, which is important for programming language design and for software verification. In this talk we discuss what could possibly replace this relationship in a context with randomness.

Anthony DeLegge. *An Epidemic Model with a Multi-Stage Vaccine.* Benedictine University. Saturday, 11:00 a.m.

In 2009, a strain of the H1N1 flu virus caused a worldwide pandemic. Many people got sick, and public health officials were very worried about the lack of immunity that most people had to it, especially if the virus mutated to a more lethal form. Fortunately, a vaccine was developed for the disease; however, the first results indicated that two doses of the vaccine would be required to be fully immune. This ended up not being the case except for children under the age of 10, but it's not surprising why so many people panicked at the initial report: the nation's vaccine supply would be effectively cut in half, and it was possible that people could get sick before receiving the second vaccine, or even forget to get it.

In this talk, we will construct an epidemic model with a multi-stage vaccine implemented in order to address the following question: If a multi-stage vaccine is required, will it be effective in stopping the spread of a disease?

William Green with Howard Dwyer. *Integrating Factors and Repeated Roots of the Characteristic Equation.* Eastern Illinois University. Saturday, 11:30 a.m.

Most texts on elementary differential equations solve homogeneous constant coefficient linear equations by introducing the characteristic equation; once the roots of the characteristic equation are known the solutions to the differential equation follow immediately, unless there is a repeated root. In this talk we show how an integrating factor can be used to find all of the solutions in the case of a repeated root without depending on an assumption about the form that these solutions will take. We also show how an integrating factor can be used to explain the "extra" power of t which appears in the trial form of the solution when using the method of undetermined coefficients on a non-homogeneous equation in the case where the right hand side is a polynomial multiple of the corresponding homogeneous solution.

Andrew Mansheim. *Easy as 2 Plus 2: A Simple Yet Thought-Provoking Investigation.* Western Illinois University. Saturday, 11:30 a.m.

Everyone knows $2 + 2 = 4$ and $2x2 = 4$. But, for what other pairs of numbers is the product equal to the sum? This question may seem straightforward, but many college students can find only a few solutions and are surprised to discover that there are actually infinitely many such pairs. Further analysis of this problem leads students from arithmetic to algebra and allows them to see the value of variables and expressions.

The goal of the presentation is to begin with an elementary arithmetic concept and extend this to generate a broader algebraic analysis appropriate for students in a general education mathematics course. The presenter will share how students typically respond to this problem and how they are challenged to provide specific and general solutions. The presenter will demonstrate how students may approach this problem and how this algebra investigation can result in an application of rational functions which then expands into inverses and composition of functions. Similar problems will be shared that help extend the use and understanding of algebra.

Student Sessions

Michael Carlyle. *Direct Sums for Graph Automorphisms.* Augustana College. Friday, 2:15 p.m.

A graph automorphism is a mapping from and to itself that preserves vertex adjacency. We will present examples of graphs whose automorphism group is easy to see, and then we will show graphs whose automorphism group is a direct sum. We will also show examples where these techniques are inadequate.

Brian Hofman. *Cryptography using Elliptic Curves.* Trinity Christian College. Friday, 2:15 p.m.

In this presentation we will introduce elliptic curve encryption, using elliptic curves produced mod a prime. We will describe the form and structure of elliptic curves as well as the algorithm for adding points, which is useful for generating the points on the curve. Hasse's theorem for the number of points on an elliptic curve mod a prime will be discussed, along with how that theorem can be used to create a curve with the desired number of points for encryption. An example of encryption with a specific elliptic curve will be demonstrated, and, if time allows, Pollard's Rho method of solving the elliptic curve discrete logarithm problem will be presented.

Sandra Tovalin-Schmidt. *Vibrational Spectroscopy and Group Theory.* Benedictine University. Friday, 2:35 p.m.

Infrared and Raman spectroscopy are two types of vibrational spectroscopy that detect particular molecular vibrations using light. The number and type of vibrational energy levels of a molecule depend on its molecular symmetry, thus this is a direct application of group theory. To understand vibrations of molecules, point groups will be defined and character tables of those point groups will be developed. Through these two key steps, character tables can then be interpreted to provide information of the two types of symmetries that are involved.

Maria Radcliffe. *A Node-Weighted Model for the Spread of a Non-Indigenous Species.* Benedictine University. Friday 2:35 p.m.

We present and analyze a model for the spread of a non-indigenous species (NIS) through a network. The model is a modified version of an SI epidemic model on a network in which the transmissibility rate from one node to another depends on the density of the NIS at the node at which the NIS is already present. This work is part of a larger project devoted to the study of the spread of the urban weed Ailanthus altissima (tree of heaven) throughout the United States.

Trisha Russo. *Reproductive Strategies Compared to a General Model of Fecundity.* Benedictine University. Friday 2:55 p.m.

