An Introduction to Discrete Mathematics in the Classroom: Latin Squares

Students' Guide

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Preface for the Student:

Hello and welcome to the wonderful world of Latin Squares! In this module, you will have the opportunity to pursue ideas in discrete mathematics that you otherwise might not come across until college! As you will discover, this module is unlike any ordinary textbook; instead, this module contains numerous investigations and problems that help you make connections and discoveries on your own. In this module, your instructor will act as a guide, leading you to develop important ideas and concepts. It is your job as the student to hypothesize, investigate, and discover these rich math concepts on your own. Good luck and have fun!

Introduction Sudoku Puzzle:

Please complete the following Sudoku Puzzle. If you are unfamiliar with the rules, in a Sudoku, each row, column, and small box must contain all the digits from 1-9 exactly once.

6			3			1	8	
5						7	6	
		1		8				9
	4		1		7	6	9	
	7	6		9		8	4	
	3	5	8		4		1	
7				3		5		
	6	2						8
	5	9			2			1

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Introduction

In this module you will learn about Latin Squares and about applications of this area of mathematics. You will quickly find out that Latin Squares are used throughout the world in the form of Sudokus, scheduling, and round robin tournaments. Furthermore Latin Squares are an important area of mathematics (where they are used in graph theory and abstract algebra) and can be enjoyed by all. In this module, you should begin to keep a journal that includes all ideas, descriptions, solutions to problems, attempts at solutions, and questions about the Latin Squares you encounter.

The Culminating Problem: Placing Students in Groups

There are 16 students in a class. The teacher wants them to work on projects throughout the year in groups of 4. How many projects would it take so that each student was in the same group as each other student in the class exactly once? Create a plan to assign students to groups and indicate the groupings for each project.

Section 1: Introduction to Latin Squares

1.1 Introduction Problem

Three students from your school, Jeff, Kyle, and Desiree, are competing in a round-robin badminton tournament with three students, Seth, Ryan, and Katie, from the rival school. That means each student from your school plays exactly one match with each student from the rival school. Set up a schedule so there are three matches at one time, until all matches are completed.

1.2 Preview

In your journal describe the patterns you see in the following arrays. What patterns do you see in the rows and columns of each array? What patterns do you see in the diagonals of each array? Are there any patterns that are consistent in all of the following arrays or in a subset of them?

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a.

1	2	3
2	3	1
3	1	2

b.

1	3	2
3	2	1
2	1	3

c.

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

d.

a	b	c
c	a	b
b	c	a

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e.

1	4	2	5	3
4	2	5	3	1
2	5	3	1	4
5	3	1	4	2
3	1	4	2	5

1.3 Discussion

a) Discuss the following terms: diagonal, order, even/odd order, and symmetric. What might these terms mean with respect to n x n arrays?

b) One term that has a variety of definitions is idempotent. For the purposes of this module idempotent is defined in a specific manner. Your instructor will show some examples of idempotent Latin Squares. As your instructor does this, try to come up with a definition of idempotent Latin Squares as a class.

1.4 Exercise

Below are some important terms for our module. Please search for definitions and examples of these terms using the internet or, if you do not have access to the internet, your teacher will provide some results from such a search so that you may form your definitions from a combination of sources. Note that these terms may have other uses outside of this module. You

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may wish to use the phrase "graph theory" to find an appropriate definition for our work. In your journal, record in your own words what you find and reflect on how this information relates to what we have been doing in class:

-Latin Square

-diagonal

-symmetric

-order

-even/odd order

1.5 Exercise

Now that you have found the definitions of the basic terms listed above, describe the following arrays using the terms that you just defined. Write down all the terms that apply to each array in your journal and explain why each term applies. Also record any other interesting observations about the arrays.

a.

1	2	3
2	3	1
3	1	2

b.

1	3	2
3	2	1
2	1	3

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c.

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

d.

1	2	3	4
2	3	4	1
4	1	2	3
2	4	1	2

e.

1	4	2	5	3
4	2	5	3	1
2	5	3	1	4
5	3	1	4	2
3	1	4	2	5

f.

