Impact of Mathematics Research on Beliefs about Mathematics, Teaching, and Learning

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The goal of the Teacher Scholar and REU programs, and teacher education in general, is to produce high-quality teachers that are prepared for the reality of the teaching profession. As the teaching profession continues to deal with issues of recruiting and retaining quality mathematics teachers in the country, we must expand our efforts to explore and investigate innovative ways to develop future teachers. In this section, we discuss the potential effect that authentic mathematical experiences have on mathematics teacher development and provide some initial data to support this argument. In particular, we discuss changes in teachers' mathematical beliefs and changes in beliefs about teaching and learning.

It has long been an adage that to be a good mathematics teacher you must have a deep and connected understanding of school mathematics. However, we claim that it is more than just a deep knowledge of mathematics, but also what a teacher believes about the nature of mathematics. For instance, what is the consequence of a teacher who believes that mathematics in mainly symbols and calculations versus a teacher who believes that mathematics is primarily reasoning? This comparison has three interesting implications. First, the mathematics that is taught by these two teachers is going to be different. If a teacher believes that mathematics is symbols and calculations, they are likely to focus on symbols and calculations. If a teacher believes that mathematics is reasoning, they are more likely to focus on reasoning. Second, the mathematical beliefs a teacher possesses are likely to impact their mathematical knowledge. For example, if a teacher believes that mathematics is primarily reasoning, they are more likely to apply that to their own mathematical learning and hence are more likely to have a deep and connected understanding of mathematics. Finally, the mathematical beliefs that a teacher holds are likely to affect instructional decisions. A teacher who believes mathematics is about making conjectures and providing justification is going to provide opportunities for students to experience these processes. A teacher who believes mathematics is mainly rules will place less emphasis on these instructional techniques. What a teacher believes about the nature of mathematics not only affects the mathematics discussed in class, but also the mathematics learned and understood by the teacher and the instructional techniques used by the teacher. Hence, what a teacher believes about the nature of mathematics is a critical component of becoming a quality mathematics teacher.

Supporting Data

To investigate the influence that authentic mathematical experiences have on the beliefs of teachers, surveys were implemented before and after the REU Site experience. Ultimately, two distinct surveys were developed: one that focused on the nature of mathematics and another that focused on the teaching and learning of mathematics. In the

later survey, belief questions were clustered into several categories (e.g. problem solving, mathematical authority, collaboration) with subquestions to discern any differences between their personal beliefs about learning, their beliefs about how students learn mathematics, and their beliefs about the teaching of mathematics.

Data for this study consisted of pre- and post-Likert belief measures. The Likert scale ranged from 1 to 5 with 1 representing strongly agree and 5 strongly disagree. The data were analyzed to uncover an overall characterization of these students, change over the duration of the program, subtleties among students, and discrepancies between their personal and teaching beliefs. The following excerpts are example items taken from the mathematics survey with the average initial score, average final score, and overall change.

Q: I can invent my own mathematics to describe patterns that I see.

$2.63 \rightarrow 1.72 \quad (-0.91)$

Q: Mathematics involves a great deal of creativity.

$2.45 \rightarrow 1.82 \quad (-0.63)$

Q: Mathematics is mainly rules.

$3.91 \rightarrow 4.55 \quad (+0.64)$

The above examples illustrate how the REU students began viewing mathematics as a more creative process based on reasoning and justification and moved away from viewing mathematics as a procedural computation.

Our next goal was to see if we could uncover subsequent changes in their beliefs about teaching and learning. The following excerpts are example survey items taken from the teaching and learning survey with the average initial score, average final score, and overall change. These examples illustrate a change in their beliefs about their role as teachers including placing more authority and responsibility on the shoulders of students and increasing the importance of developing a deep and connected student knowledge.

Q: The teacher should provide verification for mathematical arguments given in class rather than expecting students to do so.

$3.00 \rightarrow 3.73 \quad (+0.73)$

Q: Teachers should teach mathematics as an integrated whole so that students will become aware of the connections among various mathematical ideas.