Fecundity can be described as the amount of offspring produced for each individual, specifically female. Some reproductive strategies are described as semelparity and iteroparity. Semelparity is being able to only reproduce once in a lifespan, much like an annual plant. Iteroparous is being able to reproduce more than once during a lifespan, much like a perennial plant. There have been various models created and researched in order to grasp a better understanding on when semelparity or iteroparity is preferred for a certain species. During this talk, I will be discussing previous results concerning these strategies and comparing that to a general model.

Enela Aliaj. *An Edge-Weighted Model for the Spread of a Non-Indigenous Species.* Benedictine University. Friday, 2:55 p.m.

We present and analyze a model for the spread of a non-indigenous species (NIS) through a network. The model is a modified version of an SI epidemic model on a network in which the transmissibility rate from one node to another depends on a weight assigned to the edge connecting a pair of nodes. This work is part of a larger project devoted to the study of the spread of the urban weed Ailanthus altissima (tree of heaven) throughout the United States. In the Ailanthus altissima model, the weights are proportional to the number of roads and railroad lines between distinct locations.

Natalia Poniatowska. *Security of a Message Authentication Scheme Based on Quasigroups.* Benedictine University. Friday, 3:30 p.m.

A *quasigroup* is a set of elements with one binary operation whose multiplication table forms a Latin square. These algebraic structures are similar to groups, however they are not required to be associative. This non-associativity has many applications, one such area is cryptology. A widely studied and used cryptographic tool is Message Authentication Code, MAC. In this talk I discuss a type of a MAC, called QMAC, which was introduced by Meyer.

The construction of a QMAC and its security concerns will be discussed.

Miranda Henderson. *Comparison of Boolean Versus Continuous Dynamics on Three-Gene Regulatory Networks.* Benedictine University. Friday, 3:30 p.m.

We compare the dynamics of continuous models and Boolean models of certain three-gene regulatory networks, in which there are feedback loops. We also discuss the occurrence of Hopf bifurcations in the continuous model to understand the similarities between the continuous and Boolean models and their relationship to the occurrence of feedback loops. We will compare the incidence of Hopf bifurcations in several networks to further analyze the relationship between the continuous and Boolean models for three-gene regulatory networks with feedback loops.

Sabrina Allen. *On a* 2*-fold* G *Designs, Where* G *has Order at most* 4 *and Edge-Multiplicity* 2*.* University of Illinois, Urbana-Champaign. Friday, 3:50 p.m.

For a positive integer k, let ${}^2K_k$ denote the 2-fold complete mutigraph of order k. If G is a bipartite subgraph of ${}^2K_4$, we find necessary and sufficient conditions for the existence of G-decompositions of ${}^2K_n$. We also report on some results when G is tripartite.

Marihan Hegazy. *Comparison of Boolean Versus Continuous Dynamics on Four-Gene Regulatory Networks.* Benedictine University. Friday, 3:50 p.m.

We compare the dynamics of continuous models and Boolean models of certain four-gene regulatory networks in which there are feedback loops. We examine the effects of the presence of two gene feedback loops on the dynamics in both the Boolean and continuous cases. We also analyze the networks for the occurrence of a Hopf Bifurcation.

Peter Wiese. *Modeling Spiking in Neurons with a Poisson Process.* Augustana College. Friday, 4:10 p.m.

In the nervous system, nerve cells communicate through changes in ion concentrations called action potentials, or spikes. These spikes have been recorded and studied to understand the change in their distribution due to the presentation of a stimulus. By using a Poisson process, it is possible to model the distribution of spikes in time. Based on physiological properties, changes in the model are made to account for both the absolute refractory period and bursting of spikes. We will present several different models implemented on a spread sheet, both of a single neuron and of small systems of neurons.

Benjamin Studer. *Parametric Equations for Video Games.* Augustana College. Friday, 4:10 p.m.

Danmaku or "manic shooters" are a genre of video games that feature intricate patterns of bullets that the player must avoid. The motion of the bullets is determined by parametric equations of varying complexity. Using a game engine we will explore how various patterns can be produced through the use of parametric equations using both polar or Cartesian coordinate systems.

Bohdan Khomtchouck. *A Laplacian Derviation of Zipf's Law.* Benedictine University. Saturday 9:45 a.m.

Zipf's Law predicts a linear relationship between word rank and frequency in communicative systems, and is widely reported in texts yet remains enigmatic as to its origins. Computer simulations have shown that communicative systems emerge at an abrupt phase transition in the fidelity of mappings between symbols and objects. Since the phase transition approximates the Heaviside or step function, we show that Zipfian scaling is expected based on the Laplace transform which yields $1/s$.