4	5	7	9	12
5	7	9	12	4
7	9	12	4	5
9	12	4	5	7
12	4	5	7	9

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g.

a	c	b
c	b	a
b	a	c

h.

3	2	1
1	3	2
2	1	3

i.

1	3	2	4
3	2	4	1
2	4	1	3
4	1	3	2

j.

1	4	2	3
3	2	4	1
2	3	1	4
4	1	3	2

Through discussion, share and modify your descriptions of the above arrays with other classmates and your instructor.

Section 2: The "Clicking" and "Diagonal" Methods

2.1 Problem

Alvin wants to set up Christmas lights on his roof the same as last year, but he has forgotten which string of lights goes where. He wants one string of lights in each marked region, and

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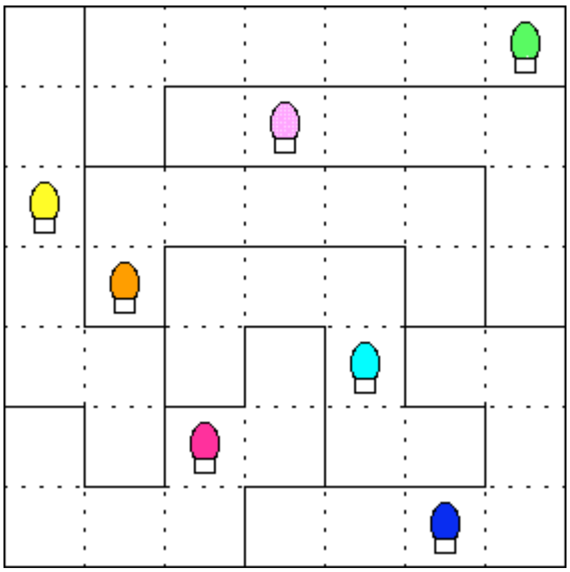
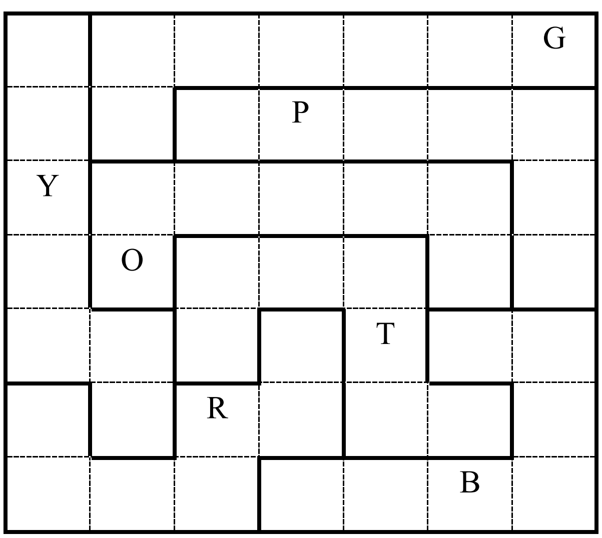
wants each row and column to have exactly one light of each color. Each string of lights consists of seven bulbs, one of each color. Note that the order of the colors on each string may vary.

Alvin remembers where one of each color should go, but you must help him place each string so that his rooftop has one light of each color in every row and column.



If you do not have a color copy of this module, the coloring is listed below: G for green, P for pink, Y for yellow, O for orange, T for teal, R for red and B for blue.



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2.2 Critical Thinking

Now, develop a strategy to create your own Latin Squares. Attempt to create different strategies for finding idempotent and/or symmetric Latin Squares. In your journal, write a full explanation of how someone might use your strategy. Write it so that a classmate or teacher could easily understand.

2.3 Problem

Create Latin Squares of order 2, 3, 4, 5, 6, and 7. Record these Latin Squares and the strategy you used to create them in your journal.

2.4 Problem

Now, if possible, create Latin Squares of orders 2, 3, 4, 5, 6, and 7 that are both idempotent and symmetric. Record your results and conjectures that you form in the process in your journal.

2.5 Problem

Jeff, Kyle, Seth, and Ryan are going to go on dates with Katie, Andrea, Keri, and Desiree. Each individual will go on four dates on four consecutive nights– one date each on Wednesday, Thursday, Friday, and Saturday. Additionally, all eight people will be on their dates at the same time. Using Latin Squares, show how the dates should be scheduled so that each boy goes on exactly one date with each girl?

