Stanton Graph Families
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Warm Up

Define "path" in your own words. Draw some paths below.

Define "star" in your own words. Draw some stars below.

Please note the following defintions/notations:

C_n: a cycle with n verticies.

S_n: a star with $n+1$ verticies.

P_n: a path with $n+1$ verticies.

Find a cyclic decomposition for K_9 using a path with four edges and then a star with four edges.

How many different edge lengths are present in K_{19}? What complete graph has 31 different edge lengths?

Lets talk λ: Decomposing λ-fold multigraphs

Let's review first.

Definition: The **edge multiplicity** is the number of edges between two verticies.

In your own words, what does it mean for a K_n vertices to be decomposed into a graph H cyclicly? What requirements must H satisfy?

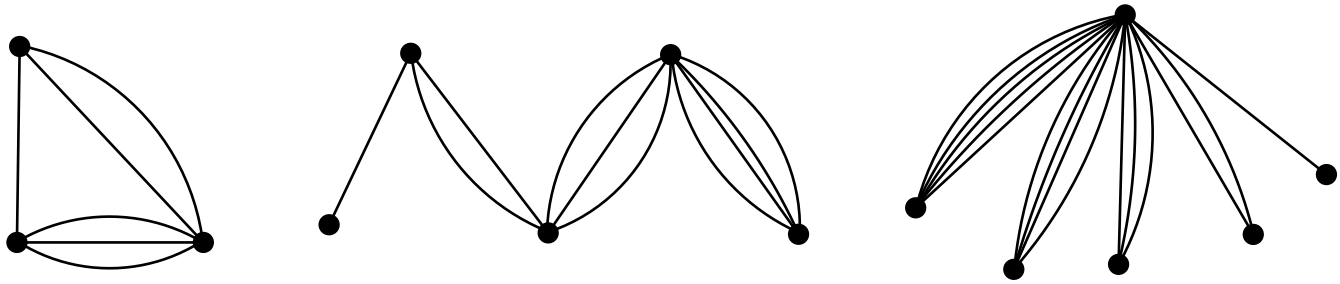
Based on this, if we were to increase the edge multiplicity of H, how would the restrictions on G change? Explore this a bit on a separate paper. One way to do this would be to draw a ${}^{\lambda}K_n$ and create a path, star, or cycle, which can cyclically decompose the complete graph. First try ${}^{2}K_5$, and once you feel that you understand how to cyclicly decompose ${}^{2}K_5$, try ${}^{3}K_5$, ${}^{3}K_7$, or a choose a higher value for λ or a different K_n and try it out!

At this point, you have probably come to the conclusion that to decompose ${}^{\lambda}K_n$ into a graph H, H must have a maximum edge multiplicity of λ. Hence, we have added a new restriction for decomposition when using graphs that have multiple-edges. Based on your findings, what would you expect the smallest value for λ to be, such that a graph G, with maximum edge multiplicty x, so that G decomposes ${}^{\lambda}K_n$?

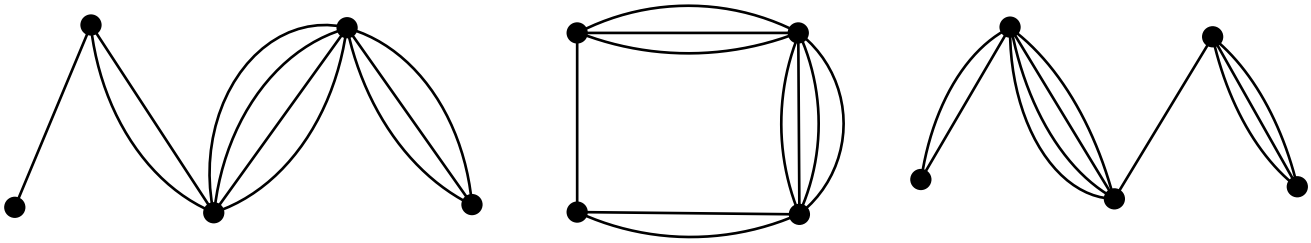
Stanton Graphs

Now we are going to explore some families of graphs. Look for patterns and compare the Stanton and non-Stanton graphs.