$2.00 \rightarrow 1.55 \quad (-0.45)$

Data analysis of the survey results uncovered several changes in the REU participants' beliefs over the course of the summer program. The table below provides a summary of the changes in their teaching and learning beliefs. The categories along the left indicate the various subcomponents of the teaching and learning survey. The headings along the top row (personal, learning, teaching) reflect the perspective of the question being asked. Hence, the participants were asked questions within each category concerning themselves, how students learn, and how they would teach. The scores were adjusted so that a lower score represented beliefs consistent with current *NCTM*

recommendations. The scores from the initial and final surveys were then averaged to identify the amount and direction of change for each category. In the table below, each category contains a label N, SB, or NSB indicating there was no change (N), a change towards standards-based beliefs (SB), or away from a standards-based belief system (NSB).

	Personal	Learning	Teaching	Total
Procedural vs. Understanding	**NSB**	SB	----	SB
Discovery	N	**NSB**	SB	SB
Problem Solving	SB	SB	SB	SB
Reasoning & Proof	SB	**NSB**	SB	SB
Connections	SB	**NSB**	SB	SB
Perseverance	SB	SB	SB	SB
Communication	SB	**NSB**	SB	**NSB**
Confidence	SB	SB	SB	SB

The participants displayed changes in many categories. The overall average for the three perspectives changed towards standards-based beliefs with an overall average initial score of 2.36 and a final average score of 1.94. In addition, the perspective of personal beliefs and teaching beliefs shifted towards beliefs that aligned more closely with current recommendations for most subcategories. However, some of the subcategories displayed more change than others. For instance, problem solving had the greatest overall change with an average initial score of 2.77 and a final average score of 2.11. Another category that displayed a large shift was perseverance. The REU program appears to have had a positive effect on the beliefs of the participants in these areas and within the different subcategories.

However, the participants' beliefs from the perspective of how students learn mathematics shifted in different directions depending on the subcategory. While beliefs related to problem solving, perseverance, confidence, and procedural versus conceptual understanding became more consistent with current standards, beliefs related to discovery, reasoning and proof, making connections, and communication of mathematics shifted away from current recommendations. It must be noted that the average score for all of the categories was less than 3, which can be interpreted to mean that their beliefs were still aligned with standards-based instruction, but simply not as strong as they were prior to the REU experience. This raises questions about why the beliefs changed and the direction of the changes. Could the experience with mathematics and teaching cause future teachers to have a more cautious view of some of the current recommendations, but encourage others as it relates to student learning? More data analysis needs to be

done to identify the different factors and reasons for the changes in beliefs about student learning during the REU program.

Overall the analysis displays the positive effective of the REU program on participants' beliefs about mathematics, teaching, and learning. In terms of mathematics, students began viewing mathematics as a more creative process based on reasoning and justification and moved away from viewing mathematics as procedural computation. In addition, changes were found in their beliefs about teaching. The participants developed and strengthened beliefs that were consistent with current teaching recommendations, especially in the areas of problem solving, learning for understanding, and perseverance. Although the data reveal an overall positive influence, there are areas that showed a shift away from current recommendations. In addition, there is no direct evidence that suggests that a change in beliefs about mathematics led to a change in beliefs about teaching and learning. Consequently, further research needs to be done to document the changes in teacher beliefs that result from authentic mathematical experiences and to uncover the connections between the various beliefs in question.

Closing Remarks

The Teacher-Scholar and REU Programs are built on a deliberate effort to increase the involvement of mathematicians in the preparation of teachers. A major purpose of this collaboration has been to change teachers' beliefs about mathematics and subsequently their beliefs about teaching and learning. By providing future teachers the opportunity to explore and do authentic mathematics, they are able to see mathematics in a new light. They have the opportunity to investigate, discover, make conjectures, generalize, communicate, test ideas, reflect, and justify. Through the experience of doing mathematics, we hope to alter or reinforce pre-service teachers' mathematical beliefs that are conducive to quality teaching.