Section 3: Half-Idempotent Latin Squares

3.1 Discussion and Review

With your instructor and classmates, discuss your results and strategies for finding Latin Squares along with how you found idempotent and symmetric Latin Squares of orders 2-7.

Find a strategy that can be extended to all Latin Squares of a certain order (odd or even).

If not already discussed, share ideas on how to solve the dating and round-robin problems from Sections 1 and 2.

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3.2 Exercise

In your journal, fill in the blanks to make the following Latin Square symmetric.

1			
	2		
		1	
			2

3.3 Discussion

The array above is an example of a half-idempotent Latin Square. How is this different than an idempotent Latin Square? What pattern do you see that might lead to the name "half-idempotent?" What does it mean to have a half-idempotent Latin Square? Discuss your thoughts. Also, what difficulties did you have in completing Problem 2.5?

3.4 Problems

In your journal, create symmetric, half-idempotent Latin Squares of orders 6 and 8. While doing so, attempt to find a pattern that you can use to create all symmetric, half-idempotent Latin Squares of even order.

If a pattern is found, try using it to create a symmetric, half-idempotent Latin Square of order 10.

Write what you have learned in your journal and be prepared to discuss your results in class.

Section 4: Mutually Orthogonal Latin Squares

4.1 Discussion and Summary

Discuss the strategies that have been discovered in order to create symmetric half idempotent Latin Squares. Review the strategies for finding symmetric, non-symmetric, idempotent, and

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half-idempotent Latin Squares. Become familiar with various strategies that can be employed to create such Latin Squares.

4.2 Activity

Use the playing cards provided and a 3x3 grid. Arrange the cards on the grid in such a way that each row and column has no two cards of the same rank (e.g., not both kings) **and** no two cards of the same suit. Attempt to find a solution to this using Latin Squares.

4.3 Exercise/Discussion

Discuss strategies for finding a solution to the Playing Card Activity.

Now, find the definition for the following term and record it in your journal.

-Mutually Orthogonal Latin Squares

Discuss how mutually orthogonal Latin Squares can be used to solve the Playing Card Activity. Also discuss strategies that can be used to develop mutually orthogonal Latin Squares.

4.4 Problem/Discussion

Nine golfers are going to meet the next four Saturdays to play rounds of 18 holes. They will always golf in groups of three people each. How should the nine golfers arrange themselves so that each golfer plays with every other golfer?

Discuss any ideas on ways to generate a solution to the golfer problem.

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4.5 Problem

Sixteen golfers are going to meet the next five Saturdays to play rounds of 18 holes. They will always golf in groups of four people each. How should the sixteen golfers arrange themselves so that each golfer plays with every other golfer?

4.6 Challenge Problem

Leonhard Euler, 1707-1783

Attempt to find two mutually orthogonal Latin Squares of order 6. Using the internet, you may want to research the findings of L. Euler on this topic.

In your journal, either write down the two mutually orthogonal Latin Squares of order 6 or provide an explanation of why two such Latin Squares do not exist.

Section 5: Affine Planes and Applications

5.1 Exercise

Find the definitions for the following terms and record them in your journal.