Gonzalo Landeros. *An Andriod App for a Homophonic Password Generator.* Benedictine University. Saturday 9:45 a.m.

We frequently use passwords in everyday life for many things, from e-mail accounts to banking transactions. The evolution of technology has made crime change as well. We now face digital or technological theft by attackers from all over the world. If we take the security of passwords for granted, we might end up giving our information to anyone, including those who are willing use it in a malicious manner. In this paper, we study the strengths and weaknesses of passwords, and then create our own homophonic cipher that generates passwords which are easy to

remember. These are at least just as secure as a typical 'complex password'. We also work towards implementing this cipher as an android application.

Jessica Klister with Ryan C. Bunge, Saad I. El-Zanati, and Catherine Kruger. *On Rosa-Type Labelings of Directed Graphs.* University of Wisconsin, LaCrosse. Saturday 10:05 a.m.

Let $\mathbf{Z}$ denote the set of integers. For integers a and b with $a \leq b$, let $[a, b] = \{x \in \mathbf{Z}: a \leq x \leq b\}$. Let K_k^* denote the complete directed graph on k vertices. Let G be a directed graph with n arcs on $\leq n+1$ vertices. A *labeling* of G is a one-to-one function $f: V(G) \mapsto [0, n]$. If f is a labeling of G and $e = (u, v) \in E(G)$, let $\bar{f}(e) = f(v) - f(u)$ if $f(v) > f(u)$ and let $\bar{f}(e) = n + 1 + f(v) - f(u)$ if $f(v) < f(u)$. A labeling f of G is a *directed ρ-labeling* of G if $\{\bar{f}(e): e \in E(G)\} = [1, n]$. It can be shown that for such a G, there exists a cyclic G-decomposition of K_{n+1}^* if and only if G admits a directed ρ-labeling. If G is bipartite, we define a labeling of G that leads to cyclic G-decompositions of K_{nx+1}^* for every positive integer x. We investigate these and other labelings of some classes of directed graphs and give the corresponding decomposition results.

Kiran Munir. *Stream Cipher and Watermarking Methods in the Medical Field.* Benedictine University. Saturday 10:05 a.m.

With the advent of complex technology and advancements in medical data transfer and storage, the need for security and privacy of patients by hospitals and physicians is essential. The transfer of important patient information, in the form of encrypted data, is accomplished by cryptographic methods and by digital watermarking, a method used to embed data onto an image. We discuss stream ciphers used in medical imaging and the advantage of using such ciphers. We further present a method that uses a stream cipher for encrypting patient information and follows it by a Discrete Cosine Transform based watermarking that embeds the encryption on the image.

Sreevidya Bodepudi. *The Dynamics of an SIR Model with Constant and Pulse Vaccination with Horizontal and Vertical Transmission.* Benedictine University. Saturday 10:25 a.m.

We discuss an SIR epidemic model for a disease that is transmitted vertically and horizontally. We compare the dynamics of this model under both a constant vaccination strategy and a pulse vaccination strategy. Pulse vaccination strategies have been successful in controlling and eliminating viral infectious diseases such as measles. We also discuss the global stability of an infection-free periodic solution.

Joseph Buchanan with Ryan C. Bunge, Erik Pelttari, Greg Rasmuson, Alexander Su, Sevasti Tagaris. *On d-Moduluar Labelings of the Union of Two-Cycles.* Illinois State University. Saturday 10:25 a.m.

For positive integers r and s, let $K_{r\times s}$ denote the complete multipartite graph with r parts of size s each. Let G be a graph with n edges, d be a positive integer such that $d|2n$ and set $c = 2n/d+1$. A *d-modular ρ-labeling* of G is a one-to-one function f: $V(G) \to [0, cd)$ such that $\{\min(|f(u)-f(v)|, cd-|f(u)-f(v)|)$: $\{u, v\} \in E(G)\} = \{1, 2, \ldots, \lfloor \frac{cd}{2} \rfloor\} \setminus c\mathbf{Z}$. It is known that if a z-partite graph G admits a d-modular ρ-labeling, then there exists a cyclic G-decomposition of $K_{c\times td}$ for every positive integer t such that $\gcd(t, (z-1)!) = 1$. We investigate d-modular labelings of the union of two vertex-disjoint cycles.

Edward Price. *Building Group Extensions Storing Only 2-Cocycles for Relators.* Benedictine University. Saturday 11:00 a.m.

In this talk, we will discuss the process of doing arithmetic in a group extension. A group extension is a group which is built from two smaller groups, namely a normal subgroup and a quotient group. In addition to these two groups we also need a map, called the 2-cocycles, on each pair of elements in the quotient group. This map describes how to lift products in the quotient group to products in the group extension. In this talk we will describe how to do arithmetic in a group extension assuming the 2-cocycles are known. We will then show that in fact we only need to know the 2-cocycles for each relator in a finite presentation for the quotient group for Q to be able to do arithmetic in a group extension.