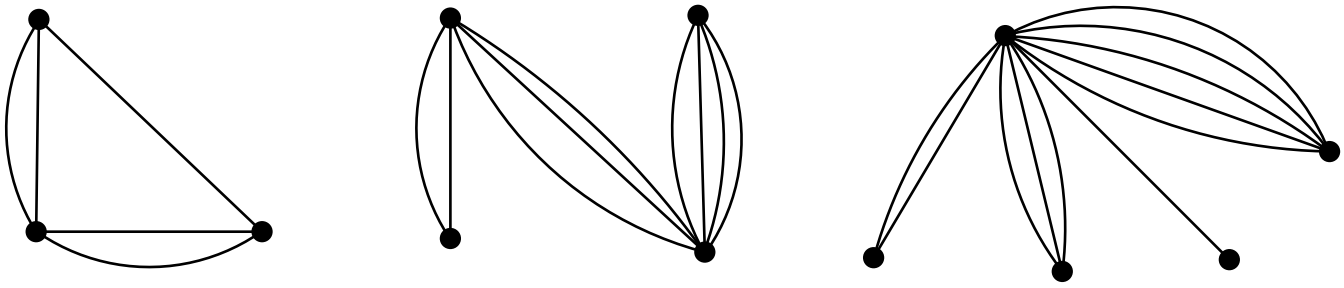
Standard Stanton Graphs



Non-Standard Stanton Graphs



Non-Stanton Graphs



Based on the example graphs above, define what a Stanton graph is in your own words.

Based on the example graphs above, define what a standard Stanton graph is in your own words.

Please note, from now on, when we refer to Stanton graphs, we are referring to standard Stanton graphs.

Draw a Stanton 4-path (SP_4), a Stanton 5-path(SP_5), and a Stanton 6-path (SP_6).

How many edges does each graph have?

SP_4:

SP_5:

SP_6:

How many edges would a SP_n have? (That is, a Stanton path on $n+1$ vertices). A SS_n? A SC_n?

Let's try to decompose some complete graphs using Stanton Graphs!

Try decomposing a K_5 using a Stanton-2 path. Can you do it? If not, why?

How could we change K_5 so that it could possibly be decomposed using a Stanton-2 path?

Investigation

We want to decompose λ-fold complete graphs into Stanton stars or paths.

Draw the Stanton 2-star and Stanton 2-path below. What do you notice about these two graphs?

What is the smallest λ you will need to be able to decompose a complete graph into copies of the Stanton 2-star?

If a graph G is going to decompose ${}^{\lambda}K_n$ cyclically, how many of each edge length will G need to have? (Hint: there are 2 cases, so you should have two answers.)

Can you get that many of each edge length with a Stanton 2-star or 2-path? (Hint: How many total edges are in this Stanton graph?) Why or why not?

If possible, find a decomposition of a λ-fold complete graph into Stanton 2-stars.

Can you decompose a higher λ complete graph using the Stanton 2-star? For example, could $\lambda = 3$? 4? What values of λ work?

We will continue the investigation with Stanton stars. Draw the Stanton 3-star (SS_3) below.

Decide (i) which λ you should be working with to decompose into copies of SS_3 cyclically and (ii) how many of each edge length you need. Is this possible with this Stanton graph?

If it is possible, decide what order of complete multigraph you will be decomposing. How many different edge lengths do you have?

Try to cyclically decompose your complete multigraph into this Stanton 3-star.

Continue investigating Stanton stars on a separate sheet of paper. See if you can find a pattern for decomposing complete graphs into Stanton stars based on their order. Is there a pattern in the order of complete multigraph you are using? Is there a pattern in the labeling you are using? Investigate whick ${}^{\lambda}K_n$ can be decomposing by SS_4, SS_5, SS_6... If you can find any patterns, give justifications for why your patterns work.

Next, you will investigate decomposing λ-fold complete graphs into Stanton *paths*. Draw the Stanton 3-path below.

Decide what value for λ you should be working with, how many different edge lengths you would need, and how many different edge lengths you have. Decide what ${}^{\lambda}K_n$ you are working in, and find a cyclic decomposition.

Try decomposing using the Stanton 4-path and greater.

Note any patterns you find. Could you find a decomposition for any Stanton n-path? Describe what you would do to create this decomposition, and give a justification for why this pattern holds.