-Affine Planes

-Parallel Classes of Affine Planes

-Euclid's Parallel Postulate

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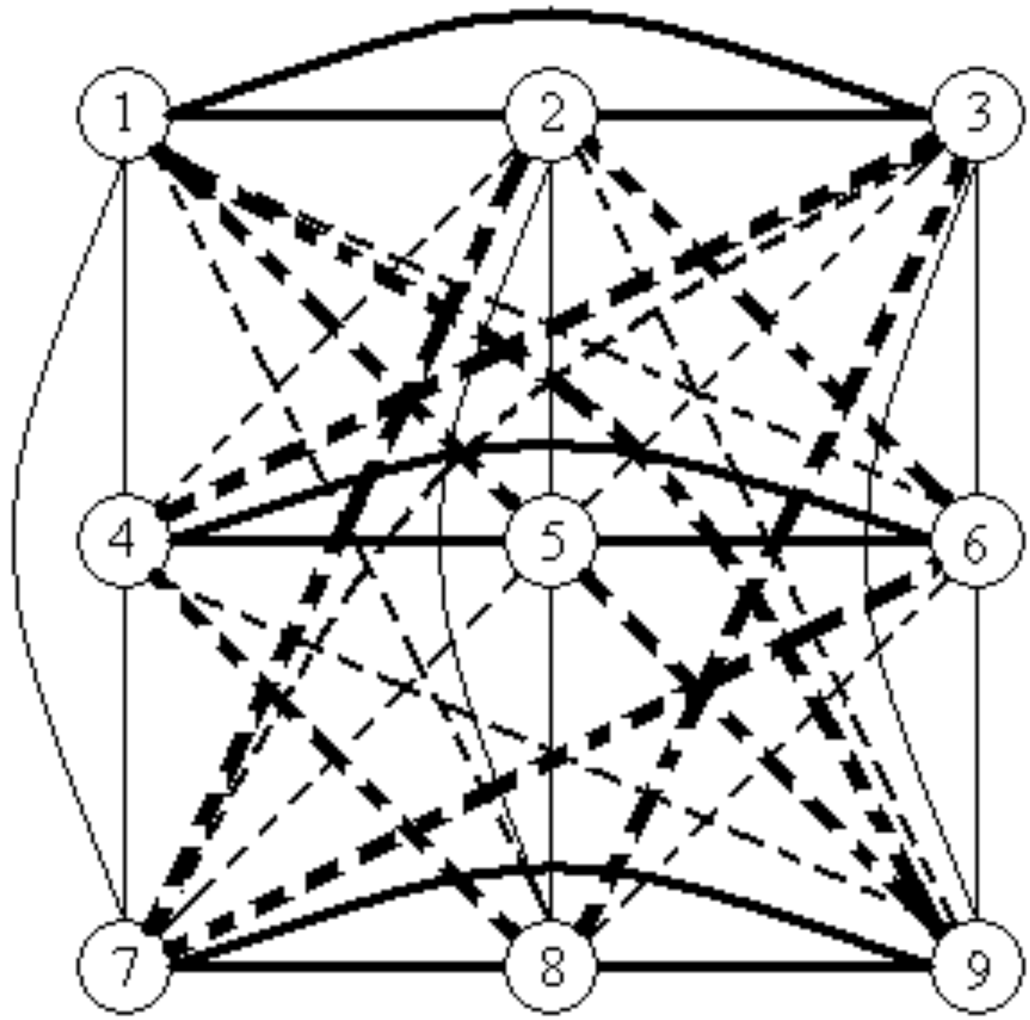

The diagram above is a drawing of an Affine Plane with 9 points labeled 1 to 9. Know that each encircled number is a point in the Affine Plane, and each closed figure (for example, the figure connecting the points 1, 4, and 7, or the triangle connecting the points 2, 6, and 7) represents one of the 12 "lines" in this Affine Plane. Also see that each parallel class is represented by a different set of lines (thin, thick, thin-dotted, and thick-dotted). If desired, confirm that all the properties of an Affine Plane are indeed satisfied by this diagram.

5.2 Discussion

Discuss how the golfer problem from Section 1.4 relates to Affine Planes. Furthermore, discuss how parallel classes of Affine Planes also relate to the golfer problem of Section 1.4 and to Euclid's Parallel Postulate.

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5.3 Possible Project Topics

1. Alphabet Latin Squares

Create three different Latin Squares of order 26, using each letter of the English alphabet. Describe the strategies used to generate each Latin Square.

2. Scheduling a Round-Robin Tournament

Use Latin Square(s) to design a schedule for 7 tennis players to play a round robin tournament in which every player plays one match with every other player in 6 rounds, 3 simultaneous matches per round (Note that one player will have a by in each round).

3. Field Problem

In what ways do Latin Squares apply to finite fields? How do the addition tables of finite fields relate to Latin Squares? Provide examples and explain your reasoning. To complete this project, you must understand finite fields. A finite field is a finite set of elements with four operations: addition, subtraction, multiplication, and division with the exception of division by zero. In a field, the result of any combination of operations on members of the field results in a member of the field.

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