Ellen Sparks with Ryan C.Bunge, Megan Cornett, Saad I. El-Zanati. *On 2-fold Graceful Labelings.* Illinois State University. Saturday, 11:00 a,m.

Let $\mathbf{Z}$ denote the set of integers and $\mathbf{N}$ denote the set of nonnegative integers. For integers a and b with $a \leq b$, let $[a,b] = \{x \in \mathbf{Z} : a \leq x \leq b\}$. For a positive integer k, let ${}^2K_k$ denote the 2-fold complete mutigraph of order k. Similarly, let ${}^2[a,b]$ denote the multiset that contains every element of $[a,b]$ exactly two times. Let G be a multigraph of size n, order at most $n+1$, and edge multiplicity at most 2. A *labeling* of G is a one-to-one function $f\colon V(G) \to \mathbf{N}$. If f is a labeling of G and $e = \{u,v\} \in E(G)$, let $\bar{f}(e) = |f(u) - f(v)|$. A 2-*fold graceful labeling* of G is a one-to-one function $f : V(G) \to [0,n]$ such that:

$$\{\bar{f}(e)\colon e \in E(G)\} = \begin{cases} {}^2[1,\frac{n}{2}] & \text{if } n \text{ is even,} \\ {}^2[1,\frac{n-1}{2}] \cup \{\frac{n+1}{2}\}, & \text{if } n \text{ is odd.} \end{cases}$$

A graph G is said to be 2-*fold graceful* if it admits a 2-*fold graceful labeling*. It can be shown that if G with n edges is 2-fold graceful, then there exists a cyclic G-decomposition of ${}^2K_{n+1}$. El-Zanati has conjectured that every tree is 2-fold graceful. We investigate 2-fold graceful labelings of various classes of graphs including several classes of trees.

Andreana Holowatyj. *Hextile Planar Isotopty and Reidemeister Moves.* Benedictine University. Saturday 11:20 a.m.

A hexagonal knot mosaic is a knot diagram that lies in a hexagonal grid in a particular way. We enumerate basic planar isotopy moves in n-hextile regions and explore methods to catalog the infinitely many basic isotopy moves on a hexagonal grid. We then show that for any Reidemeister move of a knot projected on a hexagonal grid can be decomposed into a finite sequence of irreducible hextile moves.

Seerat Hassan. *The Dynamics of Continuous and Impulsive Vaccination in SIR-SVS Epidemic Models.* Benedictine University. Saturday, 11:20 a.m.

Vaccinations play an important role in the management of some infectious diseases. However, the different methods of vaccination yield varying efficacy. We discuss the efficacy of two possible strategies: a continuous vaccination strategy (CVS) and a pulse vaccination strategy (PVS). Both strategies assume continuous vaccination for newborns. CVS utilizes continuous vaccination of a portion of individuals while PVS vaccinates a constant number of individuals over each time interval. We present the basic reproduction numbers of both models and discuss the global stability of equilibria of both models.

Alexander Sistko. Tau-N Factorizations in the Integers. Bradley University. Saturday, 11:40 a.m.

For integers a, b, and n with $n > 0$, write $a\tau_n b$ whenever $a \equiv b (\mathrm{mod}\ n)$. Then τ_n is a relation on the integers, and we call $a = (\pm 1)b_1 b_2 \cdots b_k$ a τ_n-factorization of a if $b_i \tau_n b_j$ for all i, $j \leq k$. Imitating the usual factorization of integers into primes, we discuss τ_n-atoms and τ_n-atomic domains and answer the following question: for which $n \in \mathbf{N}$ is $\mathbf{Z}$ a τ_n-atomic domain?

Katherine Hunzinger. The Dynamics of an Impulsive Two-Prey One-Predator System. Benedictine University. Saturday, 11:40 a.m.

We consider a two-prey one-predator system for which the predator species is augmented impulsively at periodic intervals. Such a model can be used to control a pest species with a predator. We discuss conditions for which there exists a globally asymptotically stable, periodic pest eradiation solution and for which there exists a permanent solution. We also present numerical simulations of the model. By using Floquet's Theorem and small amplitude perturbation we can see that there exists a globally asymptotically stable two-pest eradication periodic solution when the impulsive period is less than some critical value. It can also be proven that the system is permanent if the impulsive period is larger than some critical value while the conditions for extinction of 1/2 of the prey and permanence of the remaining two species are given. Also, there exists a stable positive periodic solution with a maximum value no larger than a given level, hence the stability of the positive periodic solution and its period can be used to control insect pests at acceptably low levels. This essentially discusses the effect of biological control by using a particular pest/organism/predator to control the populations of two different preys.